Understanding Growth through Automation: The Neoclassical Perspective

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Motivation: Uzawa's Theorem and Automation

$$\begin{cases} \text{Balanced growth (constant factor shares)} \\ \text{Capital augmenting technical progress} \end{cases} \rightarrow \quad \text{CD: } Y = A\,(ZK)^{\alpha}\,L^{1-\alpha}$$

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   - automation = capital autonomously completes a step of a production process
2. Follow neoclassical blueprint: Identify task technology consistent with the Kaldor facts
4. Develop a model of IT-powered automation
   - IT optimizes task load of producing task specific machines
   - IT application requires completion of tasks

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- BGP with constant LS possible; requires Pareto relative productivity of capital across tasks
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What We Show

- BGP with constant LS possible; requires Pareto relative productivity of capital across tasks
  - underlying technology can seen as nongeneric because of Pareto distribution (we show it explicitly by developing illustrative microfoundations)
- Provide a task-based microfoundation of the Cobb–Douglas production
- Identify key conditions for IT to be labor-share displacing

Literature

- **Growth and automation: Acemoglu and Restreppo (2018)**
  - agnostic about the link between tasks structural change
    (task space static with random churning in and out of use)
    - offer different take on "horses versus humans" analogy
      (humans fungible, churning diffuses productivity gains across tasks)
  - different story for future: model of IT in automation vs. acceleration of trends
- Foundations of task-based theory of growth: Zeira (1998, 2006)
- Microfoundations of Cobb-Douglas pf: Jones (2005), Houthakker (1955):
  - task-based microfoundation
  - no requirement of economy-wide aggregation

Model

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- Production technology:

  1. task *complexity* space: $\mathcal{Q} = \mathbb{R}_+ = [0, +\infty)$

  2. *increasing* capital requirement function: $k(q)$

     - labor input normalized to 1 (not an assumption)

  3. measure function:

  $$\mu(\mathcal{S}) := \int_{\mathcal{S}} g(q)\, dv,$$

  where $\mathcal{S} \in \mathcal{B}(\mathcal{Q})$, g density, v canonical Lebesgue measure

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- For a given partition $\{\mathcal{Q}_k, \mathcal{Q}_l\}$ of $\mathcal{Q}$, producing Y units of output requires

$$K = Y \int_{\mathcal{Q}_k} k(q)\, d\mu \quad \text{and} \quad L = Y \int_{\mathcal{Q}_l} 1 d\mu$$

Aggregation

Given technology vector $T := (k, g)$, define

$$Y(T; K, L) := \sup \left\{ \hat{Y} \in \mathbb{R}_+ : \exists_{\text{m.p: } \mathcal{Q}_l, \mathcal{Q}_k} \text{ s.t. } K \geq \hat{Y} \int_{\mathcal{Q}_k} k(q)\, d\mu, L \geq \hat{Y} \int_{\mathcal{Q}_l} 1 d\mu \right\}$$

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Assumption

There exists a partition $\{\mathcal{Q}_k, \mathcal{Q}_l\}$ of $\mathcal{Q}$ with $\mathcal{Q}_k \cap \mathcal{Q}_l = \emptyset$ and *$\mathcal{Q}_k \cup \mathcal{Q}_l = \mathcal{Q}$ such that $\int_{\mathcal{Q}_l} 1 d\mu < \infty$ and $\int_{\mathcal{Q}_k} k(q)\, d\mu < \infty$, k is positive-valued function on at least part of the domain, and g has full support.*

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Lemma

There exists a unique factor utilization cutoff $q^* \geq 0$ and $\hat{Y} > 0$ such that $K = \hat{Y} \int_0^{q^*} k(q)\, d\mu$, $L = \hat{Y} \int_{q^*}^{\infty} d\mu$, and $Y(T; K, L) = \hat{Y}$.

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Lemma

Marginal products of each factor are:

$$MPK = \frac{Y}{K} \left(1 + k(q^*) \frac{L}{K} \right)^{-1} \quad \textit{and} \quad MPL = \frac{Y}{L} (1 - \frac{K}{Y} MPK) \qquad \textit{a.e.,}$$

where q^* is the factor utilization cutoff.

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Lemma

Suppose $T_1 = \{k_1, \mu_1\} \in \mathcal{T}$ and $T_2 = \{k_2, \mu_2\} \in \mathcal{T}$ are such that there exists a μ_1, μ_2-measurable (and invertible) map $f : \mathcal{Q} \to \mathcal{Q}$ so that $k_1 \equiv k_2 \circ f^{-1}$ (a.e.) and $\mu_1 \equiv \mu_2 \circ f^{-1}$. Then, the aggregate production function associated to each technology is identical; that is, $Y(T_1; K, L) \equiv Y(T_2; K, L)$.

Discrete example

- Output: Hang a picture on a wall

1. move to location (coordinates) **[1 labor or 2 capital]**
2. take a nail from (coordinates) **[1 labor or 90 capital]**
3. move to location (coordinates) **[1 labor or 2 capital]**
4. place a nail tip against the wall (coordinates) **[1 labor or 10 units of capital]**
5. apply force to nail head until nailed in **[1 labor or .0001 capital]**
6. move to location (coordinates) **[1 labor or 2 capital]**
7. place picture hook on the nail **[1 labor of 100 units of capital]**

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Existence of BGP

GE and growth

- Given path $T_t = (k_t, \mu_t) \in \mathcal{T}$, allocation is a path of $C_t, K_t, L_t, Y_t, q_t^*$ such that

$$\max_{C_t, K_t, L_t, Y_t, q_t^*} \int_0^\infty e^{-\rho t} u\left(C_t\right) dt,$$

subject to

$$C_t + \dot{K}_t - \delta K_t = Y_t,$$

$$L_t = \bar{L},$$

where q_t^* satisfies

$$K_t = Y_t \int_0^{q_t^*} k_t\left(q\right) d\mu_t, \qquad L_t = Y_t \int_{q_t^*}^\infty d\mu_t.$$

Definition of balanced growth path

Definition

A balanced growth path with automation and constant factor shares (BGP) comprises an allocation sequence: $Y_t = Y_0 e^{\gamma_Y t}$, $C_t = C_0 e^{\gamma_C t}$, $K_t = K_0 e^{\gamma_K t}$, $q_t^* = q_0^* e^{\gamma_{q^*} t}$ *and a technology sequence* $T_t = \left\{k_t\left(q\right) = k_0\left(q\right) e^{\gamma_k t}, g_t\left(q\right) = g_0\left(q\right) e^{\gamma_g t}\right\} \in \mathcal{T}$, *such that* $\gamma_Y > 0$, $\gamma_{q^*} > 0$ *and* $\alpha \equiv \frac{K_t}{Y_t} MPK_t$ *is constant, where* γ_k, γ_g, $Y_0 > 0$, $C_0 > 0$, $K_0 > 0$, $q_0^* > 0$ *are scalars.*

- We use the standard approach of assuming "stable shape" conditions on g and k
- Not an assumption: Justified by random task churning. (Will come back to this.)

Balanced growth path existence

Proposition

If $\gamma_k < 0$ *and* $\gamma_g - \alpha\gamma_k > 0$*, BGP exists and features:*

1. *Technology*

$$T_0 = \left\{ k_0\left(q\right) = k_0 q^{\theta}, g_0\left(q\right) = g_0 q^{-\zeta - 1} \right\},$$

where $\zeta := \frac{\gamma_Y + \gamma_g}{\gamma_{q^*}}$*,* $\theta := \frac{\gamma_Y - \gamma_k}{\gamma_{q^*}}$*,*

2. *Growth rates:* $\gamma := \gamma_Y = \gamma_C = \gamma_g - \alpha\gamma_k$.

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2. *Growth rates:* $\gamma := \gamma_Y = \gamma_C = \gamma_g - \alpha\gamma_k$.

Corollary

The aggregate production function along the BGP is Cobb–Douglas; that is, output Y_t and marginal products MPK_t and MPL_t are consistent with those implied by

$$Y_t\left(K, L\right) = A_t\left(Z_t K\right)^{\alpha} L^{1-\alpha}$$

where $\alpha = \frac{\zeta}{\theta}$ *and* $A_t > 0$, $Z_t > 0$ *are scalars that grow at a constant rate.*

Proof outline

1. Standard: Y, C, K grow at same rate γ

2. Given constant growth q^*, $k(q)$ must be CES because

$$\alpha = \frac{K}{Y} MPK = \left(1 + k\left(q^*\right) \frac{L}{K}\right)^{-1}$$

3. Given output growth, labor input must decline at rate γ because

$$\bar{L} = Y \int_{q^*}^{\infty} 1 g(q) dv$$

Nongeneric nature of BGP task technology

Nongeneric nature of CD task technology

- Nongeneric to the extent that Pareto distributed capital requirements is, since

$$Pr\left(\mathbf{k} \geq k | \mathbf{k} \geq k_0\right) = Pr\left(q \geq k^{\alpha} | q \geq k_0^{\alpha}\right) = \left(\frac{k}{k_0}\right)^{-\alpha}$$

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- Example: Poisson entry of tasks with fixed productivity growth after entry:
    - capital productivity among active tasks grows at rate γ, 0 otherwise
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- Implies Pareto distributed k:

$$Pr\left(k_0 e^{\gamma\tau} \geq k\right) = Pr\left(\tau \geq \gamma^{-1} \log \frac{k}{k_0}\right) = e^{-\frac{\mu}{\gamma} \log \frac{k}{k_0}} = \left(\frac{k}{k_0}\right)^{-\frac{\mu}{\gamma}}$$

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- Churning in and out of use adds *random growth* that also leads to Pareto pdf

Comment: Issue with infinite measure

- In baseline CD model measure of tasks is infinite, i.e., $\mu(\mathcal{Q}) = \infty$
- Can work instead with the following truncation technology (for q_0 small enough):

$$T = \left(k\left(q\right) = Z^{-1}\left(q + q_0\right)^{\frac{1}{\alpha}} \frac{1-\alpha}{\alpha}, g\left(q\right) = A^{-1}\left(q + q_0\right)^{-2} \right)$$

Effect of technical change on the labor share

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Framework: Representative competitive firm

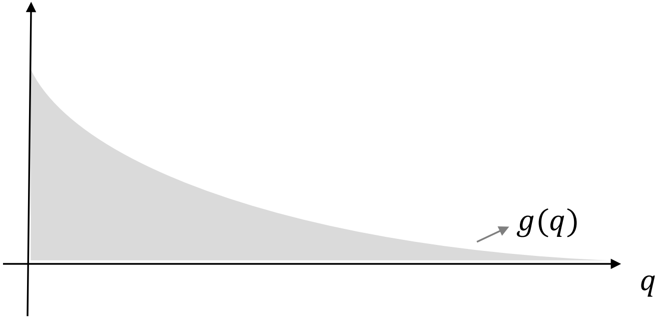
- Define prices: w (wage), r (user cost of capital), P (output)
- Profit maximization implies $P = c\left(w, r\right)$
- Cost minimization implies
  - firm chooses a cutoff q^* to minimize

$$c\left(w, r\right) := wL + rK$$

  where

$$K = \int_0^{q^*} k\left(q\right) d\mu \qquad L = \int_{q^*}^{\infty} 1 d\mu$$

Framework: Representative competitive firm



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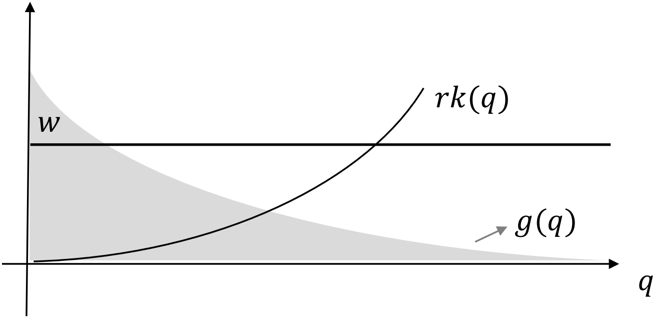

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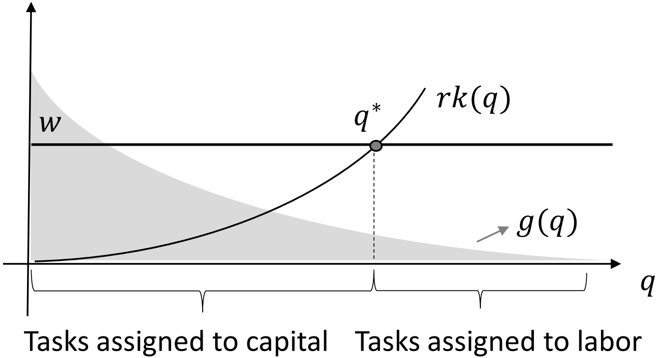

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Framework: Perturbation of technology

- Define a perturbation of technology $T = (k, g)$:

$$T_\varepsilon := (k, g) + \varepsilon(\Delta k, \Delta g)$$

- Evaluate the marginal effect ε at $\varepsilon = 0$

Framework: Labor share decomposition

- Labor share:

$$LS_{\varepsilon} := \frac{w}{P_{\varepsilon}} \frac{L}{Y}_{\varepsilon} (q_{\varepsilon}^{*}),$$

Framework: Labor share decomposition

- Labor share:

$$LS_\varepsilon := \frac{w}{P_\varepsilon}\frac{L}{Y}_\varepsilon(q^*_\varepsilon)\,,$$

- Decomposition:

$$\frac{d\log LS_\varepsilon}{d\varepsilon}|_{\varepsilon=0} = \underbrace{\frac{d\log \frac{L}{Y}_\varepsilon(q^*_\varepsilon)}{d\varepsilon}|_{\varepsilon=0}}_{\text{displacement effect DE}} + \underbrace{-\frac{d\log P_\varepsilon}{d\varepsilon}|_{\varepsilon=0}}_{\text{productivity effect PE}}$$

where

$$PE = \underbrace{-\frac{1}{P}\left(w\frac{d\frac{L}{Y}(q^*)}{dq^*} + r\frac{d\frac{K}{Y}(q^*)}{dq^*}\right)\frac{dq^*_\varepsilon}{d\varepsilon}|_{\varepsilon=0} - \frac{1}{P}\int_0^{q^*}\frac{\Delta g(q)}{g(q)}k(q)\,d\mu}_{\text{automation}}$$

$$+ \underbrace{\frac{1}{P}\int_0^{q^*} r\Delta k(q)\,d\mu}_{\text{direct technical change effect}}\,,$$

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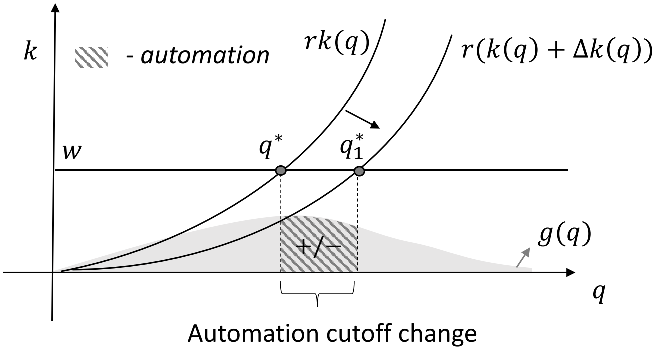
Case 1: Δk perturbation



Effect of Δk perturbation

Proposition

Δk-biased capital-augmenting technical progress changes the labor share by

$$\frac{d\log LS_{\varepsilon}}{d\varepsilon}|_{\varepsilon=0} = \underbrace{\frac{d\log \frac{L}{Y}_{\varepsilon}\left(q_{\varepsilon}^{*}\right)}{d\varepsilon}|_{\varepsilon=0}}_{\text{displacement effect DE}} + \underbrace{-\frac{d\log P_{\varepsilon}}{d\varepsilon}|_{\varepsilon=0}}_{\text{productivity effect PE}} \quad \text{(a.e.)}$$

where

$$DE = -h\left(q^{*}\right)\frac{\Delta k\left(q^{*}\right)}{k'\left(q^{*}\right)}$$

$$PE = \frac{1}{P}\int_{0}^{q^{*}} r\Delta k\left(q\right)d\mu = h\left(q^{*}\right)LS\frac{\int_{0}^{q^{*}}\Delta k\left(q\right)g\left(q\right)dv}{k\left(q^{*}\right)g\left(q^{*}\right)}$$

and where q^{*} is the initial automation cutoff such that at that cutoff point $k\left(q\right)$ is strictly increasing and differentiable, $S\left(q^{*}\right) = \int_{q^{*}}^{\infty}d\mu$ is the survival function, and $h\left(q\right) := -\frac{dS(q)}{dq} = \frac{g(q)}{S(q)}$ is the hazard rate.

Effect of Δk perturbation

Consider marginal progress: $\Delta k(q^*) > 0$, otherwise $\Delta k(q) = 0$.

Proposition

Δk-biased capital-augmenting technical progress changes the labor share by

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$$DE = -h\left(q^*\right)\frac{\Delta k\left(q^*\right)}{k'\left(q^*\right)}$$

$$PE = 0$$

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Effect of Δk perturbation

Consider fully diffused progress: $\Delta k(q) = k(q)$.

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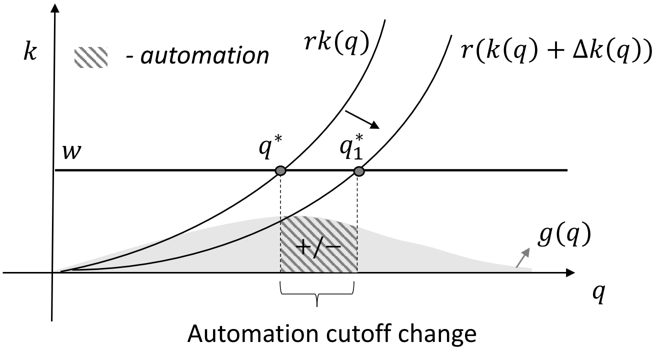
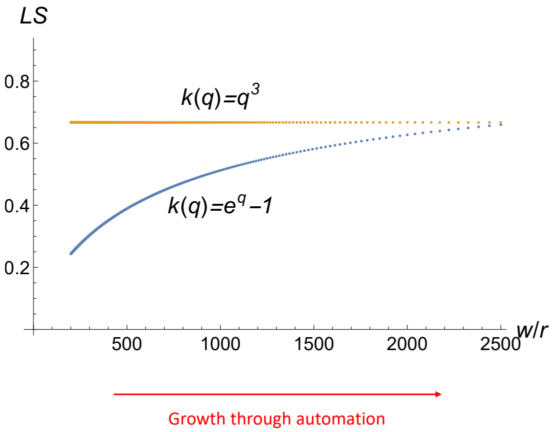
Fully diffused Δk perturbation

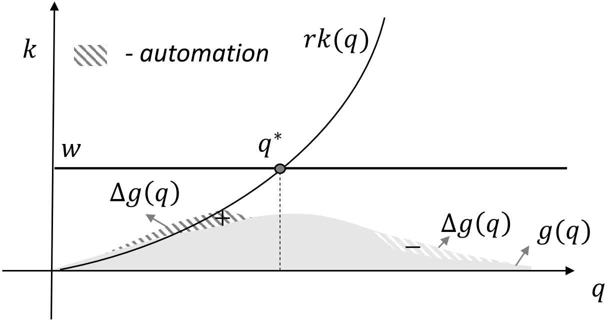


Numerical example

$$T = \left(k\,(q) = ..., g(q) = (q+.1)^{-2}\right)$$

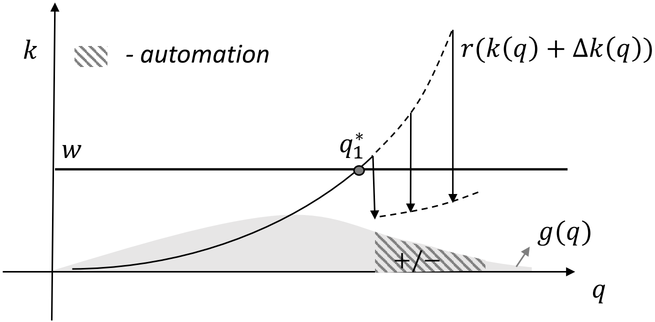


Case 2: Δg perturbation



Automation cutoff change

Equivalent to scattered jump in Δk under the automation cutoff:



Effect of Δg perturbation

Intuition: Decline in P augments income of both factors in proportion to their share.

Proposition

Δg-biased complexity reducing technical progress changes the labor share by

$$\frac{d\log LS_{\varepsilon}}{d\varepsilon}|_{\varepsilon=0} = \underbrace{\frac{d\log \frac{L}{Y}_{\varepsilon}\left(q_{\varepsilon}^{*}\right)}{d\varepsilon}|_{\varepsilon=0}}_{\text{displacement effect } DE} + \underbrace{-\frac{d\log P_{\varepsilon}}{d\varepsilon}|_{\varepsilon=0}}_{\text{productivity effect } PE} \quad \text{(a.e.)}$$

where

$$DE = \frac{d\log S\left(q_{\varepsilon}^{*}\right)}{d\varepsilon}|_{\varepsilon=0} = -h\left(q^{*}\right)\frac{\int_{0}^{q^{*}}\frac{w}{r}\frac{\Delta g(q)}{g(q)}d\mu}{g\left(q^{*}\right)k\left(q^{*}\right)}$$

$$PE = \frac{1}{P}\int_{0}^{q^{*}}\frac{\Delta g\left(q\right)}{g\left(q\right)}\left(w-rk\left(q\right)\right)d\mu = h\left(q^{*}\right)LS\left(\frac{\int_{0}^{q^{*}}\frac{\Delta g(q)}{g(q)}\left(\frac{w}{r}-k\left(q\right)\right)d\mu}{g\left(q^{*}\right)k\left(q^{*}\right)}\right)$$

and where q^{*} is the initial automation cutoff such that at that cutoff point $k\left(q\right)$ is strictly increasing and differentiable, $S\left(q^{*}\right)=\int_{q^{*}}^{\infty}d\mu$ is the survival function, and $h\left(q\right):=-\frac{dS(q)}{dq}=\frac{g(q)}{S(q)}$ is the hazard rate.

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where

$$DE + PE = -\frac{h\left(q^{*}\right)}{g\left(q^{*}\right)k\left(q^{*}\right)}\left(\int_{0}^{q^{*}} \frac{\Delta g\left(q\right)}{g\left(q\right)}\left(\frac{w}{r}\left(1-LS\right)+k\left(q\right)LS\right)d\mu\right) < 0$$

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Key lessons

- Technical progress is generally labor-share displacing if
  - Δk-change is “complexity biased” (maximizes marginal effect)
    - small for k low
    - high for k high
  - or involves Δg progress (software/digitization)

Model of IT in automation

firm

Extended setup

- Capital $\Rightarrow$ machines that are task q specific

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- Production of a machine s.t. zero profits $\Rightarrow$ endogenizes the $k(q)$ schedule:

$$Pk\left(q\right) := \min_{\{\mathcal{Q}_k, \mathcal{Q}_l\}} \left\{ r \int_{\hat{q} \in \mathcal{Q}_k} k\left(\hat{q}\right) \tilde{g}_q\left(\hat{q}\right) dv + w \int_{\hat{q} \in \mathcal{Q}_l} \tilde{g}_q\left(\hat{q}\right) dv \right\}.$$

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- Technology is $\tilde{T}_q = \{\tilde{g}_q\}$, and in goods producing sector $T = \{g\}$

Assumption

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$\tilde{g}_q(\hat{q}) = \lambda(q)\, g(\hat{q})$, *where* $g(\hat{q}) = A^{-1}\hat{q}^{-2}$ *and* $\lambda(q) = Z^{-1} q^{\frac{1}{\alpha}}$.

Aggregation in extended model

Proposition

The production function in the capital q-producing sector is Cobb–Douglas of the form:

$$Y_q\left(K,L\right) = Zq^{-\frac{1}{\alpha}}A\left(Z\left(\frac{c\left(w,r\right)}{P}\right)^{-1}\frac{\alpha}{1-\alpha}K\right)^{\alpha}L^{1-\alpha},$$

and the endogenous capital requirement function is

$$k\left(q\right) = \underbrace{Z^{-1}q^{\frac{1}{\alpha}}}_{=\lambda(q)}\frac{c\left(w,r\right)}{P},$$

where $c\left(w,r\right)$ is the unit cost of production in the capital producing sector associated with the base technology $T = \{g\}$. If, in addition, $g\left(q\right) = A^{-1}q^{-2}$, the production function in the goods sector is also Cobb–Douglas and takes the form:

$$Y\left(K,L\right) = A\left(Z\left(\frac{c\left(w,r\right)}{P}\right)^{-1}\frac{\alpha}{1-\alpha}K\right)^{\alpha}L^{1-\alpha}.$$

Model of IT in automation

Definition

A breakthrough IT automation technology comprises:

1. *A task technology* $T^{IT} = \{g^{IT}\}$.
2. *An associated strictly decreasing compression function* $\kappa : \mathbb{R}_+ \to \mathbb{R}_+$, *such that* T^{IT} *used* $n \geq 0$ *times "compresses" the task load in the production of machines of type* $q \in \mathcal{Q}$ *by factor* $\kappa(n)$, *implying transformed task density is* $\tilde{g}_{q,n}(\hat{q}) = \kappa(n)\lambda(q)g(\hat{q})$. *(Units sufficiently small to justify the use of* $n \in \mathbb{R}_+$*.)*

Assumption

$\kappa(n) = \kappa_0\beta^{-1}n^{-\beta}$, *where* $0 < \beta < \alpha^{-1} - 1$ *and* $\kappa_0 > 0$ *are scalars.*

Implies a single application of IT reduces task load by β percent.

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Effect of IT on the labor-share

Effect of IT on capital requirement function

- Application of IT transforms capital requirement $k_{old}(q) = q^{\frac{1}{\alpha}}$ to

$$k_{new}(q) = \min\left\{ q^{\frac{1}{\alpha}}, \min_{n \geq 0} \kappa(n)\, q^{\frac{1}{\alpha}} + bn \right\},$$

where $b > 0$ is cost of applying IT once

Effect of IT on capital requirement function

- Application of IT transforms capital requirement $k_{old}(q) = q^{\frac{1}{\alpha}}$ to

$$k_{new}(q) = \begin{cases} q^{\frac{1}{\alpha}} & q \leq q_{\min} \\ Cq^{\frac{1}{\alpha}\frac{1}{1+\beta}} & q \geq q_{\min} \end{cases},$$

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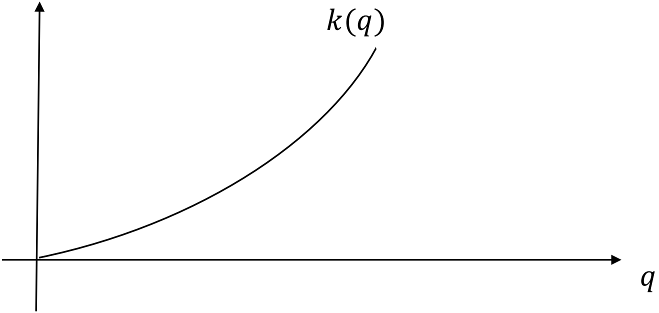
$\Rightarrow$ After dropping constants implies: $k_{old}(q) \propto q^{\frac{1}{\alpha}} \rightarrow k_{new}(q) \propto q^{\frac{1}{\alpha}\frac{1}{1+\beta}}$

Effect of IT on capital requirement function

- Optimization implies breakthrough technology applied iff $q \geq q_{\min} > 0$:



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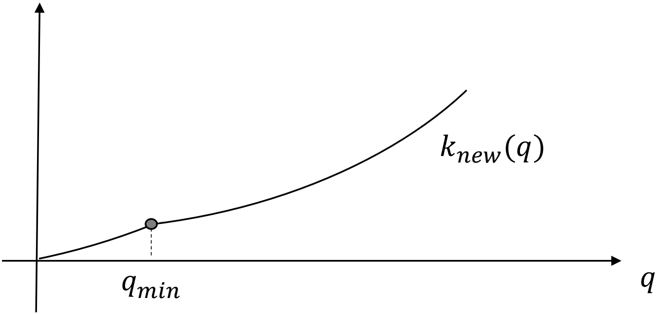

Effect of Automation Breakthroughs on Production

Proposition

The post-breakthrough labor share converges to $LS_{new} = LS_{old} - \alpha\beta$ as the economy further automates so that $q_{min}/q^* \to 0$.

Discussion

- Key properties that make IT labor-share displacing are
  - universality - applies to most tasks
  - task compression - reduces task load in proportion to the initial load
  - scalability - can be scaled up when payoff is bigger
- Interpretable under Brynjolfsson and Mcafee's (2014) view of the second machine age

skip

Discussion

Brynjolfsson McAfee (2014): automation requires *power* and *cognition*

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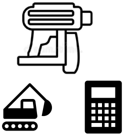
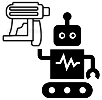
Brynjolfsson McAfee (2014): automation requires *power* and *cognition*

Tools

Mechanical Automation



IT-Powered Automation



Past: First Machine Age
Brynjolfsson McAfee (2014)

Second Machine Age

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Brynjolfsson McAfee (2014): automation requires *power* and *cognition*

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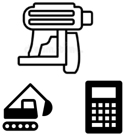

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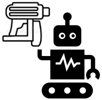

Past: First Machine Age
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Second Machine Age

$$\min\left\{\text{power}_i, \left(\text{cognition}_i^{\frac{1}{\rho_i}} + \text{mechanism}_i^{\frac{1}{\rho_i}}\right)^{\rho_i}\right\} \geq 1$$

Conclusions

- Developed theory of automation consistent with the past growth experiences
- Showed the obtained task technology can be thought of as nongeneric
- Provided a mechanism explaining labor-share displacing effect of IT-powered automation

APPENDIX

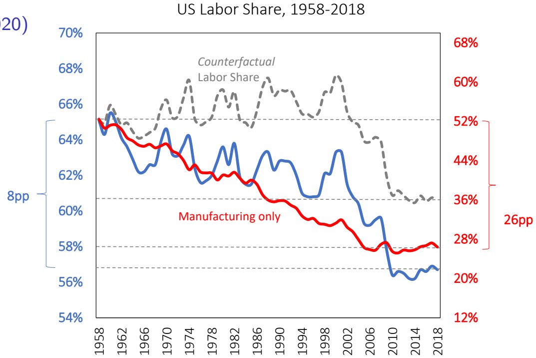
Automation a Leading Hypothesis for Labor Share Declines

- **Automation**
  Acemoglu and Restreppo (2018, 2019, 2020)
  Restreppo and Hubmer (2022)
  Graetz and Michaels (2018)
  Autor and Salomons (2018)

- **Market power**
  Autor et al. (2017)

- **Outsourcing**
  Gianonni and Mertens (2020)

- **Other**
  Basket composition: Humbner (2020)
  Taxes: Kaymak and Schott (2018)
  Measurement Gutierrez and Piton (2020), Koh et al. (2020)



Source: BEA, BLS. Labor's share in business sector's value added (excludes government, housing, farming).

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