The Trade-Comovement Puzzle*

Lukasz A. Drozd[1] Sergey Kolbin[2] Jaromir B. Nosal[3]

[1]FRB Philadelphia [2]Amazon [3]Boston College

November, 2019

*The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

Trade-comovement puzzle (Kose and Yi, 2006):

- Cross-country data: more trade, more output comovement
  - Frankel and Rose (1998), Cleark and van Wincoop (2001), Calderson Chong and Stein (2002), Otto, Voss and Willard (2001), Bordo and Helbling (2003), Baxter and Kouparitsas (2005), Kose and Yi (2006) Inklaar, Jong-A-Pin and de Haan (2008), diGiovanni and Levchenko (2010), Johnson (2014)
- Standard transmission mechanism of productivity shocks implies a weak endogenous relation at best (Backus, Kehoe and Kydland, 1995)

Trade-comovement puzzle (Kose and Yi, 2006):

- Cross-country data: more trade, more output comovement
  - Frankel and Rose (1998), Cleark and van Wincoop (2001), Calderson Chong and Stein (2002), Otto, Voss and Willard (2001), Bordo and Helbling (2003), Baxter and Kouparitsas (2005), Kose and Yi (2006) Inklaar, Jong-A-Pin and de Haan (2008), diGiovanni and Levchenko (2010), Johnson (2014)
- Standard transmission mechanism of productivity shocks implies a weak endogenous relation at best (Backus, Kehoe and Kydland, 1995)

Standard transmission mechanisms of productivity shocks *inconsistent* **with a positive effect of trade on business cycle comovement.**

What We Do

What We Do

1. Provide the foundational analysis of the trade-comovement puzzle:

   - Characterize forces linking trade and comovement in standard theory
   - Explore several straightforward extensions this analysis points toward

What We Do

1. Provide the foundational analysis of the trade-comovement puzzle:
   - Characterize forces linking trade and comovement in standard theory
   - Explore several straightforward extensions this analysis points toward
2. Among them, identify an effective resolution: *dynamic trade elasticity.*

Basic Idea

- In standard theory, elasticity between home and foreign goods is *static:*

$$c + i = \left(\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

where $\rho \approx 1$, consistent with business cycle frequency elasticity estimates.

Basic Idea

- In standard theory, elasticity between home and foreign goods is *static:*

$$c + i = \left(\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

where $\rho \approx 1$, consistent with business cycle frequency elasticity estimates.

- We make it dynamic:

$$c + i = \left(\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} - \frac{\phi}{2}\left\{\left(\frac{f/d}{f_{-1}/d_{-1}} - 1\right)^2\right\}$$

where $\rho = 15$ and $\phi > 0$, consistent with *low* business cycle trade elasticity.

Basic Idea

- In standard theory, elasticity between home and foreign goods is *static:*

$$c + i = \left(\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

where $\rho \approx 1$, consistent with business cycle frequency elasticity estimates.

- We make it dynamic:

$$c + i = \left(\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} - \frac{\phi}{2}\left\{\left(\frac{f/d}{f_{-1}/d_{-1}} - 1\right)^2\right\}$$

where $\rho = 15$ and $\phi > 0$, consistent with *low* business cycle trade elasticity.

  - Approach *firmly* grounded in evidence: low SRE but high LRE

Basic Idea

- In standard theory, elasticity between home and foreign goods is *static:*

$$c + i = \left(\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

  where $\rho \approx 1$, consistent with business cycle frequency elasticity estimates.

- We make it dynamic:

$$c + i = \left(\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} - \frac{\phi}{2}\left\{\left(\frac{f/d}{f_{-1}/d_{-1}} - 1\right)^2\right\}$$

  where $\rho = 15$ and $\phi > 0$, consistent with *low* business cycle trade elasticity.

  - Approach *firmly* grounded in evidence: low SRE but high LRE
  - Neutral for theory's business cycle properties

Basic Idea

- In standard theory, elasticity between home and foreign goods is *static:*

$$c + i = \left(\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

where $\rho \approx 1$, consistent with business cycle frequency elasticity estimates.

- We make it dynamic:

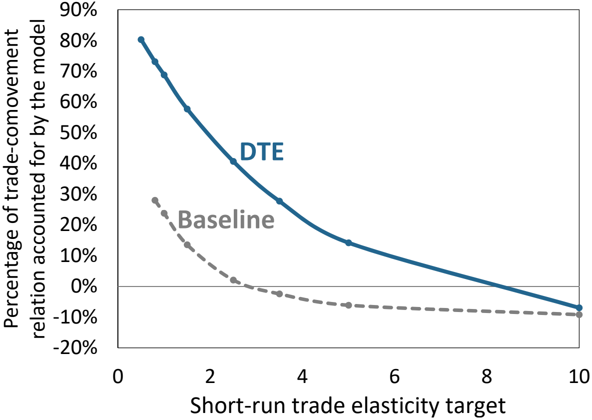
$$c + i = \left(\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} - \frac{\phi}{2}\left\{\left(\frac{f/d}{f_{-1}/d_{-1}} - 1\right)^2\right\}$$

where $\rho = 15$ and $\phi > 0$, consistent with *low* business cycle trade elasticity.

  - Approach *firmly* grounded in evidence: low SRE but high LRE
  - Neutral for theory's business cycle properties
  - Other desirable properties: see Drozd and Nosal (2012)

Preview of Main Substantive Result

- Result from an exercise a la Kose and Yi (2006):



Literature

1. Puzzle: Kose and Yi (2006)

2. Successful attempts to address the puzzle:

   - Liao and Santacrue (2015), Johnson (2014): TFP comovement correlated with trade

   - de Soyres (2017): Markups-implied transmission of terms of trade shocks

   - **But:** Johnson (2013) shows that industry level TFPs are insufficiently correlated to resolve the puzzle.

Roadmap

1. Theory of Trade-Comovement Puzzle
   - baseline results assuming full depreciation of capital
   - extended results
   - effect of dynamic trade elasticity
2. Quantitative Analysis and Results

Theory of Trade-Comovement Puzzle

Canonical IRBC model (BKK, '95)

1. Two symmetric countries (home/foreign)
2. Goods differentiated by country of origin and tradable
3. RBC supply side (random productivity, endogenous capital and labor)
4. Complete asset markets

Canonical IRBC model (BKK, '95)

1. Two symmetric countries (home/foreign)
2. Goods differentiated by country of origin and tradable
3. RBC supply side (random productivity, endogenous capital and labor)
4. Complete asset markets

Baseline: Capital depreciates within a period, no time to build

Canonical IRBC model (BKK, '95)

1. Two symmetric countries (home/foreign)
2. Goods differentiated by country of origin and tradable
3. RBC supply side (random productivity, endogenous capital and labor)
4. Complete asset markets

Baseline: Capital depreciates within a period, no time to build

Full model: three countries, durable capital w/ convex adjustment cost

NOTE: NO DYNAMIC ELASTICITY YET!

Physical Environment

Firms:

$y = Ak^{\alpha} l^{1-\alpha}$

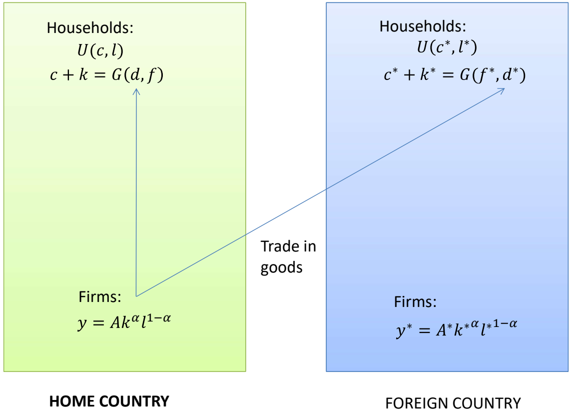
HOME COUNTRY

Firms:

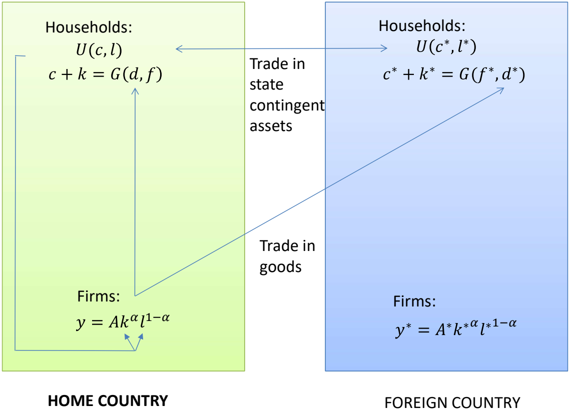
$y^* = A^* {k^*}^{\alpha} {l^*}^{1-\alpha}$

FOREIGN COUNTRY

Physical Environment



Physical Environment



Household problem

Representative household solves:

$$\sum_t \sum_{s^t} \beta^t Prob(s^t) u(c(s^t), l(s^t))$$

subject to

$$c(s^t) + k(s^t) = G(d(s^t), f(s^t)) := (\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}}$$

$$d(s^t)+f(s^t)/p(s^t)+\sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1}) = B(s^t)+w(s^t)l(s^t)+r(s^t)k(s^t)$$

where $p(s^t)$ is the price of d in f, s^t is history of shocks and

$$u(c,l) := \frac{(c^\eta(1-l)^{1-\eta})^{1-\sigma}}{1-\sigma}.$$

Household problem

Representative household solves:

$$\sum_t \sum_{s^t} \beta^t Prob(s^t) u(c(s^t), l(s^t))$$

subject to

$$c(s^t) + k(s^t) = G(d(s^t), f(s^t)) := (\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}}$$

$$d(s^t) + f(s^t)/p(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}) B(s^{t+1}) = B(s^t) + w(s^t) l(s^t) + r(s^t) k(s^t)$$

where $p(s^t)$ is the price of d in f, s^t is history of shocks and

$$u(c, l) := \frac{(c^\eta (1-l)^{1-\eta})^{1-\sigma}}{1-\sigma}.$$

go back 1 go back 2

Household problem

Demand for individual goods:

$$p = \frac{G_d(d, f)}{G_f(d, f)}$$

Labor-leisure choice:

$$wG_d(d, f) = -\frac{u_l(c, l)}{u_c(c, l)}$$

Consumption-capital choice:

$$rG_d(d, f) = 1$$

Efficient risk-sharing

$$\underbrace{\frac{u_c(c, l)}{u_c^*(c^*, l^*)}}_{IMRS} - \underbrace{\frac{c^* + k^*}{d^* + f^*/p} \frac{d + f/p}{c + k}}_{\text{real exchange rate} =: q} = 0$$

Firm problem

Firms maximize profits:

$$\Pi(s^t) = A(s^t)k(s^t)^\alpha l(s^t)^{1-\alpha} - w(s^t)l(s^t) - r(s^t)k(s^t)$$

Equilibrium profits are zero: $\Pi(s^t) = 0$, hence

$$r = \alpha A(\frac{l}{k})^{1-\alpha} \quad w = (1-\alpha)A(\frac{k}{l})^\alpha$$

Feasibility and Market Clearing

Feasibility

$$d(s^t) + d^* = y(s^t) := A(s^t)k(s^t)^{\alpha}l(s^t)^{1-\alpha}$$

$$f(s^t) + f^*(s^t) = y^*(s^t) := A(s^t)k(s^t)^{\alpha}l(s^t)^{1-\alpha}$$

Assets are zero net supply globally:

$$B(s^t) + B^*(s^t) = 0$$

Competitive equilibrium defined as usually.

Planning Problem

By welfare theorems, allocation solves:

$$\max\{u(c,l) + u(c^*,l^*)\}$$

subject to *aggregation*

$$\begin{aligned} c + k &= G(d,f) := (\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}} \\ c^* + k^* &= G(f^*, d^*) \end{aligned}$$

and *production feasibility*

$$\begin{aligned} d + d^* &= A k^{\alpha} l^{1-\alpha} \\ f + f^* &= A^* k^{*\alpha} l^{*1-\alpha}. \end{aligned}$$

where A^* and A follow an AR1 process.

Conceptual Framework

Define trade-comovement relation

Show how transmission of shocks is affected by trade

Characterize the effect of trade on comovement

Definitions

Steady-state level of trade:

$$\bar{x} := \frac{\bar{f}}{\bar{y}} = \frac{\bar{f}}{\bar{f} + \bar{d}}.$$

which, here, implies $\bar{x} = 1 - \omega$.

Definitions

Steady-state level of trade:

$$\bar{x} := \frac{\bar{f}}{\bar{y}} = \frac{\bar{f}}{\bar{f} + \bar{d}}.$$

which, here, implies $\bar{x} = 1 - \omega$.

Output elasticity to foreign shock ($\mathcal{S}$):

$$\mathcal{S}(\bar{x}) := \left(\frac{\partial \log y(A, A^*)}{\partial \log A^*} \right) \left(\frac{\partial \log y(A, A^*)}{\partial \log A} + \frac{\partial \log y(A, A^*)}{\partial \log A^*} \right)^{-1}$$

Definitions

Steady-state level of trade:

$$\bar{x} := \frac{\bar{f}}{\bar{y}} = \frac{\bar{f}}{\bar{f} + \bar{d}}.$$

which, here, implies $\bar{x} = 1 - \omega$.

Output elasticity to foreign shock ($\mathcal{S}$):

$$\mathcal{S}(\bar{x}) := \left(\frac{\partial \log y(A, A^*)}{\partial \log A^*}\right)\left(\frac{\partial \log y(A, A^*)}{\partial \log A} + \frac{\partial \log y(A, A^*)}{\partial \log A^*}\right)^{-1}$$

Theory-implied trade-comovement relation ($\mathcal{L}$)

$$\mathcal{L}(\bar{x}) := \frac{d\mathcal{S}(\bar{x})}{d\bar{x}}.$$

Decomposition of Shock Transmission

Decomposition of Shock Transmission

Define zero-sum transfers between the two countries:

$$T := \underbrace{(1 - 1/p(s^t))f(s^t)}_{=:T_p} + B(s^t) - \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1})$$

Decomposition of Shock Transmission

Define zero-sum transfers between the two countries:

$$T := \underbrace{(1 - 1/p(s^t))f(s^t)}_{=:T_p} + B(s^t) - \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1})$$

Household budget constraint implies:

$$d(s^t) + f(s^t)/p(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1}) = B(s^t) + w(s^t)l(s^t) + r(s^t)k(s^t)$$

HH problem

Decomposition of Shock Transmission

Define zero-sum transfers between the two countries:

$$T := \underbrace{(1 - 1/p(s^t))f(s^t)}_{=:T_p} + B(s^t) - \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1})$$

Household budget constraint implies:

$$d(s^t) + f(s^t) = y(s^t) + [B(s^t) - \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1}) + f(s^t) - f(s^t)/p(s^t)]$$

HH problem

Decomposition of Shock Transmission

Define zero-sum transfers between the two countries:

$$T := \underbrace{(1 - 1/p(s^t))f(s^t)}_{=:T_p} + B(s^t) - \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1})$$

Household budget constraint implies:

$$d(s^t) + f(s^t) = y(s^t) + T(s^t)$$

$$d^*(s^t) + f^*(s^t) = y^*(s^t) - T(s^t)$$

HH problem

Decomposition of Shock Transmission

Define zero-sum transfers between the two countries:

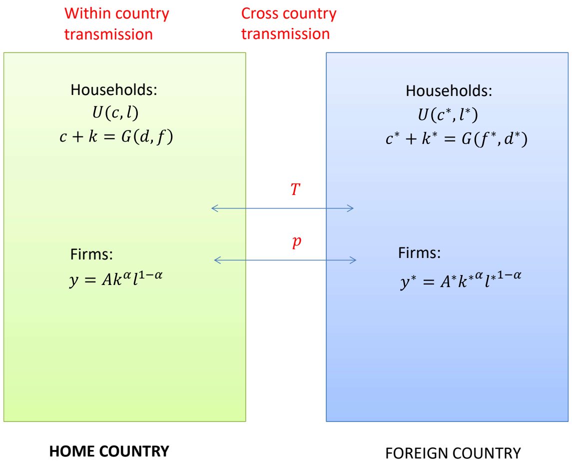
$$T := \underbrace{(1 - 1/p(s^t))f(s^t)}_{=:T_p} + B(s^t) - \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1})$$

Within country transmission: Log-linearize equilibrium system by assuming p and T exogenous processes.

Cross-country transmission: Close the equilibrium system for p and T.

HH problem

Decomposition of Shock Transmission



Conditions We Log-linearize

1. Aggregation constraint: $c + k = G(d, f)$
2. Labor-leisure choice: $wG_d(d, f) = -\frac{u_l(c,l)}{u_c(c,l)}$
3. Consumption-capital choice: $rG_d(d, f) = 1$
4. Demand for individual goods: $p = \frac{G_d(d,f)}{G_f(d,f)}$
5. Budget constraint involving T: $d + f = Ak^{\alpha}l^{1-\alpha} + T$
6. Factor prices: $r = \alpha A(\frac{l}{k})^{1-\alpha}$, $w = (1-\alpha)A(\frac{k}{l})^{\alpha}$

HH problem

Within Country Transmission

Log-deviation of home output from the steady state is:

$$\hat{y}(\hat{A};\hat{p},\hat{T}) := \frac{\hat{A}}{1-\alpha} + \underbrace{\frac{1+\alpha-\eta}{1-\alpha}\bar{x}\hat{p}}_{\text{substitution effect}} - \underbrace{\frac{1-\eta}{1-\alpha}\hat{T}}_{\text{income effect}}$$

Other conditions

Within Country Transmission

Log-deviation of home output from the steady state is:

$$\hat{y}(\hat{A};\hat{p},\hat{T}) := \frac{\hat{A}}{1-\alpha} + \underbrace{\frac{1+\alpha-\eta}{1-\alpha}\bar{x}\hat{p}}_{\text{substitution effect}} - \underbrace{\frac{1-\eta}{1-\alpha}\hat{T}}_{\text{income effect}}$$

$\Rightarrow$ Trade determines exposure to terms of trade via substitution effect:

$p \uparrow \rightarrow G_d(d,f) \uparrow \rightarrow l,k \uparrow$ *proportionally to trade*

Other conditions

Cross-Country Transmission

Step 1: Log-linearization of market clearing condition

$$\frac{f + f^* - y^*}{\bar{y}} = 0,$$

gives

$$\hat{p}(\hat{A}, \hat{A}^*; \hat{T}) := \frac{-(\hat{y}(\hat{A}, \hat{A}^*; \hat{T}) - \hat{y}^*(\hat{A}, \hat{A}^*; \hat{T})) + (\frac{1}{\bar{x}} - 2)\hat{T}}{2\rho(1 - \bar{x})}$$

Cross-Country Transmission

Step 1: Log-linearization of market clearing condition

$$\frac{f + f^* - y^*}{\bar{y}} = 0,$$

gives

$$\hat{p}(\hat{A}, \hat{A}^*; \hat{T}) := \frac{-\frac{\hat{A}-\hat{A}^*}{1-\alpha} + (\frac{1}{\bar{x}} + 2\frac{\alpha-\eta}{1-\alpha})\hat{T}}{2(\rho(1-\bar{x}) + \bar{x}\frac{1-\eta+\alpha}{1-\alpha})}$$

Cross-Country Transmission

Step 1: Log-linearization of market clearing condition

$$\frac{f + f^* - y^*}{\bar{y}} = 0,$$

gives

$$\hat{p}(\hat{A}, \hat{A}^*; \hat{T}) := \frac{-\frac{\hat{A} - \hat{A}^*}{1-\alpha} + \left(\frac{1}{\bar{x}} + 2\frac{\alpha - \eta}{1-\alpha}\right)\hat{T}}{2\left(\rho(1-\bar{x}) + \bar{x}\frac{1-\eta+\alpha}{1-\alpha}\right)}$$

$\Rightarrow$ Trade affects the impact of transfers on market clearing & hence p:

$T \uparrow \rightarrow$ excess supply of good $f \rightarrow p \uparrow$ *inversely proportionally to trade*

Cross-Country Transmission

Step 2: Log-linearization of risk-sharing condition

$$\frac{u_c(c,l)}{u_c^*(c^*,l^*)} - \underbrace{\frac{c^*+k^*}{d^*+f^*/p}\frac{d+f/p}{c+k}}_{=:q} = 0.$$

gives

$$\hat{T}(\hat{A},\hat{A}^*;\hat{p}) := \frac{-(1+\eta(\sigma-1))\frac{\hat{A}-\hat{A}^*}{1-\alpha} - (1-2\bar{x}\frac{\eta-\alpha}{1-\alpha})\hat{p}}{2\sigma\eta/(1-\alpha)}.$$

Cross-Country Transmission

Step 2: Log-linearization of risk-sharing condition

$$\frac{u_c(c,l)}{u_c^*(c^*,l^*)} - \underbrace{\frac{c^*+k^*}{d^*+f^*/p}\frac{d+f/p}{c+k}}_{=:q} = 0.$$

gives

$$\hat{T}(\hat{A},\hat{A}^*;\hat{p}) := \frac{-(1+\eta(\sigma-1))\frac{\hat{A}-\hat{A}^*}{1-\alpha} - (1-2\bar{x}\frac{\eta-\alpha}{1-\alpha})\hat{p}}{2\sigma\eta/(1-\alpha)}.$$

Step 2: Log-linearization of risk-sharing condition

$$\frac{u_c(c,l)}{u_c^*(c^*,l^*)} - \underbrace{\frac{c^*+k^*}{d^*+f^*/p}\frac{d+f/p}{c+k}}_{=:q} = 0.$$

gives

$$\hat{T}(\hat{A},\hat{A}^*;\hat{p}) := \frac{-(1+\eta(\sigma-1))\frac{\hat{A}-\hat{A}^*}{1-\alpha} - (1-2\bar{x}\frac{\eta-\alpha}{1-\alpha})\hat{p}}{2\sigma\eta/(1-\alpha)}.$$

$\Rightarrow$ Trade has little direct effect on risk sharing, especially for $\alpha \approx \eta$.

Cross-Country Transmission Summary

Trade affects general equilibrium feedback mechanism ($\hat{A} = 0$):

$$\hat{p}(\hat{A}, \hat{A}^*; \hat{T}) := \frac{\frac{\hat{A}^*}{1-\alpha} + (\frac{1}{\bar{x}} + 2\frac{\alpha-\eta}{1-\alpha})\hat{T}}{2(\rho(1-\bar{x}) + \bar{x}\frac{1-\eta+\alpha}{1-\alpha})}$$

$$\hat{T}(\hat{A}, \hat{A}^*; \hat{p}) := \frac{(1+\eta(\sigma-1))\frac{\hat{A}^*}{1-\alpha} - (1 - 2\bar{x}\frac{\eta-\alpha}{1-\alpha})\hat{p}}{2\sigma\eta/(1-\alpha)}$$

Intuition follows from the efficient risk sharing condition:

$$\underbrace{\frac{u_c(c,l)}{u_c^*(c^*,l^*)}}_{IMRS} - \underbrace{\frac{c^*+k^*}{d^*+f^*/p}\frac{d+f/p}{c+k}}_{\text{real exchange rate:}\hat{q}=(1-2\bar{x})\hat{p}} = 0$$

Decomposition of Trade-Comovement Relation

Trade-comovement relation $\mathcal{L}$ is

$$\mathcal{L} = \underbrace{(1-\eta+\alpha)\left(\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} + \bar{x}\frac{\partial^2 \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}}\right)}_{\text{substitution effect channel } \mathcal{L}_S} - \underbrace{(1-\eta)\frac{\partial^2 \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}}}_{\text{income effect channel } \mathcal{L}_I}.$$

where functions $\hat{p}(\hat{A},\hat{A}^*), \hat{T}(\hat{A},\hat{A}^*)$ pertain to the fixed point of:

$$\hat{p}(\hat{A},\hat{A}^*;\hat{T}) := \frac{\frac{\hat{A}^*}{1-\alpha} + \left(\frac{1}{\bar{x}} + 2\frac{\alpha-\eta}{1-\alpha}\right)\hat{T}}{2\left(\rho(1-\bar{x}) + \bar{x}\frac{1-\eta+\alpha}{1-\alpha}\right)}$$

$$\hat{T}(\hat{A},\hat{A}^*;\hat{p}) := \frac{(1+\eta(\sigma-1))\frac{\hat{A}^*}{1-\alpha} - \left(1-2\bar{x}\frac{\eta-\alpha}{1-\alpha}\right)\hat{p}}{2\sigma\eta/(1-\alpha)}$$

Decomposition of Trade-Comovement Relation

Trade-comovement relation $\mathcal{L}$ is

$$\hat{y}(\hat{A};\hat{p},\hat{T}) := \frac{\hat{A}}{1-\alpha} + \underbrace{\frac{1+\alpha-\eta}{1-\alpha}\bar{x}\hat{p}}_{\text{substitution effect}} - \underbrace{\frac{1-\eta}{1-\alpha}\hat{T}}_{\text{income effect}}$$

$$\mathcal{L} = \underbrace{(1-\eta+\alpha)\left(\frac{\partial\hat{p}(\hat{A},\hat{A}^*)}{\partial\hat{A}^*} + \bar{x}\frac{\partial^2\hat{p}(\hat{A},\hat{A}^*)}{\partial\hat{A}^*\partial\bar{x}}\right)}_{\text{substitution effect channel } \mathcal{L}_S} - \underbrace{(1-\eta)\frac{\partial^2\hat{T}(\hat{A},\hat{A}^*)}{\partial\hat{A}^*\partial\bar{x}}}_{\text{income effect channel } \mathcal{L}_I}.$$

where functions $\hat{p}(\hat{A},\hat{A}^*), \hat{T}(\hat{A},\hat{A}^*)$ pertain to the fixed point of:

$$\hat{p}(\hat{A},\hat{A}^*;\hat{T}) := \frac{\frac{\hat{A}^*}{1-\alpha} + (\frac{1}{\bar{x}} + 2\frac{\alpha-\eta}{1-\alpha})\hat{T}}{2(\rho(1-\bar{x}) + \bar{x}\frac{1-\eta+\alpha}{1-\alpha})}$$

$$\hat{T}(\hat{A},\hat{A}^*;\hat{p}) := \frac{(1+\eta(\sigma-1))\frac{\hat{A}^*}{1-\alpha} - (1-2\bar{x}\frac{\eta-\alpha}{1-\alpha})\hat{p}}{2\sigma\eta/(1-\alpha)}$$

Results: Effect of Trade on Comovement

Assumptions

Assumption (*RBC*)

$\alpha = \eta = 1/3.$

Assumption (*Home-bias*)

$0 < \bar{x} \leq \min\{1/(1+\sigma/2), 1/3\}$

Substitution effect channel of trade

Proposition

$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > 0$ *and* $\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} + \bar{x}\frac{\partial^2 \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} > 0$*, hence* $\mathcal{L}_S > 0$*.*

Substitution effect channel of trade

Proposition

$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > 0$ *and* $\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} + \bar{x}\frac{\partial^2 \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} > 0$*, hence* $\mathcal{L}_S > 0$*.*

- Terms of trade appreciates after the shock (foreign good is normal)
- Trade determines exposure of relative price of home good relative to terms of trade

Decomposition of Trade-Comovement Relation

Trade-comovement relation $\mathcal{L}$ is

$$\hat{y}(\hat{A};\hat{p},\hat{T}) := \frac{\hat{A}}{1-\alpha} + \underbrace{\frac{1+\alpha-\eta}{1-\alpha}\bar{x}\hat{p}}_{\text{substitution effect}} - \underbrace{\frac{1-\eta}{1-\alpha}\hat{T}}_{\text{income effect}}$$

$$\mathcal{L} = \underbrace{(1-\eta+\alpha)\left(\frac{\partial\hat{p}(\hat{A},\hat{A}^*)}{\partial\hat{A}^*} + \bar{x}\frac{\partial^2\hat{p}(\hat{A},\hat{A}^*)}{\partial\hat{A}^*\partial\bar{x}}\right)}_{\text{substitution effect channel } \mathcal{L}_S} - \underbrace{(1-\eta)\frac{\partial^2\hat{T}(\hat{A},\hat{A}^*)}{\partial\hat{A}^*\partial\bar{x}}}_{\text{income effect channel } \mathcal{L}_I}.$$

where functions $\hat{p}(\hat{A},\hat{A}^*), \hat{T}(\hat{A},\hat{A}^*)$ pertain to the fixed point of:

$$\hat{p}(\hat{A},\hat{A}^*;\hat{T}) := \frac{\frac{\hat{A}^*}{1-\alpha} + \left(\frac{1}{\bar{x}} + 2\frac{\alpha-\eta}{1-\alpha}\right)\hat{T}}{2\left(\rho(1-\bar{x}) + \bar{x}\frac{1-\eta+\alpha}{1-\alpha}\right)}$$

$$\hat{T}(\hat{A},\hat{A}^*;\hat{p}) := \frac{(1+\eta(\sigma-1))\frac{\hat{A}^*}{1-\alpha} - \left(1-2\bar{x}\frac{\eta-\alpha}{1-\alpha}\right)\hat{p}}{2\sigma\eta/(1-\alpha)}$$

Income effect channel of trade

Proposition

If $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$, *then* $\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > 0$ *and* $\frac{\partial^2 \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} > 0$, *hence* $\mathcal{L}_I < 0$.

Income effect channel of trade

Proposition

If $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$, *then* $\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > 0$ *and* $\frac{\partial^2 \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} > 0$, *hence* $\mathcal{L}_I < 0$.

- Home country receives a net transfer from the foreign country
- Transfers rise with trade because trade attenuates their impact on relative price of consumption

Decomposition of Trade-Comovement Relation

Trade-comovement relation $\mathcal{L}$ is

$$\hat{y}(\hat{A};\hat{p},\hat{T}) := \frac{\hat{A}}{1-\alpha} + \underbrace{\frac{1+\alpha-\eta}{1-\alpha}\bar{x}\hat{p}}_{\text{substitution effect}} - \underbrace{\frac{1-\eta}{1-\alpha}\hat{T}}_{\text{income effect}}$$

$$\mathcal{L} = \underbrace{(1-\eta+\alpha)\left(\frac{\partial\hat{p}(\hat{A},\hat{A}^*)}{\partial\hat{A}^*} + \bar{x}\frac{\partial^2\hat{p}(\hat{A},\hat{A}^*)}{\partial\hat{A}^*\partial\bar{x}}\right)}_{\text{substitution effect channel } \mathcal{L}_S} - \underbrace{(1-\eta)\frac{\partial^2\hat{T}(\hat{A},\hat{A}^*)}{\partial\hat{A}^*\partial\bar{x}}}_{\text{income effect channel } \mathcal{L}_I}.$$

where functions $\hat{p}(\hat{A},\hat{A}^*), \hat{T}(\hat{A},\hat{A}^*)$ pertain to the fixed point of:

$$\hat{p}(\hat{A},\hat{A}^*;\hat{T}) := \frac{\frac{\hat{A}^*}{1-\alpha} + (\frac{1}{\bar{x}} + 2\frac{\alpha-\eta}{1-\alpha})\hat{T}}{2(\rho(1-\bar{x}) + \bar{x}\frac{1-\eta+\alpha}{1-\alpha})}$$

$$\hat{T}(\hat{A},\hat{A}^*;\hat{p}) := \frac{(1+\eta(\sigma-1))\frac{\hat{A}^*}{1-\alpha} - (1-2\bar{x}\frac{\eta-\alpha}{1-\alpha})\hat{p}}{2\sigma\eta/(1-\alpha)}$$

Trade-Comovement Puzzle

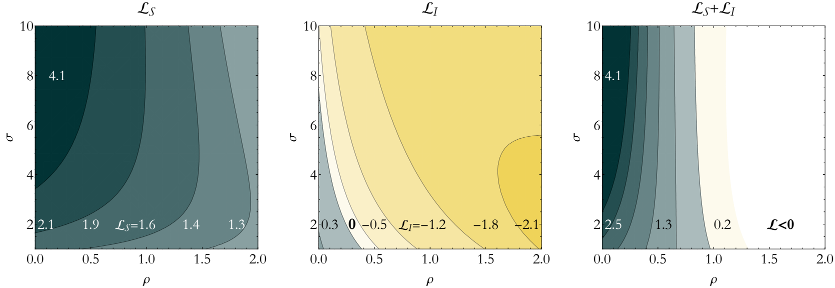


Figure: Model-implied trade-comovement relation $\mathcal{L}$ for $\bar{x} = 5\%$.

Trade-Comovement Puzzle

- Risk-sharing contributes to weak relation between trade and comovement
- Models that shut down risk-sharing potentially promising in resolving the puzzle:
  - Financial autarky (generally fails)
  - GHH preferences (sort of work with higher Frisch elasticity)
  - Very low elasticity ρ (only Leontief improves notably)
  - Dynamic trade elasticity

Generalization

Extended Model

Consider a two-period planning problem of the form:

$$\max\{u(c,l) + u(c^*,l^*) + u(c_{+1},l_{+1}) + u(c^*_{+1},l^*_{+1})\}$$

subject to

$$\begin{aligned} c + \delta\bar{k} + \Delta k + \psi\Delta k^2 &= G(d,f) \\ c_{+1} + \delta\bar{k} - (1-\delta)\Delta k &= G(d_{+1},f_{+1}) \\ d + d^* &= A(\bar{k} + \Delta k)^\alpha l^{1-\alpha} \\ d_{+1} + d^*_{+1} &= \bar{k}^\alpha l_{+1}^{1-\alpha}, \end{aligned}$$

and same constraints for the foreign country, where $\bar{k}$ is stationary level such that $\Delta k = 0$ for $A = A^* = 1$.

Extended Model: Results

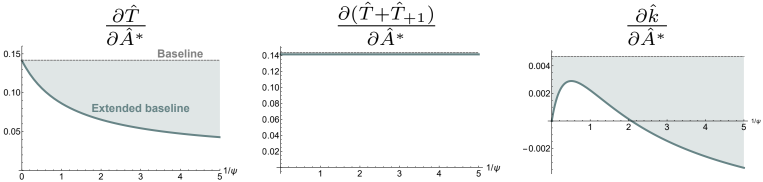


Figure: Transfers and capital accumulation in extended baseline model.

Notes: The figure assumes the following parameter values: $\rho = 5/4$, $\sigma = 2$, $\delta = 1/20$, $\alpha = \eta = 1/3$ and $\bar{x} = 5\%$.

Extended Model: Results

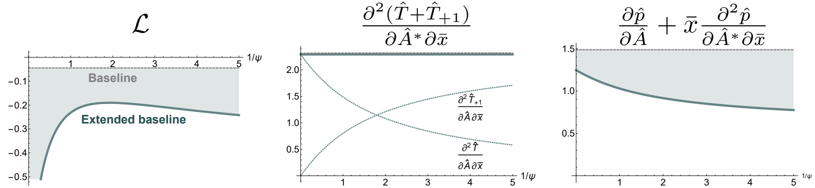


Figure: Decomposition of trade-comovement relation in extended model.

Notes: The figure assumes $\rho = 5/4$, $\sigma = 2$, $\delta = 1/20$, $\alpha = \eta = 1/3$ and $\bar{x} = 5\%$.

Dynamic Trade Elasticity

Prototypical Dynamic Elasticity Model (DTE)

Consider a two-period planning problem of the form:

$$\max\{u(c,l) + u(c^*,l^*) + u(c_{+1},l_{+1}) + u(c^*_{+1},l^*_{+1})\}$$

subject to

$$\begin{aligned} c + \delta\bar{k} + \Delta k + \psi\Delta k^2 &= G(d,f) \\ c_{+1} + \delta\bar{k} - (1-\delta)\Delta k &= d_{+1} + f_{+1} \\ d + d^* &= A(\bar{k}+k)^{\alpha} l^{1-\alpha} \\ d_{+1} + d^*_{+1} &= \bar{k}^{\alpha} l^{1-\alpha}_{+1}, \end{aligned}$$

and same constraints for the foreign country, where $\bar{k}$ is stationary level such that $\Delta k = 0$ for $A = A^* = 1$.

Effect of Dynamic Trade Elasticity

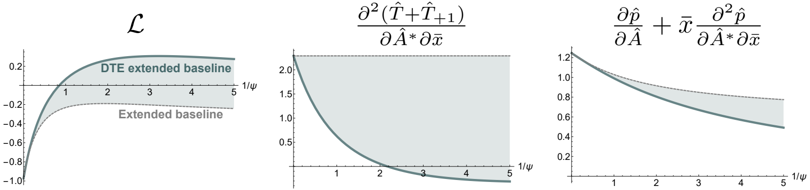


Figure: Effect of DTE for $\delta = 1/20, \sigma = 2, \rho = 5/4$ and $\bar{x} = 5\%$.

Effect of Dynamic Trade Elasticity

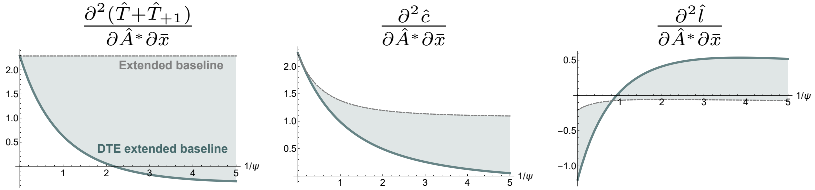


Figure: Effect of dynamic trade elasticity on income effect channel in extended baseline model.

Key Intuition

Consider a storage economy

$$\max\{u(c) + u(c^*) + u(c_{+1}) + u(c^*_{+1})\}$$

subject to

$$\begin{aligned} c + \Delta k &= G(d, f) \\ c_{+1} - \Delta k &= d_{+1} + f_{+1} \\ d + d^* &= A \\ d_{+1} + d^*_{+1} &= 1, \end{aligned}$$

and same constraints for the foreign country.

Key Intuition

Proposition

Consider any positive productivity shock abroad $\Delta A^* > 0, \Delta A = 0$:
Then, $c = c_{+1} = c^* = c^*_{+1}$, and

$$\frac{d\sigma(\Delta k)}{d\bar{x}} \frac{\bar{x}}{\sigma(\Delta k)} \times 100 = \frac{4\bar{x}}{1 - 4\bar{x}^2} \times 100$$

- For $\bar{x} = 5\%$, a one percent increase in volume of trade raises the volatility of Δk by 20%!
    - In extended baseline DTE with $\psi = 0$, this number is from 24% to 26% depending on $\rho \in [1, 2]$

Key Intuition

- How to smooth out a surge in supply of good f by some $\Delta > 0$ without changing d/f and d^*/d^* in first period?

$$\begin{bmatrix} 1-\bar{x} & \bar{x} \\ \bar{x} & 1-\bar{x} \end{bmatrix} \begin{bmatrix} \Delta k \\ \Delta k^* \end{bmatrix} = \begin{bmatrix} 0 \\ \Delta \end{bmatrix}$$

Inversion implies:

$$\begin{bmatrix} \Delta k \\ \Delta k^* \end{bmatrix} = \begin{bmatrix} -\frac{\bar{x}}{1-2\bar{x}} \\ \frac{1-\bar{x}}{1-2\bar{x}} \end{bmatrix} \Delta$$

Key Intuition

- Storage technology can smooth out the shock
- But as trade rises, volatility of Δk increases
- In our model volatility of capital is penalized by the convex adjustment cost and decreasing marginal product of capital
- The result is a countervailing effect of trade on transfers

Quantitative Results from Exercise a la Kose and Yi

Figure: Slope of trade-comovement relation: Fraction explained.

Model	Model slope relative to data est.
Baseline	**20%**
DTE	**64%**
DTE low SRE target	**80%**
FA	25%
GHH baseline Frisch	29%
GHH high Frisch 2	50%

Notes: The table reports the implied slope between trade and comovement (output correlation) by each model variant relative to the corresponding value for the data. Data value is derived from the OLS regression. The slope value for the models has been calculated by increasing bilateral trade intensity from the median value to 90th percentile, and accordingly adjusting trade openness with rest of the world.

Conclusions

- Trade-comovement puzzle associated with a sizable effect of trade on income (wealth) effect of shocks
- Modeling low short- and high long-run trade elasticity largely resolves the puzzle
- Trade-comovement puzzle is best interpreted as imposing empirically viable parametric and structural restrictions on the standard transmission mechanism as opposed to rejecting it outright.

Relation to First-Order Conditions

$$
\begin{aligned}
\hat{k} &= \bar{x}\hat{p} + \hat{y} && \leftarrow rG_d(d,f) = 1, r = \alpha A(\frac{l}{k})^{1-\alpha} \\
\hat{l} &= \frac{1-\eta}{1-\alpha}\bar{x}\hat{p} - \frac{1-\eta}{1-\alpha}\hat{T} && \leftarrow wG_d(d,f) = -\frac{u_l(c,l)}{u_c(c,l)} \\
\hat{d} &= \rho\bar{x}\hat{p} + \hat{y} + \hat{T} && \leftarrow p = \frac{G_d(d,f)}{G_f(d,f)}, d+f = y+T, c+k = G(d,f) \\
\hat{f} &= (1-\bar{x})\rho\hat{p} + \hat{y} + \hat{T}
\end{aligned}
$$

go back

Allocation Satisfies Static Planning Problem

By welfare theorems, allocation solves:

$$\max\{u(c,l) + u(c^*,l^*)\}$$

subject to *aggregation*

go back

$$c + k = G(d,f) := (\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}}$$
$$c^* + k^* = G(f^*, d^*)$$

and *production feasibility*

$$d + d^* = Ak^{\alpha} l^{1-\alpha}$$
$$f + f^* = A^* k^{*\alpha} l^{*1-\alpha}.$$

where A^* and A follow an AR1 process.