

# Credit Enforcement Cycles

Lukasz A. Drozd<sup>1</sup>   Ricardo Serrano-Padial<sup>2</sup>

<sup>1</sup>FRB Philadelphia

<sup>2</sup>Drexel University

July, 2017

**The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.**

NEW YORK (CNNMoney, 2012) – Borrowers facing foreclosure are learning that they can stay in their homes for years (...) Among the tactics: Challenging the bank's actions, waiting to file paperwork right up until the deadline, requesting the lender dig up original paperwork or, in some extreme cases, declaring bankruptcy. Nationwide, the average time it takes to process a foreclosure has climbed to 674 days from 253 days just four years ago (...).

**Credit supply**  
(borrowing constraints/credit spreads)

**Credit supply**  
(borrowing constraints/credit spreads)



**Default decisions**  
+  
**Ability to enforce repayment /  
seize collateral**

**Credit supply**  
(borrowing constraints/credit spreads)



**Default decisions**  
+  
**Ability to enforce repayment /  
seize collateral**



**Endogenous to economic environment:**  
**1. Current and future economic conditions**  
**2. Efficacy of enforcement institutions**

**Credit supply**  
(borrowing constraints/credit spreads)



**Default decisions**  
+  
**Ability to enforce repayment /  
seize collateral**



**Literature =>** **Endogenous to economic environment:**

- 1. Current and future economic conditions**
- 2. Efficacy of enforcement institutions**

**Credit supply**  
(borrowing constraints/credit spreads)



**Default decisions**

+

**Ability to enforce repayment /  
seize collateral**



**Literature =>**

**This paper =>**

- Endogenous to economic environment:**
- 1. Current and future economic conditions**
  - 2. Efficacy of enforcement institutions**

# Motivation

## 1. Enforcement of credit contracts a time consuming process

- enforcement of contracts (OECD high-income) 532 days (WB,2017)
- resolution of insolvency in OECD high-income: 432 days (WB, 2017)



# Motivation

## 1. Enforcement of credit contracts a time consuming process

- enforcement of contracts (OECD high-income) 532 days (WB,2017)
- resolution of insolvency in OECD high-income: 432 days (WB, 2017)

## 2. Widespread defaults lead to major enforcement delays

- US (06-09): foreclosure timelines 8 ↑ 15 months (Calem, 2014)
- Italy (07-11): loan enforcement 4 ↑ 6+ years (Bank of Italy, 2014)
- Spain(07-15): commercial loan enforcement 2.5 ↑ 5 years (est.)

# Motivation

## 1. Enforcement of credit contracts a time consuming process

- enforcement of contracts (OECD high-income) 532 days (WB,2017)
- resolution of insolvency in OECD high-income: 432 days (WB, 2017)

## 2. Widespread defaults lead to major enforcement delays

- US (06-09): foreclosure timelines 8 ↑ 15 months (Calem, 2014)
- Italy (07-11): loan enforcement 4 ↑ 6+ years (Bank of Italy, 2014)
- Spain(07-15): commercial loan enforcement 2.5 ↑ 5 years (est.)

## 3. Numerous micro-level studies now document causal link to credit

- court congestion ⇒ efficacy of enforcement (Iverson, 2015)
- efficacy of enforcement ⇒ strategic default (Schianterlli, 2016)
- court congestion ⇒ credit supply (Japelli et al., 2005; Safavian and Sharma, 2007; Ponticelli, 2015; Rodano, 2016; Chan et al., 2014):

# Motivation

## 1. Enforcement of credit contracts a time consuming process

- enforcement of contracts (OECD high-income) 532 days (WB,2017)
- resolution of insolvency in OECD high-income: 432 days (WB, 2017)

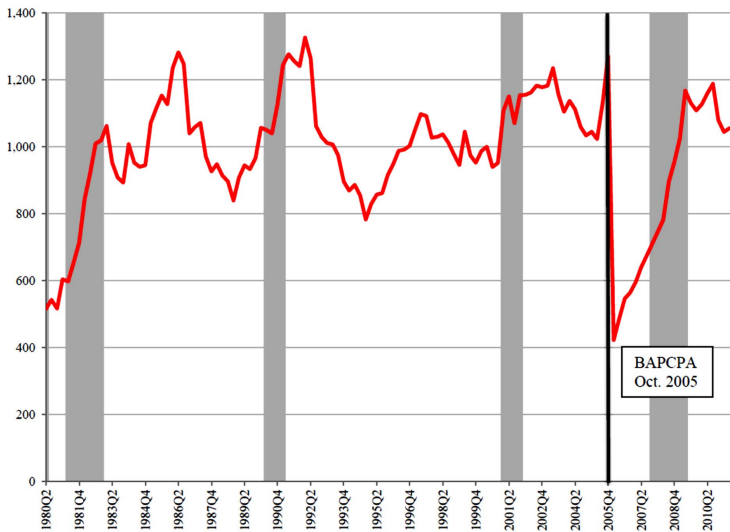
## 2. Widespread defaults lead to major enforcement delays

- US (06-09): foreclosure timelines 8 ↑ 15 months (Calem, 2014)
- Italy (07-11): loan enforcement 4 ↑ 6+ years (Bank of Italy, 2014)
- Spain(07-15): commercial loan enforcement 2.5 ↑ 5 years (est.)

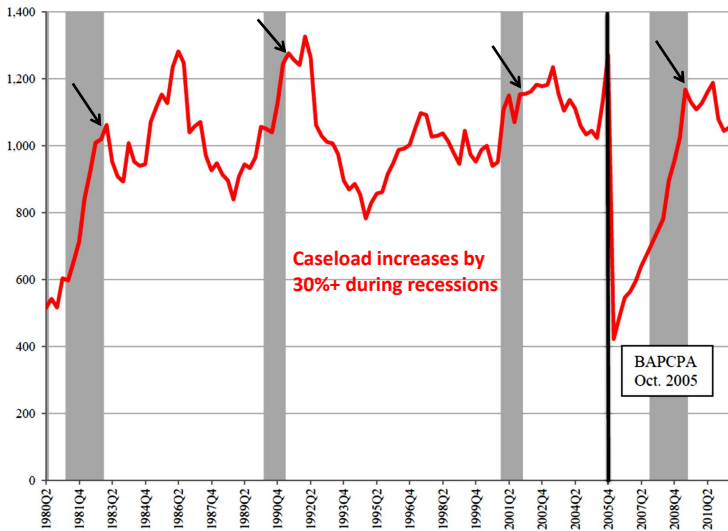
## 3. Numerous micro-level studies now document causal link to credit

- court congestion ⇒ efficacy of enforcement (Iverson, 2015)
- efficacy of enforcement ⇒ strategic default (Schianterlli, 2016)
- court congestion ⇒ credit supply (Japelli et al., 2005; Safavian and Sharma, 2007; Ponticelli, 2015; Rodano, 2016; Chan et al., 2014):

## Weighted Caseload per Judge in the U.S. (Iverson, 2015)



## Weighted Caseload per Judge in the U.S. (Iverson, 2015)



## Diff-and-Diff BAPCPA Results (Iverson, 2015)

- Business cycle 300h extra caseload per judge associated with:
  - probability of ch.11 bankruptcy filing dismissal up by 8%, conversions to ch.7 up by 11% for SME
  - increased loan losses on C&I up by  $\approx 50\%$
  - re-filing of dismissed cases doubles (recidivism)

## What We Do

- Build a model of credit supply in which enforcement is a depletable resource

## What We Do

- Build a model of credit supply in which enforcement is a depletable resource
- Use the model to inspect the mechanism:

shock  $\uparrow \Rightarrow$  enforcement  $\downarrow \Rightarrow$  default  $\uparrow \Rightarrow$  credit  $\downarrow \dots$



## What We Do

- Build a model of credit supply in which enforcement is a depletable resource
- Use the model to inspect the mechanism:

shock  $\uparrow \Rightarrow$  enforcement  $\downarrow \Rightarrow$  default  $\uparrow \Rightarrow$  credit  $\downarrow \dots$

- Contribution:
  - analysis of enforcement externality with credit & heterogeneous agents in GG-setup

## Related Literature

- Enforcement externality:
  - **Bond and Rai (2009)**, Arellano and Kocherlakota (2009), tax evasion and crime literature.
- Global Games
  - Carlsson and Van Damme (1993), Morris and Shin (1998, 2003), Frankel, Morris and Pauzner (2003), **Sakovics and Steiner (2012)**

# Model

## Model

- Two agents (a la Gale and Hellwig, 1985):
  - Entrepreneurs:
    - seek loans to finance idea/project
  - Lenders:
    - provide funds subject to zero profits

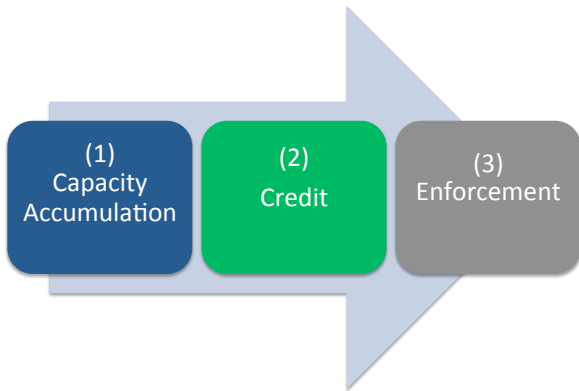
## Model

- Two agents (a la Gale and Hellwig, 1985):
  - Entrepreneurs:
    - seek loans to finance idea/project
  - Lenders:
    - provide funds subject to zero profits
- Debt defaultable; seizing collateral requires enforcement that is capacity constrained

## Model

- Two agents (a la Gale and Hellwig, 1985):
  - Entrepreneurs:
    - seek loans to finance idea/project
  - Lenders:
    - provide funds subject to zero profits
- Debt defaultable; seizing collateral requires enforcement that is capacity constrained
- Enforcement capacity accumulated ex- ante by a planner

## Timing



# Enforcement



## Entrepreneurs

- Measure one of risk-neutral entrepreneurs with loan  $b$  from lender(s)
  - invest  $y + b$  and receive  $(y + b)w$ ,  $w \in [0, \infty)$  is private info,  $w \sim F$   
 $F$  unrestricted, but in presentation single-peaked (log-normal, Pareto)

## Entrepreneurs

- Measure one of risk-neutral entrepreneurs with loan  $b$  from lender(s)
  - invest  $y + b$  and receive  $(y + b)w$ ,  $w \in [0, \infty)$  is private info,  $w \sim F$   
 $F$  unrestricted, but in presentation single-peaked (log-normal, Pareto)
- 2. Simultaneously decide whether to repay  $\bar{b} \equiv \bar{w}(y + b)$  or default
  - if default and
    - face enforcement, lose the project and get 0
    - not enforced, get a share  $\gamma$  of liquidation value  $\mu(y + b)w$

## Entrepreneurs

- Measure one of risk-neutral entrepreneurs with loan  $b$  from lender(s)
  - invest  $y + b$  and receive  $(y + b)w$ ,  $w \in [0, \infty)$  is private info,  $w \sim F$   
 $F$  unrestricted, but in presentation single-peaked (log-normal, Pareto)
- 2. Simultaneously decide whether to repay  $\bar{b} \equiv \bar{w}(y + b)$  or default
  - if default and
    - face enforcement, lose the project and get 0
    - not enforced, get a share  $\gamma$  of liquidation value  $\mu(y + b)w$

### Lemma

*Entrepreneurs default iff  $E(P) \geq \theta_{\bar{w}}$ , where  $\theta_{\bar{w}}(w) := 1 - \frac{1}{\mu\gamma} \left(1 - \frac{\bar{w}}{w}\right)$ .*

## Enforcement Technology

- Enforcement of defaulted loans is limited by capacity fixed capacity  $X$ :  
default rate  $\psi \leq$  enforcement capacity ( $X$ )  $\Rightarrow P \leq X/\psi$

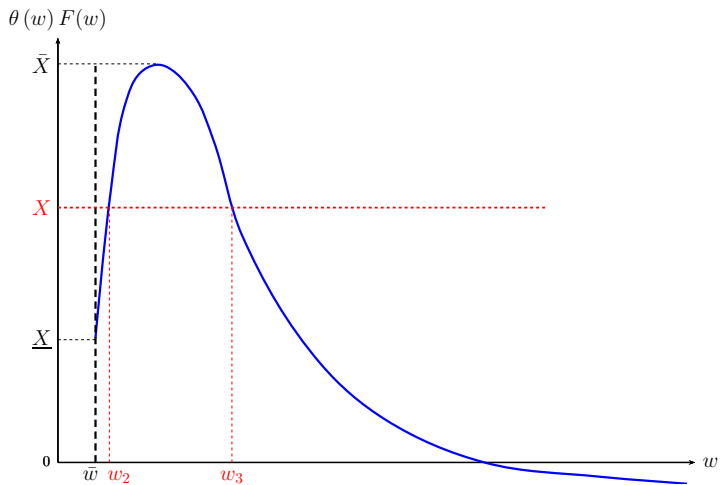
## Common Knowledge-Equilibrium

- Higher  $\psi$  lower  $P \Rightarrow$  Strategic complementarities  $\Rightarrow$  Multiple equilibria

## Common Knowledge-Equilibrium

- Higher  $\psi$  lower  $P \Rightarrow$  Strategic complementarities  $\Rightarrow$  Multiple equilibria
  - let  $\hat{w}$  be threshold type indifferent between defaulting or not
  - observe default rate is  $\psi = F(\hat{w})$
  - for equilibrium must have:  $\theta(\hat{w}) = P = X/F(\hat{w}) \rightarrow \theta(\hat{w})F(\hat{w}) = X$

## Multiplicity under Common-Knowledge



## Not a Satisfactory Equilibrium Concept?



## Not a Satisfactory Equilibrium Concept?

- Multiplicity relies on common knowledge of  $\psi$  (higher order beliefs)

## Not a Satisfactory Equilibrium Concept?

- Multiplicity relies on common knowledge of  $\psi$  (higher order beliefs)
- **Small** uncertainty regarding  $X$  can eliminate multiplicity
  - creates strategic uncertainty ( $\psi$  no longer common knowledge)
  - strategic uncertainty tampers coordination  $\Rightarrow$  uniqueness
- GG-Equilibrium: agents receive a noisy signal:  $x = X + \nu\eta$   
 $\nu > 0$  scale factor,  $\eta \in [-1/2, 1/2]$  i.i.d. with distribution  $H$

GG intuition

## GG-Equilibrium

### Proposition (*uniqueness*)

*The enforcement game has a unique limit equilibrium characterized by a (weakly) decreasing threshold  $k(w)$  on signal  $x$  such that:*

- *if  $x \geq k(w)$ , agents choose to repay ( $a = 1$ )*
- *if  $x < k(w)$ , agents choose to default ( $a = 0$ )*

solution

## GG-Equilibrium

### Proposition (*uniqueness*)

The enforcement game has a unique limit equilibrium characterized by a (weakly) decreasing threshold  $k(w)$  on signal  $x$  such that:

- if  $x \geq k(w)$ , agents choose to repay ( $a = 1$ )
- if  $x < k(w)$ , agents choose to default ( $a = 0$ )

- **Equilibrium fully characterized by indifference conditions:**

$$\lim_{\nu \rightarrow 0} \mathbb{E}[P|x = k(w)] = \theta_{\bar{w}}(w) \quad \forall w \text{ s.t. } \theta_{\bar{w}}(w) \in [0, 1]$$

solution

## GG-Equilibrium Strategy

### Proposition (*equilibrium strategy*)

In the limit  $k(w)$  is given by:

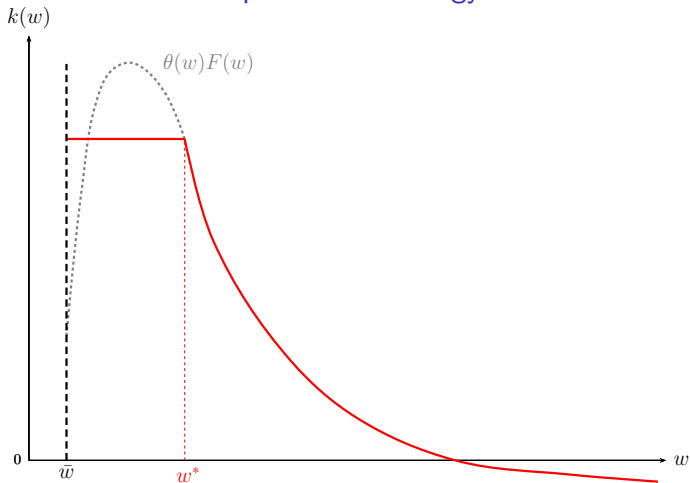
$$k(w) = \begin{cases} \theta(w^*)F(w^*) & \text{for all } w \in (\bar{w}, w^*] \\ \theta(w)F(w) & \text{for all } w > w^* \end{cases}$$

where  $w^* \geq \bar{w}$  and  $w^*$  has two possible values:

- 1 If  $\bar{w} \geq w_{\max}$ ,  $w^* = \bar{w}$
- 2 If  $\bar{w} < w_{\max}$ ,  $w^*$  is the unique solution to

$$\theta(w^*)F(w^*) (1 - \log \theta(w^*)) - F(\bar{w}) = \int_{\bar{w}}^{w^*} \theta(w) dF(w)$$

## GG-Equilibrium Strategy



$$\theta(w^*)F(w^*) (1 - \log \theta(w^*)) - F(\bar{w}) = \int_{\bar{w}}^{w^*} \theta(w)f(w)dw$$

## Intuition Behind Clustering

① Consider two types:

- $m_h = 60\%$  have  $\theta_h$
- $m_l = 20\%$  have  $\theta_l < \theta_h$  (but close enough)

## Intuition Behind Clustering

① Consider two types:

- $m_h = 60\%$  have  $\theta_h$
- $m_l = 20\%$  have  $\theta_l < \theta_h$  (but close enough)

② What makes  $k_l \searrow k_h$ ?



## Intuition Behind Clustering

① Consider two types:

- $m_h = 60\%$  have  $\theta_h$
- $m_l = 20\%$  have  $\theta_l < \theta_h$  (but close enough)

② What makes  $k_l \searrow k_h$ ?

## Intuition Behind Clustering

① Consider two types:

- $m_h = 60\%$  have  $\theta_h$
- $m_l = 20\%$  have  $\theta_l < \theta_h$  (but close enough)

② What makes  $k_l \searrow k_h$ ?

- When  $h$  receives  $k_h$ , she thinks  $\psi = \frac{1}{2}m_l = 30\%$
- When  $l$  receives  $k_l > k_h$ , she thinks  $\psi = \frac{1}{2}m_l + m_h/2 = 70\%$

## Intuition Behind Clustering

① Consider two types:

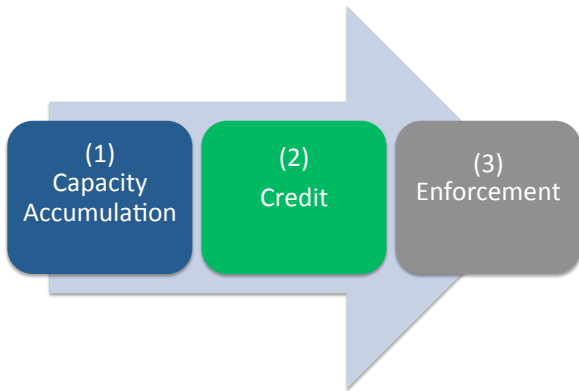
- $m_h = 60\%$  have  $\theta_h$
- $m_l = 20\%$  have  $\theta_l < \theta_h$  (but close enough)

② What makes  $k_l \searrow k_h$ ?

- When  $h$  receives  $k_h$ , she thinks  $\psi = \frac{1}{2}m_l = 30\%$
- When  $l$  receives  $k_l > k_h$ , she thinks  $\psi = \frac{1}{2}m_l + m_h/2 = 70\% \Rightarrow P_l < P_h$

# Endogenous Credit

## Timing



## Endogenous Credit Setup

- Lenders issue credit contracts  $(b, \bar{w})$ 
  - bertrand competition: loan maximizes agents' payoffs s.t. zero profits.

## Endogenous Credit Setup

- Lenders issue credit contracts  $(b, \bar{w})$ 
  - bertrand competition: loan maximizes agents' payoffs s.t. zero profits.

Optimization Problems

## Comparative Statics

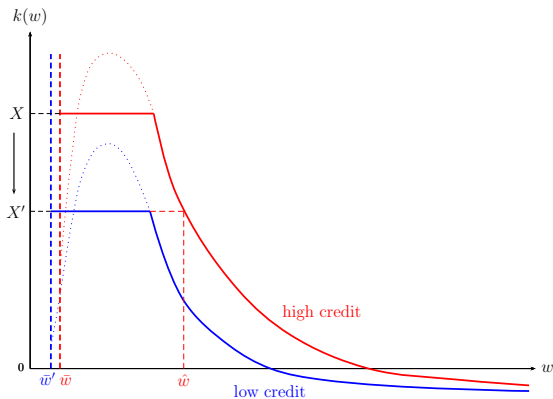
- Shock tangles measure  $s$  of enforcement capacity:  $X' = X - s$ 
  - equivalent to an exogenous shift in distribution of returns



# Comparative Statics

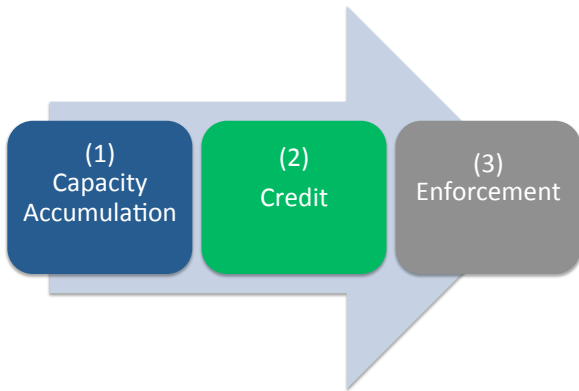
## Proposition

If  $X \geq k(w^*)$  at the optimal contract then  $b$  and  $\bar{w}$  increase (decrease) with  $X$ .



# Calibrated Numerical Example

## Timing



## Endogenous Enforcement Setup

- Planner chooses enforcement capacity  $X$  at a cost  $c(X)$ 
  - maximizes expected agent payoffs net of capacity costs

## Calibrated Numerical Example

1. Calibrate to U.S. data (large shock, small shock)

Example

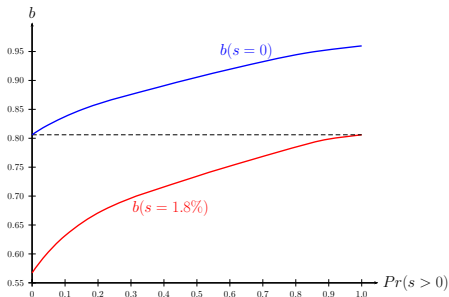
2. Show:

- shock that lowers  $X$  (or affects  $F$ ) leads to a severe credit crunch
- propagation even if there is ample capacity to accommodate the shock.

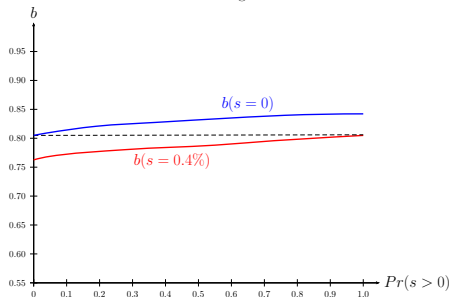
## Transmission of Calibrated Shocks

- Binary shock  $s$  reduces capacity to  $X = X_o - s$  before credit market opens ( $\uparrow$  default rate on existing loans  $\Rightarrow$   $\downarrow$  residual capacity for new loans)

Crisis Shock



Average Shock



- Credit enforcement cycle*: (potential dynamics)

$$\uparrow \psi \Rightarrow \downarrow X \Rightarrow \downarrow b \Rightarrow \downarrow \psi \dots$$

## Conclusions

- Framework to study endogenous enforcement
  - Focus on link between enforcement institutions and credit fluctuations
  - Approach applicable to other default spillovers (e.g., endogenous collateral values)
- Developed method to deal with equilibrium indeterminacy under heterogeneity, suitable for quantitative work
  - Economic fragility despite substantial heterogeneity

NEW YORK (CNNMoney, 2012) – Borrowers facing foreclosure are learning that they can stay in their homes for years (...) Among the tactics: Challenging the bank's actions, waiting to file paperwork right up until the deadline, requesting the lender dig up original paperwork or, in some extreme cases, declaring bankruptcy. Nationwide, the average time it takes to process a foreclosure has climbed to 674 days from 253 days just four years ago (...).



## BACKUP SLIDES

## Related Literature

### 1 Law and Finance: enforcement and credit

- La Porta et al (1990), Djankov et al (2007, 2008)
- Our paper: (endogenous) enforcement and credit **volatility**

### 2 Enforcement externalities:

- Bond and Rai (2009), Arellano and Kotcherlakota (2009), tax evasion and crime literature.

### 3 Global Games

- Carlsson and Van Damme (1993), Morris and Shin (1998, 2003), Frankel, Morris and Pauzner (2003), **Sakovics and Steiner (2012)**

## Lender and Planner Problems

- Lenders: ( $a = 1$  denotes repay)

$$V(X) := \max_{b, \bar{w}, P} \left[ \int_{\{w:a=1\}} (y+b)(w-\bar{w})dF + (1-P) \int_{\{w:a=0\}} \gamma\mu(y+b)wdF \right]$$

$$\text{s.t. } P \leq \min \left\{ \frac{X}{\psi}, 1 \right\} \text{ and}$$

$$b \leq \int_{\{w:a=1\}} (y+b)\bar{w}dF + P \int_{\{w:a=0\}} \mu(y+b)wdF \\ + (1-P) \int_{\{w:a=0\}} (1-\gamma)\mu(y+b)wdF$$

go back

## Intuition For Clustering

- ① Thresholds must be decreasing in  $w$  (lower propensity to default  $\Rightarrow$  lower  $k$ )
- ②  $k(\cdot)$  strictly decreasing  $\Rightarrow$  As  $\nu \rightarrow 0$  agent believes  $\psi = F(w)$  at  $x = k(w)$

## Intuition For Clustering

- ① Thresholds must be decreasing in  $w$  (lower propensity to default  $\Rightarrow$  lower  $k$ )
- ②  $k(\cdot)$  strictly decreasing  $\Rightarrow$  As  $\nu \rightarrow 0$  agent believes  $\psi = F(w)$  at  $x = k(w)$
- ③  $k(w) \searrow k(w')$  for  $w < w' < w_{max}$

## Intuition For Clustering

- ① Thresholds must be decreasing in  $w$  (lower propensity to default  $\Rightarrow$  lower  $k$ )
- ②  $k(\cdot)$  strictly decreasing  $\Rightarrow$  As  $\nu \rightarrow 0$  agent believes  $\psi = F(w)$  at  $x = k(w)$
- ③  $k(w) \searrow k(w')$  for  $w < w' < w_{max}$   
 $\theta \downarrow$  but  $\psi \uparrow\uparrow$  (mass concentrated to the left of  $w_{max}$ )

## Intuition For Clustering

- 1 Thresholds must be decreasing in  $w$  (lower propensity to default  $\Rightarrow$  lower  $k$ )
- 2  $k(\cdot)$  strictly decreasing  $\Rightarrow$  As  $\nu \rightarrow 0$  agent believes  $\psi = F(w)$  at  $x = k(w)$
- 3  $k(w) \searrow k(w')$  for  $w < w' < w_{max}$

$\theta \downarrow$  but  $\psi \uparrow\uparrow$  (mass concentrated to the left of  $w_{max}$ )

$\Downarrow$

$\mathbb{E}(P|x = k) \Downarrow\downarrow < \theta \downarrow$

## Intuition For Clustering

- 1 Thresholds must be decreasing in  $w$  (lower propensity to default  $\Rightarrow$  lower  $k$ )
- 2  $k(\cdot)$  strictly decreasing  $\Rightarrow$  As  $\nu \rightarrow 0$  agent believes  $\psi = F(w)$  at  $x = k(w)$
- 3  $k(w) \searrow k(w')$  for  $w < w' < w_{max}$

$\theta \downarrow$  but  $\psi \uparrow\uparrow$  (mass concentrated to the left of  $w_{max}$ )

$\Downarrow$

$\mathbb{E}(P|x = k) \Downarrow\Downarrow < \theta \downarrow$

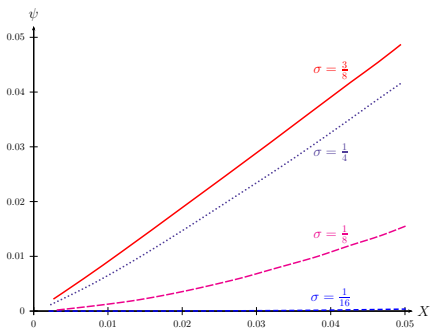
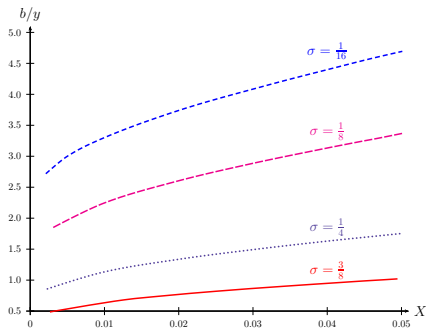
$\Downarrow$

Snowballing: agent with  $w'$  wants to default at higher signals,  $k(w') \uparrow$



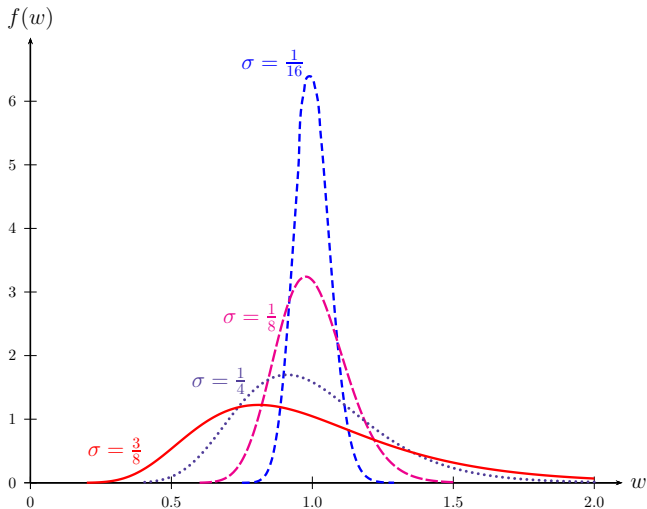
## The Impact of Heterogeneity

$F = \text{Lognormal}$ ,  $Ew = 1.02$ ,  $\mu = 0.88$  (Bernanke et al; 1999),  $y = 1$ ,  $\gamma = 0.25$



concentrated returns  $\Rightarrow$  negligible insolvency rate, cluster too large (60%)

## The Impact of Heterogeneity



[go back](#)

## Optimal capacity: Borrowing constraint

$$\sigma = 3/8, c(X) = 0.088X$$

Statistic	Value	Target
$b/y$	0.8	0.5 - 1
$\psi$	2.3%	2.3%
ROE	3.4%	
$c(X_o)/ROE$	0.06	
Utilization ( $\frac{\psi}{X}$ )	95%	
Cluster	1.7%	
% Strategic	6.9%	

Borrowing constraint:  $b$  is 20% lower than without externality ( $\gamma = 0$ )

go back

## Uniqueness

- If agent receives  $x$  then  $X \in [x - \nu/2, x + \nu/2]$  and other agents' signals are in  $[x - \nu, x + \nu]$
- Proof sketch:
  - 1 There is a lowest and highest equilibria, both in threshold strategies (resp.  $\underline{k}$  and  $\bar{k}$ )

## Uniqueness

- If agent receives  $x$  then  $X \in [x - \nu/2, x + \nu/2]$  and other agents' signals are in  $[x - \nu, x + \nu]$
- Proof sketch:
  - ① There is a lowest and highest equilibria, both in threshold strategies (resp.  $\underline{k}$  and  $\bar{k}$ )
  - ② Assume that  $\bar{k}(w) = \underline{k}(w) + \Delta$  for all  $w$
  - ③ Agent beliefs when  $x = \bar{k}(w)$  under  $\bar{k}$ , relative to  $x = \underline{k}(w)$  under  $\underline{k}$ :
    - $X$  went up by  $\Delta$  on average

## Uniqueness

- If agent receives  $x$  then  $X \in [x - \nu/2, x + \nu/2]$  and other agents' signals are in  $[x - \nu, x + \nu]$
- Proof sketch:
  - 1 There is a lowest and highest equilibria, both in threshold strategies (resp.  $\underline{k}$  and  $\bar{k}$ )
  - 2 Assume that  $\bar{k}(w) = \underline{k}(w) + \Delta$  for all  $w$
  - 3 Agent beliefs when  $x = \bar{k}(w)$  under  $\bar{k}$ , relative to  $x = \underline{k}(w)$  under  $\underline{k}$ :
    - $X$  went up by  $\Delta$  on average
    - strategic beliefs about  $\psi$  are the same! (translation invariant)

## Uniqueness

- If agent receives  $x$  then  $X \in [x - \nu/2, x + \nu/2]$  and other agents' signals are in  $[x - \nu, x + \nu]$
- Proof sketch:
  - 1 There is a lowest and highest equilibria, both in threshold strategies (resp.  $\underline{k}$  and  $\bar{k}$ )
  - 2 Assume that  $\bar{k}(w) = \underline{k}(w) + \Delta$  for all  $w$
  - 3 Agent believes when  $x = \bar{k}(w)$  under  $\bar{k}$ , relative to  $x = \underline{k}(w)$  under  $\underline{k}$ :
    - $X$  went up by  $\Delta$  on average
    - strategic beliefs about  $\psi$  are the same! (translation invariant)
  - 4 Either  $\underline{k}$  or  $\bar{k}$  violates indifference conditions
  - 5 Argument generalizes to unequal  $\Delta \Rightarrow$  eq. uniqueness

NEW YORK (CNNMoney, 2012) – Borrowers facing foreclosure are learning that they can stay in their homes for years (...) Among the tactics: Challenging the bank's actions, waiting to file paperwork right up until the deadline, requesting the lender dig up original paperwork or, in some extreme cases, declaring bankruptcy. Nationwide, the average time it takes to process a foreclosure has climbed to 674 days from 253 days just four years ago (...).



## BACKUP SLIDES

# Enforcement Delays

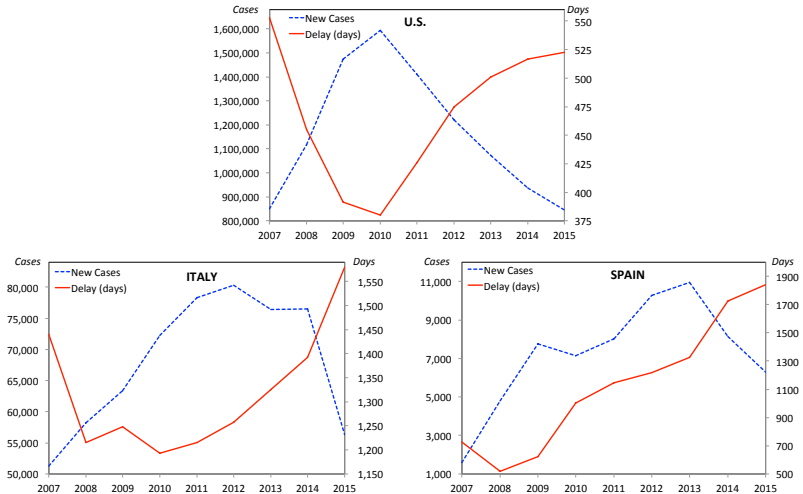


Figure : Default Cases (left axis) and Enforcement Delay (right axis)

## How We Solve the Game

- For  $\lim_{\nu \rightarrow 0} k(w)$  equilibrium satisfies:

$$\mathbb{E}[P|x = k(w)] = \theta(w) \quad \forall w \text{ s.t. } \theta(w) \in (0, 1)$$

- Beliefs about  $X$  approximate true  $X$  as  $\nu \rightarrow 0$
- How about strategic beliefs about  $\psi$ ?

## How We Solve the Game

- For  $\lim_{\nu \rightarrow 0} k(w)$  equilibrium satisfies:

$$\mathbb{E}[P|x = k(w)] = \theta(w) \quad \forall w \text{ s.t. } \theta(w) \in (0, 1)$$

- Beliefs about  $X$  approximate true  $X$  as  $\nu \rightarrow 0$
- How about strategic beliefs about  $\psi$ ?
- *Laplacian Property*: if  $k(w) = k$  for all  $w \Rightarrow \psi|x = k$  uniformly distributed
- Under heterogeneous  $k(w)$  the Laplacian property holds 'on average' (Sakovics-Steiner, 2012 )
  - Cluster of thresholds converging to the same limit  $\Rightarrow$  average the indifference conditions and replace the average belief by the uniform distribution
  - Use monotonicity of  $\theta$  to identify the cluster bounds

## The Belief Constraint (Sakovics-Steiner, 2012)

- **Equilibrium fully characterized by indifference conditions:**

$$\lim_{\nu \rightarrow 0} \mathbb{E}[P|x = k(w)] = \theta_{\bar{w}}(w) \quad \forall w \text{ s.t. } \theta_{\bar{w}}(w) \in [0, 1]$$

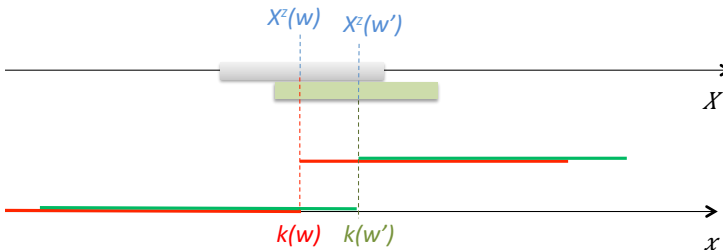
### Lemma (*belief-constraint*)

Let  $\psi(W')$  be the default rate in some measurable set  $W' \subseteq [0, \infty)$ . Then, for any  $z \in [0, 1]$ ,

$$\frac{1}{\int_{W'} f(w)dw} \int_{W'} \mathbb{P}_w(\psi(W') \leq z | x = k(w)) f(w)dw = z,$$

where  $\mathbb{P}_w(\cdot | x = k(w))$  is the probability assessment of the default rate in  $W'$  by an agent whose signal  $x$  is equal to her threshold  $k(w)$ .

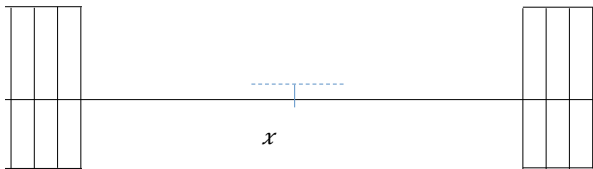
## The Belief Constraint



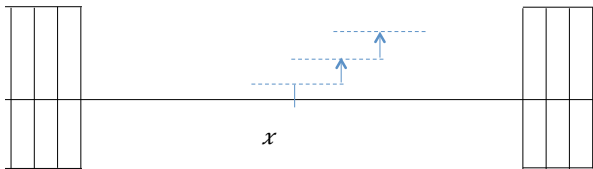
- Belief asymmetry between  $w$  and  $w'$  regarding each other actions in  $[k(w), k(w')]$  averages out

go back

## Basic Idea



## Basic Idea



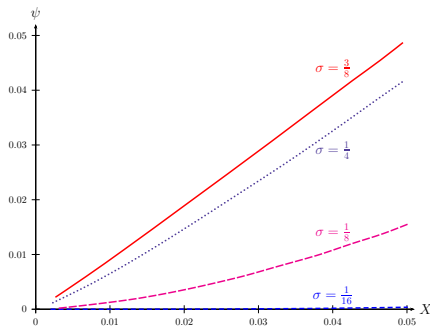
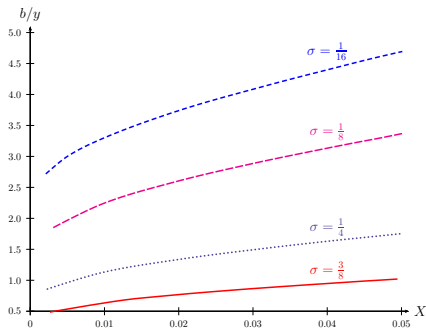
go back



# Identification

$F = \text{Lognormal}$ ,  $Ew = 1.02$ ,  $\mu = 0.88$  (Bernanke et al; 1999),  $y = 1$ ,  $\gamma = 0.25$

Return Distributions



concentrated returns  $\Rightarrow$  negligible insolvency rate, cluster too large (60%)

## Extended Intuition For Clustering

- ① Thresholds must be decreasing in  $w$  (lower propensity to default  $\Rightarrow$  lower  $k$ )
- ②  $k(\cdot)$  strictly decreasing  $\Rightarrow$  As  $\nu \rightarrow 0$  agent believes  $\psi = F(w)$  at  $x = k(w)$

## Extended Intuition For Clustering

- ① Thresholds must be decreasing in  $w$  (lower propensity to default  $\Rightarrow$  lower  $k$ )
- ②  $k(\cdot)$  strictly decreasing  $\Rightarrow$  As  $\nu \rightarrow 0$  agent believes  $\psi = F(w)$  at  $x = k(w)$
- ③  $k(w) \searrow k(w')$  for  $w < w' < w_{max}$

## Extended Intuition For Clustering

- ① Thresholds must be decreasing in  $w$  (lower propensity to default  $\Rightarrow$  lower  $k$ )
- ②  $k(\cdot)$  strictly decreasing  $\Rightarrow$  As  $\nu \rightarrow 0$  agent believes  $\psi = F(w)$  at  $x = k(w)$
- ③  $k(w) \searrow k(w')$  for  $w < w' < w_{max}$   
 $\theta \downarrow$  but  $\psi \uparrow\uparrow$  (mass concentrated to the left of  $w_{max}$ )

## Extended Intuition For Clustering

- ① Thresholds must be decreasing in  $w$  (lower propensity to default  $\Rightarrow$  lower  $k$ )
- ②  $k(\cdot)$  strictly decreasing  $\Rightarrow$  As  $\nu \rightarrow 0$  agent believes  $\psi = F(w)$  at  $x = k(w)$
- ③  $k(w) \searrow k(w')$  for  $w < w' < w_{max}$

$\theta \downarrow$  but  $\psi \uparrow\uparrow$  (mass concentrated to the left of  $w_{max}$ )

$\Downarrow$

$\mathbb{E}(P|x = k) \Downarrow\Downarrow < \theta \downarrow$

## Extended Intuition For Clustering

- 1 Thresholds must be decreasing in  $w$  (lower propensity to default  $\Rightarrow$  lower  $k$ )
- 2  $k(\cdot)$  strictly decreasing  $\Rightarrow$  As  $\nu \rightarrow 0$  agent believes  $\psi = F(w)$  at  $x = k(w)$
- 3  $k(w) \searrow k(w')$  for  $w < w' < w_{max}$

$\theta \downarrow$  but  $\psi \uparrow\uparrow$  (mass concentrated to the left of  $w_{max}$ )

$\Downarrow$

$\mathbb{E}(P|x = k) \Downarrow\Downarrow < \theta \downarrow$

$\Downarrow$

Snowballing: agent with  $w'$  wants to default at higher signals,  $k(w') \uparrow$

## Sketch of Proof of Uniqueness

- If agent receives  $x$  then  $X \in [x - \nu/2, x + \nu/2]$  and other agents' signals are in  $[x - \nu, x + \nu]$
- Proof sketch:
  - 1 There is a lowest and highest equilibria, both in threshold strategies (resp.  $\underline{k}$  and  $\bar{k}$ )

## Sketch of Proof of Uniqueness

- If agent receives  $x$  then  $X \in [x - \nu/2, x + \nu/2]$  and other agents' signals are in  $[x - \nu, x + \nu]$
- Proof sketch:
  - 1 There is a lowest and highest equilibria, both in threshold strategies (resp.  $\underline{k}$  and  $\bar{k}$ )
  - 2 Assume that  $\bar{k}(w) = \underline{k}(w) + \Delta$  for all  $w$
  - 3 Agent believes when  $x = \bar{k}(w)$  under  $\bar{k}$ , relative to  $x = \underline{k}(w)$  under  $\underline{k}$ :
    - $X$  went up by  $\Delta$  on average



## Sketch of Proof of Uniqueness

- If agent receives  $x$  then  $X \in [x - \nu/2, x + \nu/2]$  and other agents' signals are in  $[x - \nu, x + \nu]$
- Proof sketch:
  - 1 There is a lowest and highest equilibria, both in threshold strategies (resp.  $\underline{k}$  and  $\bar{k}$ )
  - 2 Assume that  $\bar{k}(w) = \underline{k}(w) + \Delta$  for all  $w$
  - 3 Agent believes when  $x = \bar{k}(w)$  under  $\bar{k}$ , relative to  $x = \underline{k}(w)$  under  $\underline{k}$ :
    - $X$  went up by  $\Delta$  on average
    - strategic beliefs about  $\psi$  are the same! (translation invariant)

## Sketch of Proof of Uniqueness

- If agent receives  $x$  then  $X \in [x - \nu/2, x + \nu/2]$  and other agents' signals are in  $[x - \nu, x + \nu]$
- Proof sketch:
  - 1 There is a lowest and highest equilibria, both in threshold strategies (resp.  $\underline{k}$  and  $\bar{k}$ )
  - 2 Assume that  $\bar{k}(w) = \underline{k}(w) + \Delta$  for all  $w$
  - 3 Agent believes when  $x = \bar{k}(w)$  under  $\bar{k}$ , relative to  $x = \underline{k}(w)$  under  $\underline{k}$ :
    - $X$  went up by  $\Delta$  on average
    - strategic beliefs about  $\psi$  are the same! (translation invariant)
  - 4 Either  $\underline{k}$  or  $\bar{k}$  violates indifference conditions
  - 5 Argument generalizes to unequal  $\Delta \Rightarrow$  eq. uniqueness