

# On the Optimality of Zero APR on Credit Cards: An Analytical Framework

Lukasz A. Drozd<sup>1</sup>    Michal Kowalik<sup>2</sup>

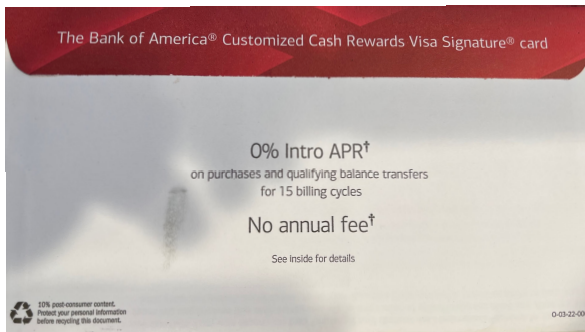
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July, 2022

## **DISCLAIMER:**

**The views expressed in this paper are solely those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia, Federal Reserve Bank of Boston or the Federal Reserve System.**

## A TYPICAL CREDIT CARD OFFER



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## DISCLOSURE SUMMARY

### †Details of Rate, Fee and Other Cost Information

Accounts will receive the initial terms of this firm offer. Other than those terms set forth under "Preselcted Offer Minimum Terms,"<sup>8</sup> Account terms are not guaranteed for any period of time. All terms, including fees and APRs for new transactions, may change in accordance with the Credit Card Agreement and applicable law based on information in your credit report, market conditions, business strategies, or for any reason. Please review all of these materials so that you are fully informed about the terms of this credit card offer.

#### Interest Rates and Interest Charges

<b>Annual Percentage Rate (APR) for Purchases</b>	<b>0%</b> Introductory APR for the first 15 Statement Closing Dates following the opening of your account.  After that, your APR will be <b>15.99% to 23.99%</b> , based on your creditworthiness when you open your account. This APR will vary with the market based on the Prime Rate.
<b>APR for Balance Transfers</b>	<b>0%</b> Introductory APR for the first 15 Statement Closing Dates following the opening of your account for transactions made within 60 days of opening your account.  After that, your APR will be <b>15.99% to 23.99%</b> , based on your creditworthiness when you open your account. This APR will vary with the market based on the Prime Rate.
<b>APR for Cash Advances</b>	<b>18.99% to 26.99%</b> , based on your creditworthiness when you open your account, for Direct Deposit and Check Cash Advances, and <b>28.99%</b> for Bank Cash Advances. See footnote <sup>1</sup> for explanation.  These APRs will vary with the market based on the Prime Rate.

**BANK OF AMERICA** 

Take advantage of your low Introductory APR<sup>†</sup> offer.

You'll get a **0% Intro APR<sup>†</sup> for 15 billing cycles for purchases, and for any balance transfers made in the first 60 days.** After the Intro APR offer ends, a Variable APR that's currently **15.99% - 23.99%** will apply. A **3% fee (min. \$10)** applies to all balance transfers.

## CONTRIBUTIONS

1. Document the impact of promos on the pricing of credit card debt
2. Develop a normative theory of how rates should be set intertemporally

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## OUTLINE

- Empirical patterns
- Basic theory of promo pricing
- Extensions (endogenous default, hidden savings, hyperbolic preferences)

## Empirical patterns

## DATA SOURCE AND DESCRIPTION

- Data collection from all bank holding companies under DFAST (Y14M)
  - excludes banks with assets below \$100 billion
- Panel of general purpose credit card accounts with all credit-related attributes
  - est. to cover 70% of all credit card accounts
  - no link to bureau records (only credit scores)
- Proprietary but replicable within FRS

## FACT 1

### 1. About a quarter of card debt is on promo; most on 0 APR for well over a year.

TABLE: Promotional debt and the duration of promotional spells.

Statistic <sup>a</sup> [in % unless otherwise noted]	2019	2018
Fraction of debt with promo rate <sup>b</sup>	22.3	22.4
Fraction of prime debt with promo rate <sup>c</sup>	27.3	27.0
Average duration of promo spell <sup>d</sup> [in months]	19.8 (15.7)	20.4 (16.5)
Average time to promo expiration <sup>d</sup> [in months]	9.6 (8.3)	8.3 (7.5)
Fraction of zero APR promos <sup>a</sup>	80.4	83.3
Fraction of promos with APR $\leq 3\%$	84.1	85.7
Fraction of promos with APR $\leq 6\%$	88.1	89.6

<sup>a</sup>We calculate each respective statistic for each month in 2018 and 2019 and then average them over each respective year. <sup>b</sup>We define debt as credit card balances that are carried over for at least one cycle. We calculate it on the account level in each month  $t$  by taking the difference between the balances in month  $t - 1$  net of payments made by the borrower in month  $t$ . <sup>c</sup>Prime debt includes accounts with prime credit score (e.g., minimum 670 credit scores on the account). <sup>d</sup>Debt-weighted, unweighted values are in the parentheses.

## FACT 2

### 2. Promo expirations imply large rate hikes for borrowers.

TABLE: APR on promotional accounts and APR promotional discounts.

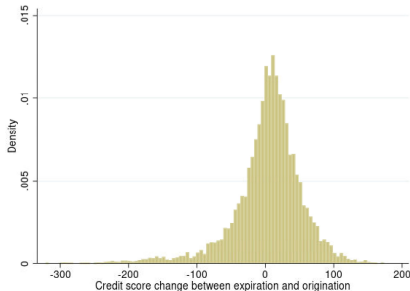
Statistic [in %]	2019	2018
Average APR hike associated with promos <sup>b</sup>	16.8	17.1
Average APR on non-promo accounts <sup>b</sup>	18.7	18.0
Average credit score on all promo accounts	727	728
Average credit score on 0 APR promo accounts	731	726
Average credit score on nonpromo accounts	696	698

Notes to previous tables apply. <sup>a</sup> Fraction of APR as posted on the accounts regardless of the amount borrowed (that is, this measure includes accounts with no debt). <sup>b</sup> Debt-weighted average. APR discount is the difference between the promotional APR on the account and the nonpromotional reset rate on the same account.

## FACT 3

### 3. Change in default risk orthogonal to rate hikes built into promo contracts.

Figure 2: Histogram of credit score changes between promo origination and expiration.



Notes: The figure plots the histogram of the changes in credit scores across promo accounts during the promo period. To calculate it, we take the average score on the account over the first 3 months after the expiration of the promotion and subtract the average score on the same account over the first 3 months after the origination of the promotion. The score change is an unweighted statistic calculated across all promo accounts throughout the sample period. Source: Federal Reserve System, Y14M.

## FACT 4

### 4. Delinquency on promo cards about average; lenders initially appear to “subsidize” aggressively prices promo cards.

TABLE: Delinquency rates on credit card debt.

Statistic [in %]	30+ dpd <sup>a</sup>	120+ dpd
<i>All promo accounts:</i>		
- 2 months after expiration of promo	9.2	5.6
- 5 months after expiration of promo	11.3	7.0
<i>0 APR accounts with 3% or less BT fee:</i>		
- 2 months after expiration of promo	9.2	5.6
- 5 months after expiration of promo	11.3	7.0
All accounts <sup>c</sup>	6.7	3.5

Notes to previous tables apply. <sup>a</sup> 30 or more days past due credit card debt that has not been written off by the lender. Delinquent credit card debt is generally written off after 180 days past due and after debt is discharged in bankruptcy court. <sup>b</sup> This category includes most aggressively priced promo accounts; that is, those with zero APR and 3 percent or less balance transfer fee <sup>c</sup>

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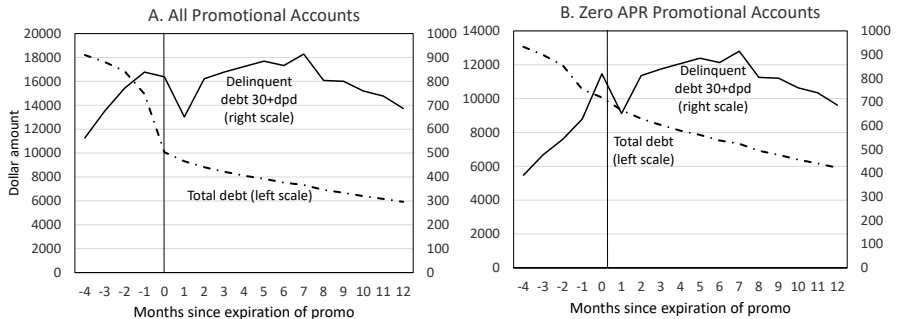
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Delinquency rate is higher... but this is due to debt paydowns after expiration (next slide)

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Figure 1: Delinquent debt and total debt at promo flag expiration.

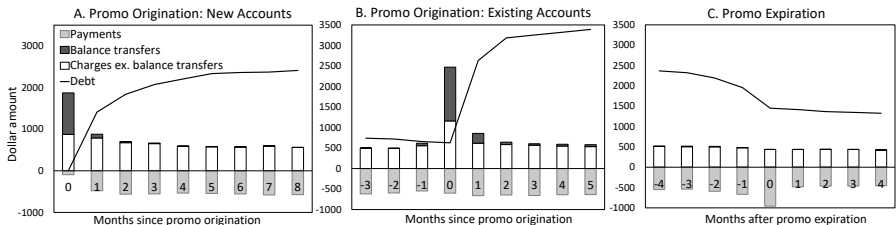


Notes: The figure plots debt and delinquent debt that is 30+ days past due and has not (yet) been written off (typically after 180 days past due or after bankruptcy discharge). The left panel includes all promotional cards and the right panel reports the same for the most aggressively discounted promotional cards (0 APR cards with 3 percent or less balance transfer fee). The pool of accounts is fixed and they come from different time periods in 2018 and 2019, all centered around the expiration of the promo period (“0” on the horizontal axis). Source: Federal Reserve System, Y14M.

## FACT 5

**5. Refinancing of card debt via promos is a prevalent phenomenon; suggests “cat and mouse” game between borrowers, lenders or incumbent lenders and new lenders.**

Figure 3: Charges and payments over the life cycle of promo cards.



Notes: The figure shows the life cycle of new promotional accounts and newly promotional existing accounts. We plot monthly charges excluding balance transfers, such as fees, purchases, cash advances (white bar), inbound balance transfers (black bar), and balance (re)payments (grey bar). Accumulated debt is the cumulation of charges, balance transfers and payments. Source: Federal Reserve System, Y14M.

## Basic theory of promo pricing

## ENVIRONMENT

- A large number of *lenders* and *consumer families*:
  - *lenders* have deep pockets and face zero cost of funds (in expectation)
  - *consumer family* = mass 1 of identical *members* who fully share risk

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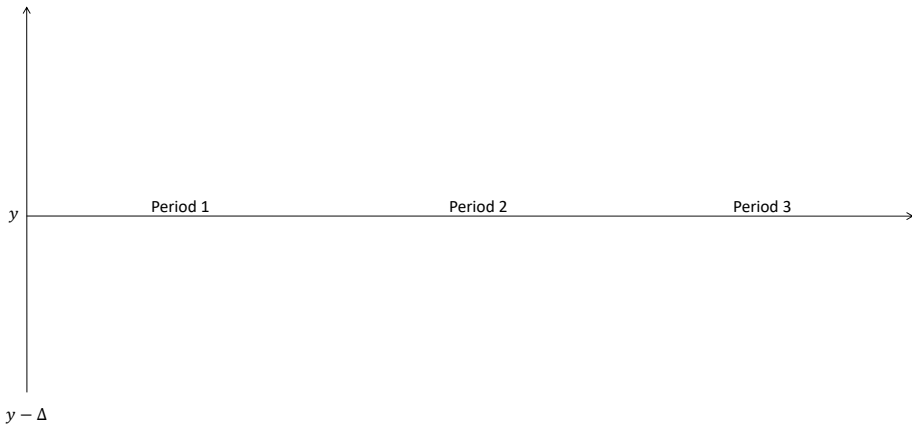
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- Members hold 1 card at a time but can refinance existing lines with other lenders
- Bertrand competition determines contract terms
  - equilibrium contracts solve “max utility s.t. zero profits”
- Focus on type-identical allocation (members make the same decisions)

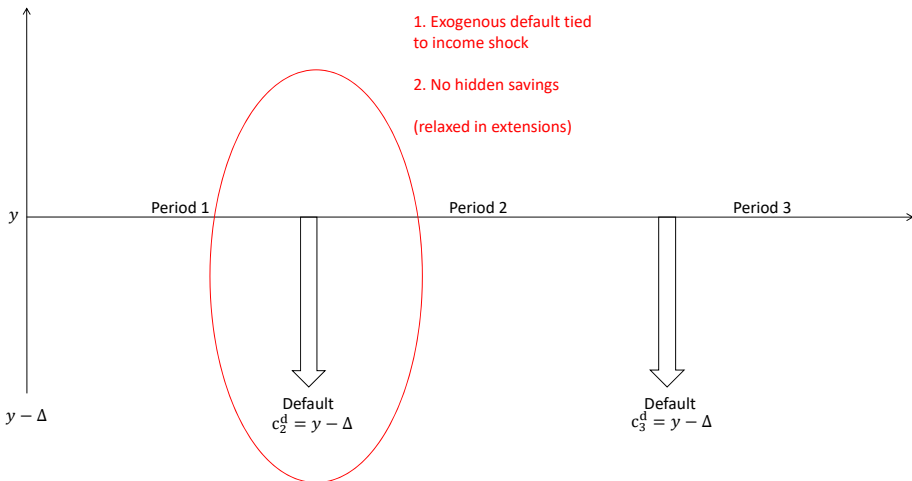
# TIMING, CONSTRAINTS AND DECISIONS

Income state of family

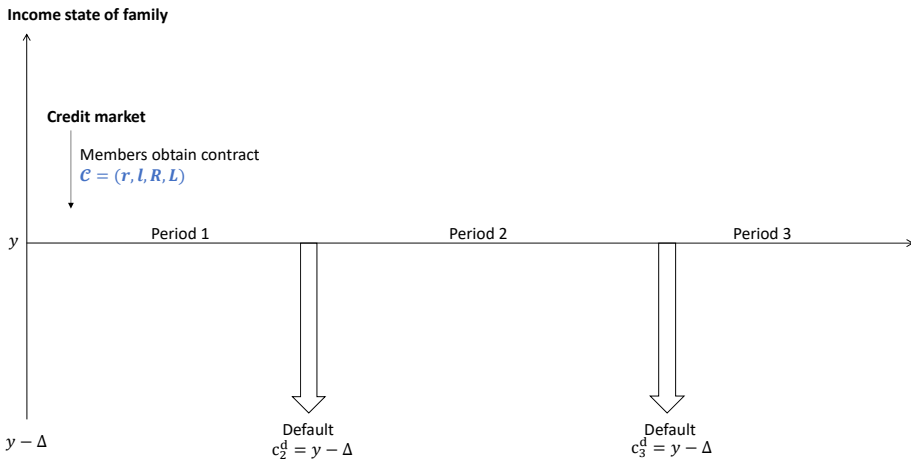


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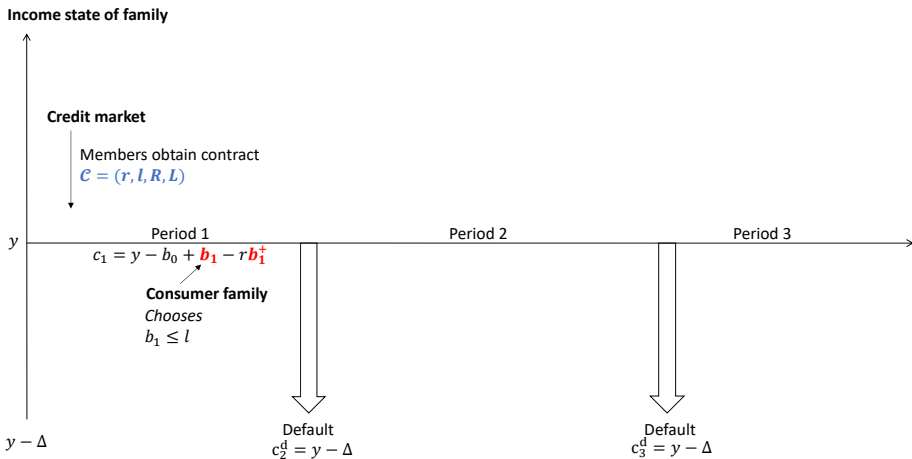
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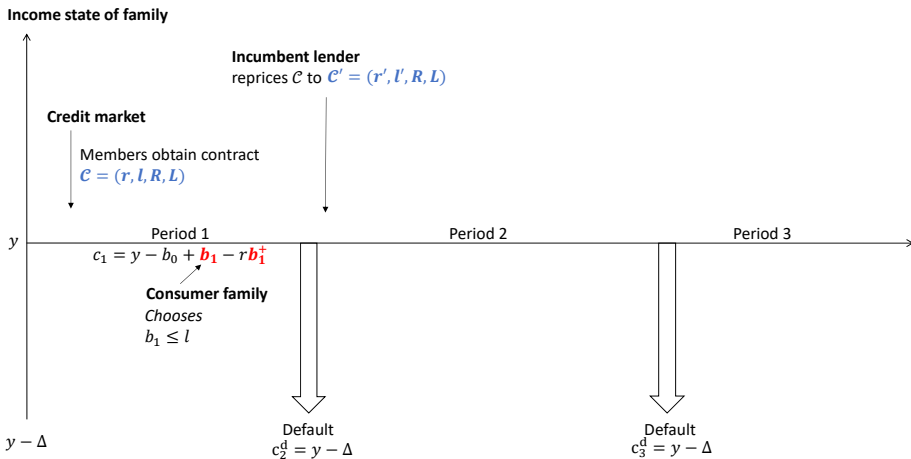
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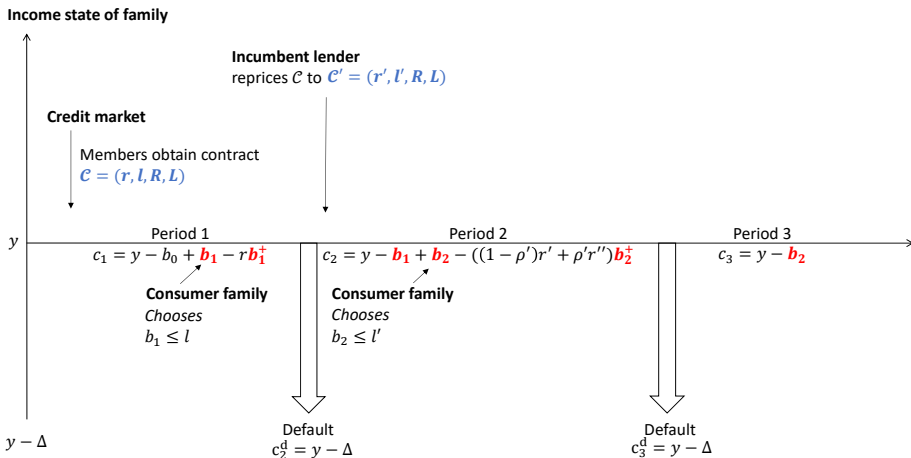
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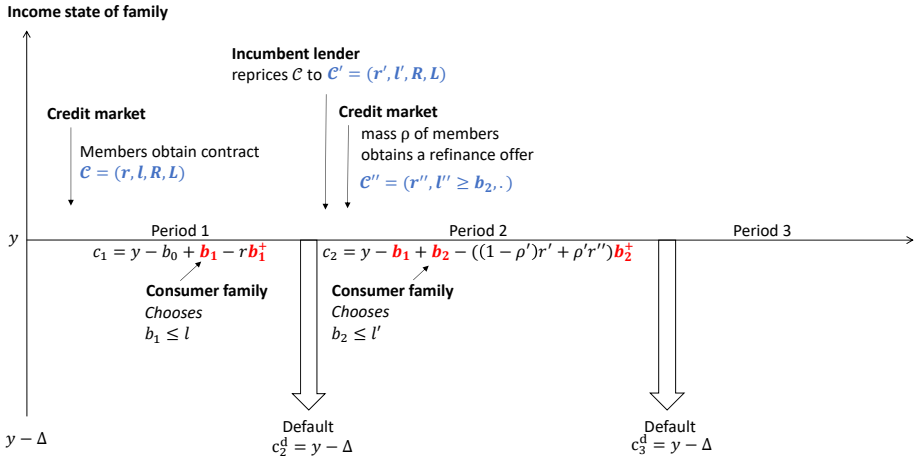
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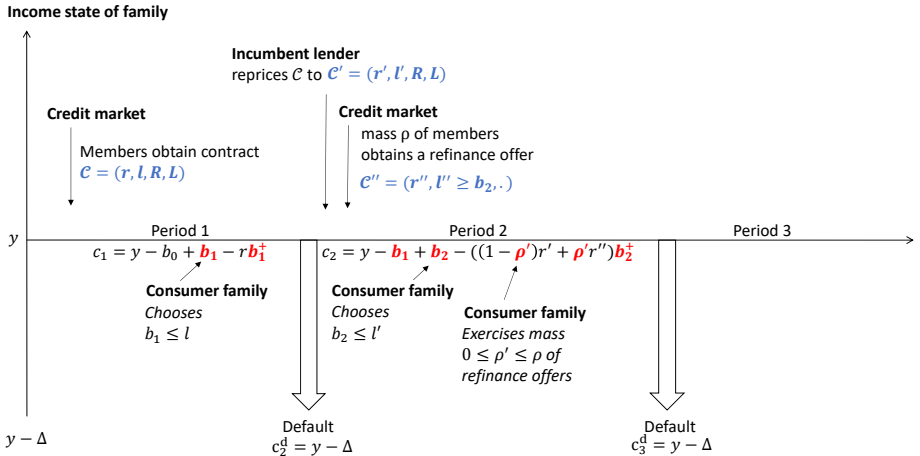
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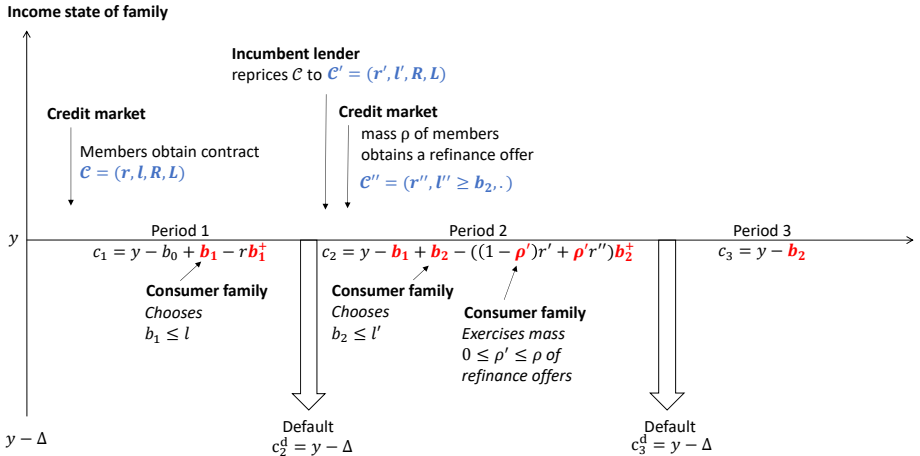
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## Equilibrium characterization

## PRELIMINARIES

1. The second period lender's zero profit condition implies

$$r''b_2^+(\cdot) = 0 \Rightarrow r'' = p$$

and for any credit limit (hence our assumption  $l'' \geq b_2$  was wlog)

2. Wlog can restrict attention to first period contracts that aren't repriced ex post, i.e.:

$$C'(\cdot) = (R, L, \cdot)$$

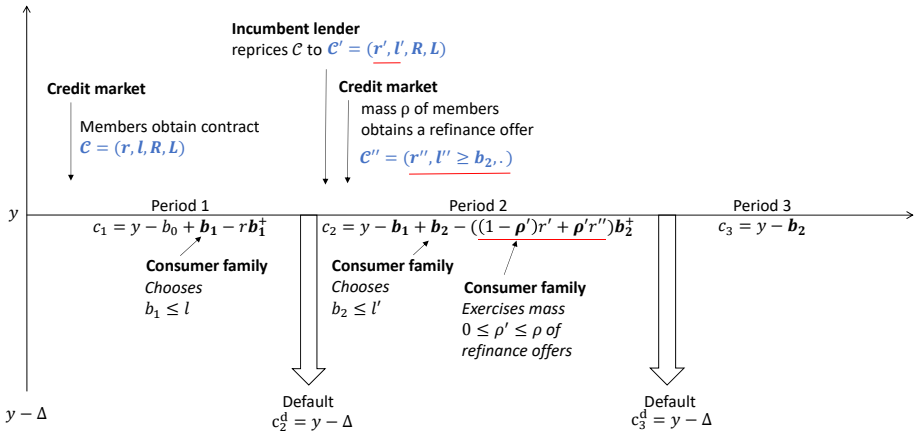
3. Refinancing decision is bang bang:  $\rho' = \rho$  if  $R > p$ , otherwise  $\rho' = 0$ .

4. Can recast as a “single lender” lender problem by applying monotone transformation:

$$\mathcal{R}(\hat{R}) = \frac{1}{1-\rho} \max\{\hat{R} - p, 0\} + \min\{\hat{R}, p\}$$

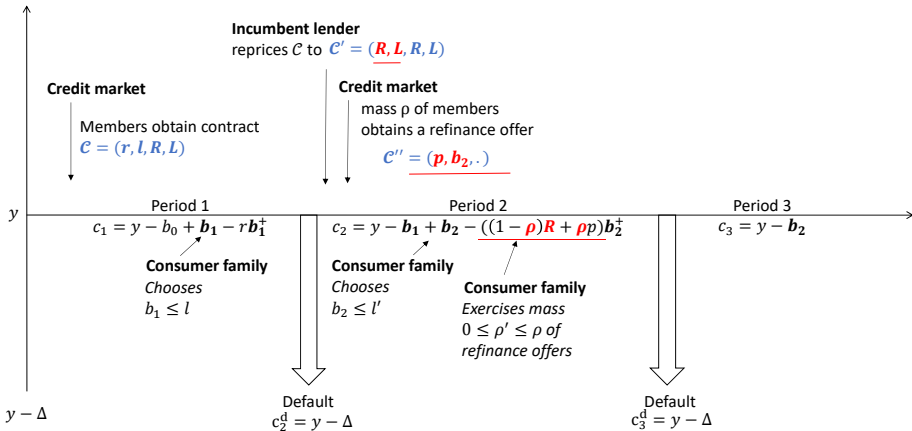
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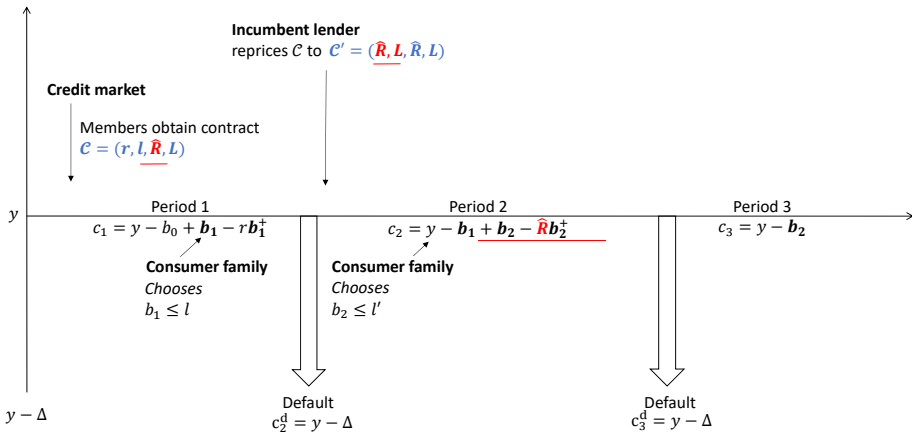
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# EQUILIBRIUM CONTRACT DEFINITION (EQ)

## LEMMA

$C = (r, l, R, L)$  is an equilibrium contract iff there exist  $\hat{R}$  and  $b_1, b_2$  such that

$$\textcircled{1} \quad R = \mathcal{R}(\hat{R}) := \frac{1}{1-\rho} \max\{\hat{R} - p, 0\} + \min\{\hat{R}, p\}.$$

$$\textcircled{2} \quad (c_1, c_2, c_3, r, l, \hat{R}, L, b_1, b_2) \text{ solves}$$

$$EQ : \max u(c_1) + \beta(1-p)u(c_2) + \beta^2(1-p)^2u(c_3) + \beta U^d$$

subject to

$$IC_1 \quad : \quad \begin{aligned} (u'(c_1)(1-r) - \beta(1-p)u'(c_2)) \mathbf{1}_{b_1=l} &\geq 0 \\ (u'(c_1)(1-r) - \beta(1-p)u'(c_2)) \mathbf{1}_{b_1 < l} &= 0 \end{aligned}$$

$$IC_2 \quad : \quad \begin{aligned} (u'(c_2)(1-\hat{R}) - \beta(1-p)u'(c_3)) \mathbf{1}_{b_2=L} &\geq 0 \\ (u'(c_2)(1-\hat{R}) - \beta(1-p)u'(c_3)) \mathbf{1}_{b_2 < L} &= 0 \end{aligned}$$

$$ZP \quad : \quad (r-p)b_1^+ + p(\hat{R}-p)b_2^+ = 0,$$

$$CL \quad : \quad b_1^+ \leq l, \quad b_2^+ \leq L$$

$$BC \quad : \quad c_1 = y - b_0 + b_1 - rb_1^+, \quad c_2 = y - b_1 + b_2 - \hat{R}b_2^+ \text{ and } c_3 = y - b_2.$$

$$\textcircled{3} \quad \text{The lender does not find it strictly profitable to reprice } r, l, \hat{R}, L \text{ ex post.}$$

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## SOLVING FOR EQUILIBRIUM CONTRACT

- Consider a planning problem of choosing  $(c_1, c_2, c_3, T_1, T_2, T_3)$ :

$$PL : \max u(c_1) + \beta(1-p)u(c_2) + \beta^2(1-p)^2u(c_3) + \beta U^d$$

subject to

$$RC : T_1 + (1-p)T_2 + (1-p)^2T_3 = 0,$$

and

$$BCPL : c_1 = y - b_0 + T_1, \quad c_2 = y + T_2, \quad c_3 = y + T_3.$$

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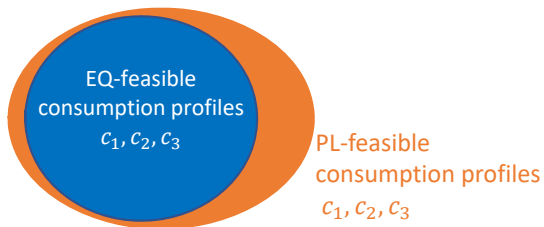
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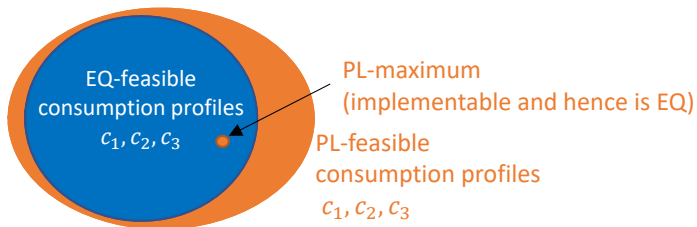
⇒ Key result: Solution to PL coincides with EQ

- the supporting contract retrieved from IC constraints

## BASIC IDEA



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## FORMAL RESULT FOR CROSSWALK BETWEEN EQ AND PL

### LEMMA

*In equilibrium, the consumer borrows in both periods; that is,  $b_1 > 0$  and  $b_2 > 0$ .*

### LEMMA

- $\Rightarrow$  *Let  $(c_1, c_2, c_3, r, l, \hat{R}, L, b_1, b_2)$  satisfy all the constraints of EQ and suppose  $b_1 > 0, b_2 > 0$ . Then, the implied transfers  $T_1 = c_1 - (y - b_0)$ ,  $T_2 = c_2 - y$ ,  $T_3 = c_3 - y$  that sustain the same level of consumption under PL are also feasible under PL (i.e., satisfy RC).*
- $\Leftarrow$  *Conversely, let  $(c_1, c_2, c_3, T_1, T_2, T_3)$  satisfy all the constraints of PL; furthermore, suppose there exists a contract  $(r, l, \hat{R}, L)$  such that  $l \geq -T_2 - T_3 (1 - \hat{R})$ ,  $L \geq -T_3$  and, for  $b_1 := -T_2 - T_3 (1 - \hat{R})$  and  $b_2 := -T_3$ ,  $IC_1, IC_2$  and ZP are satisfied. Then,  $(c_1, c_2, c_3, r, l, \hat{R}, L, b_1, b_2)$  is feasible under EQ (i.e., satisfies the constraints of EQ).*

# MAIN RESULT

## PROPOSITION

*Equilibrium contract features  $r = p = R$ , and nonbinding limits  $l, L$ .*

## PROOF

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$$PL : \max_{T_1, T_2, T_3} u(\underbrace{c_1^a + T_1}_{c_1}) + \beta (1 - p) u(\underbrace{c_2^a + T_2}_{c_2}) + \beta^2 (1 - p)^2 u(\underbrace{c_3^a + T_3}_{c_3}) + \beta U^d$$

$$RC : T_1 + (1 - p) T_2 + (1 - p)^2 T_3 = 0.$$

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## PROOF

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(Note:  $R = \mathcal{R}(p) = p$ .)

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$\Rightarrow$  L binding: invalidates condition 3 of equilibrium contract definition

( $\hat{R} > p$  means relaxing  $L$  ex post yields strictly positive profit to the lender)

## EQUILIBRIUM CONTRACT (EQ)

### LEMMA

$C = (r, l, R, L)$  is an equilibrium contract iff there exist  $\hat{R}$  and  $b_1, b_2$  such that

$$① \quad R = \mathcal{R}(\hat{R}) := \frac{1}{1-\rho} \max\{\hat{R} - p, 0\} + \min\{\hat{R}, p\}.$$

$$② \quad (c_1, c_2, c_3, r, l, \hat{R}, L, b_1, b_2) \text{ solves}$$

$$EQ : \max u(c_1) + \beta(1-p)u(c_2) + \beta^2(1-p)^2u(c_3) + \beta U^d$$

subject to

$$IC_1 : \quad \begin{aligned} (u'(c_1)(1-r) - \beta(1-p)u'(c_2)) \mathbf{1}_{b_1=l} &\geq 0, \\ (u'(c_1)(1-r) - \beta(1-p)u'(c_2)) \mathbf{1}_{b_1 < l} &= 0, \end{aligned}$$

$$IC_2 : \quad \begin{aligned} (u'(c_2)(1 - \hat{R}) - \beta(1-p)u'(c_3)) \mathbf{1}_{b_2=L} &\geq 0, \\ (u'(c_2)(1 - \hat{R}) - \beta(1-p)u'(c_3)) \mathbf{1}_{b_2 < L} &= 0 \end{aligned}$$

$$ZP : \quad (r-p)b_1^+ + p(\hat{R}-p)b_2^+ = 0,$$

$$CL : \quad b_1^+ \leq l, \quad b_2^+ \leq L,$$

$$c_1 = y - b_0 + b_1 - rb_1^+, \quad c_2 = y - b_1 + b_2 - \hat{R}b_2^+ \text{ and } c_3 = y - b_2.$$

③ *The lender does not find it strictly profitable to reprice  $r, l, \hat{R}, L$  ex post.*

## INTUITION

- Consumers understand that—this way or the other—they must pay for defaulting
  - lenders must break even in equilibrium
- After accepting the contract, borrowing only depends on applicable rates (EQ Euler)
  - if rates not “right,” borrowing levels do not maximize ex ante utility (PL Euler)

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- ⇒ Ex ante, consumers seek contracts to ensure utility maximizing borrowing ex post
- marginal rates must reflect current default risk, implying  $r = p = R$
- ⇒ Side note: allocation is constrained efficient and hence “undistorted”
- this result has nothing to do with “tax smoothing” theorem in public finance

## Extensions

## EXTENSIONS

1. Income fixed at  $y$  but default endogenously generated by a random “stigma” shock  $\Delta$

- default occurs in 2nd period when  $U_2(.) \leq u(y) - \beta D - \Delta$
- default occurs in 3rd period when  $U_3(.) \leq u(y) - \Delta$

⇒ Reinforces the result ( $r > p$ , if possible).

2. The consumer can borrow and save and consume saved funds after default

- subject to constraint  $b_1^d \geq \tau b_1$ , which can be binding or not

⇒ Same result applies.

3. T periods instead of 3 periods?

⇒ Same result applies (think about the last 3 periods).

## HYPERBOLIC DISCOUNTING AS A POTENTIAL EXPLANATION

- Preferences as of first period:  $u(c_1) + \beta\eta(u(c_2) + \beta u(c_3))$ 
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- Sophisticated case: consumer is aware of time inconsistency problem

⇒ Promos arise in both cases but for different reasons

## A NOTE ON THE LITERATURE (OR LACK OF THEREOF)

- Drozd and Kowalik (2022)
  - examine the collapse of promo lending's contribution to Great Recession
- Ausbel and Shui (2013)
  - evidence from an experiment of mailing offers to consumers showing revealed preference for low early (interest/fee) payments
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- Promotional lending prevalent in data
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- Raises a question of what drives promos in the data and what it means in terms of modeling and regulation
- Is hyperbolic discounting the only possibility?