Online Appendix

Spending the Anchor: Modeling Monetary Policy Response to the Covid Inflation

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This appendix documents the compact supply transition, the delivery-consistent communication problem and its calibration, the numerical algorithm, additional fragility exercises, the data construction, and a reproduction guide.

A Supply Block and Mapping to the Parent Model

The supply transition is inherited from DT26b. Producer types $j = 1, \ldots, N_J$ have gross-output weights ω_j, shortfalls q_{jt}, and rebuilding h_{jt}. The 2020 destruction is $q_{j0} = \Delta e_j$. A warm-pool share ψ returns without retraining,

$$q_{j1} = (1 - \psi) q_{j0} - h_{j1}, \tag{1}$$

$$q_{jt} = q_{j,t-1} - h_{jt}, \qquad t \geq 2. \tag{2}$$

Rebuilding minimizes

$$\sum_t \beta^t \left[\frac{\Gamma}{3} h_{jt}^3 + L(q_{jt}, x_t) \right], \qquad L_q(q, x) = -\xi(q + bx),$$

which gives the complementarity condition

$$F_{jt} = \Gamma h_{jt}^2 - \beta \Gamma h_{j,t+1}^2 - \xi(q_{jt} + b x_t) \geq 0, \tag{3}$$

$$h_{jt} \geq 0, \qquad h_{jt} F_{jt} = 0. \tag{4}$$

The aggregate supply states are

$$Q_t = \sum_j \omega_j q_{jt}, \qquad \mathcal{W}_t = \sum_j \omega_j \Gamma h_{jt}^2.$$

The application stores the calibrated transition in `data/input/supply_reference_path.csv`. The delivery-consistent communication solver treats this nonlinear path as conditioning information rather than replicating a full mode-specific delivery tree for each producer type.

B Delivery-Consistent Communication Problem

B.1 Information and the private persistence signal

The current supply pressure u_t is public. The bank observes

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v, \qquad \varepsilon_t^v \sim \mathcal{N}(0, \sigma_v^2),$$

and maps it into the conditional persistence probability

$$\rho_t^u = \Lambda(\alpha_v + \gamma_v v_t).$$

Here $\Lambda(z) \equiv (1 + e^{-z})^{-1}$. The signal-implied forecast is generated recursively by

$$\widehat{u}_{t+1|t}^{CB} = \rho_t^u \widehat{u}_{t|t-1}^{CB}.$$

The application treats this forecast as disclosed with the announcement, as in a published central-bank projection. Hence the informational (Delphic) part of the forecast is observed, and the scalar guidance wedge remains the one-dimensional institutional message. In the baseline, the residual supply innovation is realized only after the conditional center has been set; the fragility exercise instead lets the public anticipate part of the subsequent persistence before implementation. The baseline begins with

$v_0 = -0.28$, which implies $\rho_0^u = 0.344$. The realized ratio $u_{2022}/u_{2021} = 0.883$ is an ex-post innovation.

B.2 Message posterior and the delivery technology

The inherited default center is $\bar{a}_t = \phi_{\mathrm{IT}} \widehat{u}^{CB}_{t+1|t}$, with ϕ_{IT} the targeting rule's loading on the disclosed forecast, and the guidance wedge is $g_t = a_t - \bar{a}_t$. The message posterior is

$$q_t = \frac{\lambda_t f_R(g_t)}{(1-\lambda_t) f_G(g_t) + \lambda_t f_R(g_t)},$$

with centered Gaussian densities for the ordinary mode G and the override mode R, $\sigma_R > \sigma_G$, and λ_t the prior probability of the override mode.

The announcement a_t applies at $t+1$. Given next-period expectation e_{t+1} and supply pressure u_{t+1}, enforcing the letter requires

$$x^H_{t+1} = \frac{a_t - \beta e_{t+1} - u_{t+1}}{\kappa}.$$

The static discretionary choice, given the capacity-related target activity $\bar{x}_{t+1}$ (with κ the Phillips-curve slope and ϑ the weight on activity), solves

$$\min_x \left\{ \frac{1}{2}(\beta e_{t+1} + \kappa x + u_{t+1})^2 + \frac{\vartheta}{2}(x - \bar{x}_{t+1})^2 \right\},$$

so

$$x^d_{t+1} = \frac{\vartheta \bar{x}_{t+1} - \kappa(\beta e_{t+1} + u_{t+1})}{\kappa^2 + \vartheta}, \tag{5}$$

$$d_{t+1} = \beta e_{t+1} + \kappa x^d_{t+1} + u_{t+1}. \tag{6}$$

Delivery of the due promise is disciplined by the continuous graduated audit of the companion theory. Shading the letter by w draws the expected sanction

$$\mathcal{K}_j(w) = \chi_j \bar{F} \left[1 - \frac{\tau}{\sqrt{V}} e^{-w^2/2V} \right] + \frac{\bar{\varrho}}{2} w^2, \qquad V = \tau^2 + \sigma_\pi^2,$$

in fixed-cost units, with $\chi_G = 1 > \chi_R = 0.50 > 0$ the hidden mode exposures, $\bar{F} = 4.50$ the bounded formal consequence, $\tau = 0.24$ the audit tolerance, $\sigma_\pi = 0.24$ control noise (percentage points), and $\bar{\varrho} = 0.15$ the reputational curvature: the formal and reputational penalties are additive, and posterior damage keeps growing where the

bounded formal audit saturates. Mode j delivers the center that settles the tension between the letter and the desire,

$$\theta^j_{t+1} = \operatorname*{argmin}_{\theta} \frac{\Psi}{2}(\theta - d_{t+1})^2 + \mathcal{K}_j(\theta - a_t), \qquad \Psi = 11.6,$$

where Ψ is the continuation-adjusted temptation curvature (the Schur complement of the branch Hessian in (θ, g) after the new announcement is reoptimized; reoptimization lowers it below the period-loss value $A_\theta = 1 + \vartheta/\kappa^2$). Writing the letter–desire gap as $D_t = |a_t - d_{t+1}|$, the solution is $\theta^j_{t+1} = d_{t+1} + \operatorname{sign}(a_t - d_{t+1})P_j(D_t)$ with P_j the delivered-pull map of the first-order condition—the same branch condition, share, and ceiling as the companion theory. Because the formal sanction is bounded, the minimization has a global-minimum switch: past an enforcement threshold in D_t, delivery collapses from the high-share branch to near-complete shading, leaving only the reputational tether. The solver tabulates P_j by global minimization on a dense grid. The delivered-center losses and shares are

$$L^j_{t+1} = \frac{1}{2}(\theta^j_{t+1})^2 + \frac{\vartheta}{2}(x^j_{t+1} - \bar{x}_{t+1})^2, \tag{7}$$

$$s_{j,t} = \frac{P_j(D_t)}{D_t}, \qquad s_t = (1 - q_t)s_{G,t} + q_t s_{R,t}. \tag{8}$$

B.3 The consistency condition

The central restriction is

$$e_t = (1 - q_t)\theta^G_{t+1} + q_t\theta^R_{t+1} = s_t a_t + (1 - s_t)d_{t+1}, \tag{9}$$

and the same posterior weights and delivered centers enter welfare, together with the expected net sanction of shading:

$$\mathcal{J}_{t+1} = (1 - q_t)\left[L^G_{t+1} + \mathcal{K}^{\text{net}}_G\right] + q_t\left[L^R_{t+1} + \mathcal{K}^{\text{net}}_R\right], \qquad \mathcal{K}^{\text{net}}_j = \mathcal{K}_j(w^*_j) - \mathcal{K}_j(0). \tag{10}$$

Here $w^*_j = \theta^j_{t+1} - a_t$ is mode j's optimal shading. Equation (9) is solved jointly with the delivered centers and shares. The planner therefore internalizes the risk that an aggressive announcement must be delivered in an unfavorable state; the sanction term also removes the corner in which the planner would issue an undeliverable letter to neutralize its own communication at no cost.

C Numerical Algorithm

The solver is `code/continuous_delivery_jax.py`. The decision vector contains one announcement for each year except the terminal one. For every candidate announcement path, the program performs the following fixed-point iteration:

1. initialize expected inflation and the institutional prior path;
2. compute static discretionary centers at every date;
3. compute the mode-specific delivered centers, output gaps, and delivered losses when each announcement comes due, interpolating the tabulated delivered-pull maps;
4. update message posteriors and the mode-specific delivered shares;
5. update expected inflation using (9);
6. update the next institutional prior and repeat until convergence.

The outer problem minimizes the discounted posterior-weighted delivered loss plus the expected net delivery sanction, rebuilding resources, and the authorization cost $\frac{\varphi_g}{2} g_t^2$ of the guidance wedge. JAX supplies exact derivatives and SciPy's L-BFGS-B routine solves the bounded problem from multiple interior initializations, keeping the best local optimum (the bounded sanction embeds a nonconvex delivery technology, so the issuance objective is nonconvex). The terminal announcement and terminal expectation are zero.

The supply transition is a single nonlinear reference path generated by the companion capacity calibration. Thus the solver is exact for the communication problem conditional on the supply path and certainty-equivalent with respect to the effect of delivery on future organizational capital.

C.1 Validation checks

The replication validator verifies:

- ordered enforcement exposures $\chi_G > \chi_R > 0$ and a positive bounded sanction;
- both forms of the expectation identity (9);
- equality between the posterior weights used in expectations and welfare;

- the letter-enforcement and delivered-center Phillips-curve identities;
- bounded posteriors and delivered shares;
- convergence of the expectation–belief fixed point;
- reproduction of all macros, tables, and figures;
- successful LaTeX compilation and PDF preflight.

D Additional Quantitative Results

D.1 Baseline path

Table D.1: Baseline delivery-consistent transition

Year	Supply	Forecast	Announcement	$\widetilde{E}_t \pi_{t+1}$	Inflation	Del. share
2021	4.48	4.48	3.01	3.12	7.40	0.687
2022	3.96	1.54	1.65	1.77	5.53	0.689
2023	2.58	0.58	1.26	1.37	3.78	0.693
2024	1.53	0.23	1.21	1.31	2.68	0.696
2025	0.74	0.09	1.29	1.37	1.96	0.699
2026	0.24	0.04	1.40	1.46	1.57	0.701

The optimal 2021 announcement is 3.01 percent. This is above target and below the inherited default center. It is therefore a disinflationary guidance wedge relative to the state-contingent targeting rule, but not an unconditional promise of immediate two-percent inflation.

D.2 Fragility and information about persistence

The fragility exercise separates audit sharpness from beliefs about persistence. With a sharp audit but full acceptance of the low-persistence signal, the delivered share remains high. The main sensitivity gives the public partial weight on the subsequently realized high persistence and allows the actual hidden mode to switch to R in 2021. The baseline announcement and activity path remain fixed.

Table D.2: Fixed-policy fragility experiments

Environment	Del. share	Expected inflation	Inflation	x_{2022}, letter	x_{2022}, delivered
Robust optimum	0.687	3.12	7.40	-2.1	-1.0
Sharp audit; public accepts v_0	0.911	3.04	7.33	-17.6	-1.0
Partial see-through; hidden switch	0.0128	3.87	8.12	-18.4	-1.0
Full see-through (polar)	0.0128	9.24	13.28	-44.2	-2.7

The full-see-through row is a polar case. It illustrates the nonlinear limit of the mechanism and is not used as a historical calibration.

D.3 Calendar paths

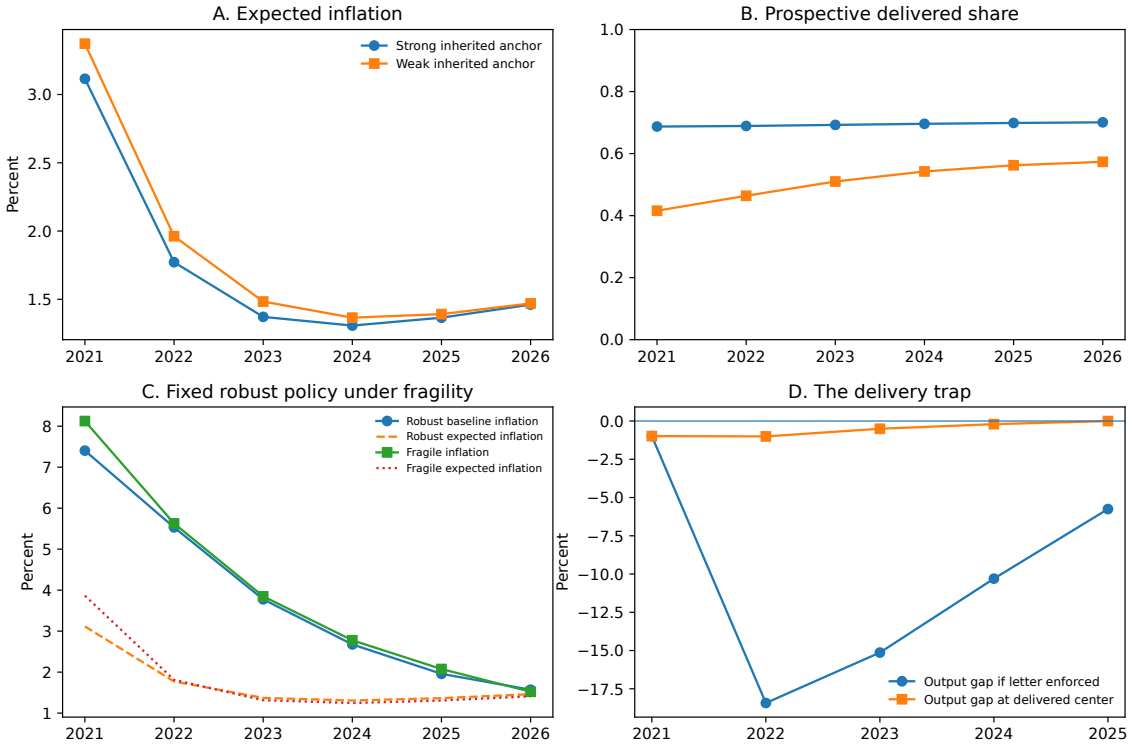

Figure D.1: Delivery-consistent fragility paths

E Data and Market Diagnostic

The inflation series are quarterly-average year-over-year CPI inflation and the Federal Reserve Bank of Cleveland model-based one-year expected-inflation measure. The FOMC-day diagnostic uses daily changes in two-year nominal Treasury yields and

fitted two-year inflation compensation. The statistic is

$$R_z(E) = \frac{\mathrm{sd}(\Delta z_t \mid t \text{ is an FOMC day in } E)}{\mathrm{sd}(\Delta z_t \mid t \text{ is an ordinary trading day in } E)}.$$

The code in `build_traction_exhibit.py` reproduces the event-study data and the corresponding figure. The market evidence is descriptive; no communication parameter is estimated from the event-day ratios.

F Reproduction Guide

From the package root, run

```
./run_all.sh
```

The script regenerates the delivery-consistent solution, tables, figures, empirical diagnostic, both PDFs, and validation report. A faster manuscript-only rebuild is

```
./run_all.sh --skip-empirical
```

Detailed file descriptions and software requirements are in `README.md` and `documentation.md`.

References

Drozd, Lukasz A. and Marina M. Tavares, "Capacity Wall: Inflation After COVID," July 2026. Working paper, Federal Reserve Bank of Philadelphia.