Rethinking Markups and Inventories over the Business Cycle

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- Theory of inventory dynamics calls for countercyclical markups:

    1. Inventory falls in demand recessions $\Rightarrow$ productive inventories
    2. Inventory-to-sales ratio rises $\Rightarrow$ countercyclical markups

(As shown by **Kryvtsov and Midrigan (2012)** and Bils and Kahn (2000))

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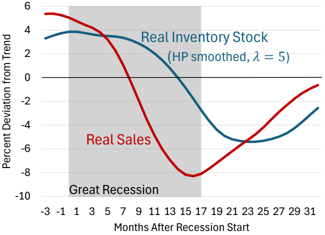
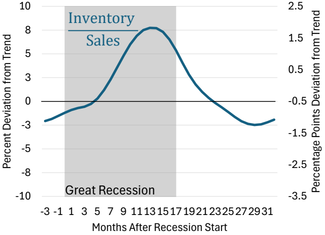
- This paper: Delayed response of inventories, *not* markups

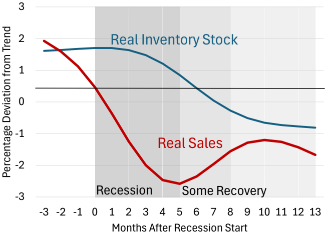
Data (1/2)

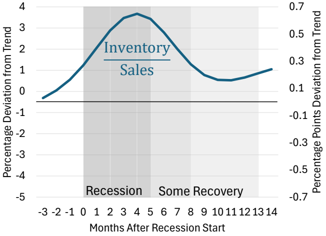
A. Great Recession:





B. Average for U.S. recessions, 1979–2007:





What We Do

- Draw on literature on the role of intangibles in bringing customers

  ≈ 10% to 20% of firms' costs are Selling, General and Admin. (SG&A)

  Drozd and Nosal (2012), Gourio and Rudanko (2014), Bai et al. (2024)
  Crouzet and Eberly (2023), Argente et al. (2024), He et al. (2024), others

- Propose a mechanism that delays response of inventories

  1. Customers & customer-hailing capital investment is *irreversible*

     ⇒ firms do not *liquidate* capital during *transient* recessions

  2. Inventory complementary to capital in hailing customers

     ⇒ inventories do not fall as much sales

Why Not Markups?

- Profits and profit margins are both strongly procyclical

  Christiano et al. (1997), Bilbie and Kanzig (2023), Broer et al. (2019)

Data (2/2)

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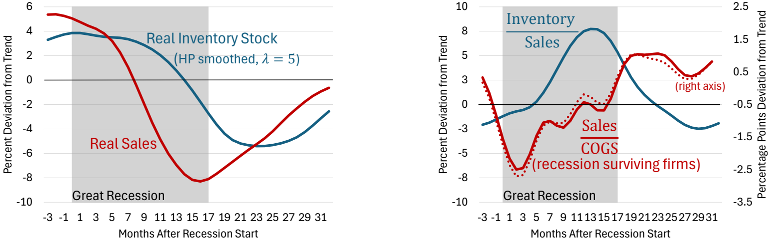

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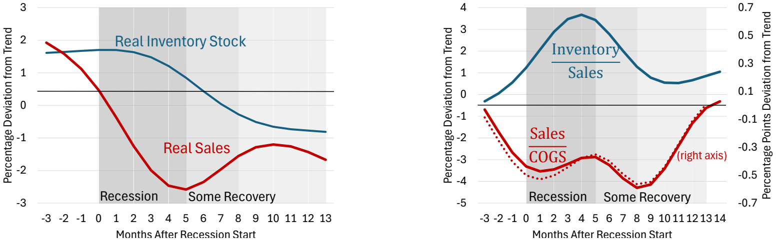

Why Not Markups?

- Profits and profit margins are both strongly procyclical

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- Major departure from the canonical model of production needed to flip the correlation between *gross margins* and *markups*

  (Think: Cobb-Douglas in labor & materials, prices of inputs taken as given)

  - Lack of evidence:
    - Nekarda and Ramey (2023), DeLoecker, Eeckhout, Unger (2020)
  - Broadly consequential for macro modeling:
    - steep marginal costs imply low employment volatility
    - does not deliver monetary non-neutrality (Broer et al., 2019)

Related Literature

- Profit margin/markup cyclicality in data:

  Christian et al. (2007), Bilbie and Kanzig (2023), Broer et al. (2019), Nekarda and Ramey (2023), Bils (1987), Rotemberg and Woodford (1999), DeLoecker et al. (2020), Gali et al. (2007)

- Inventory dynamics:

  Kahn (1987), Bils and Kahn (2000), Kryvstov and Midrigan (2013), Thomas and Khan (2007)

- Targeted search with information acquisition:

  Wolinsky (1986), Anderson and Renault (1999), Menzio (2007), Cheremukhin and Restrepo-Echavarria (2020), Lester (2011)

Data (proxy SVAR)

Gertler and Kehradi (2015) with Inventories and Margins

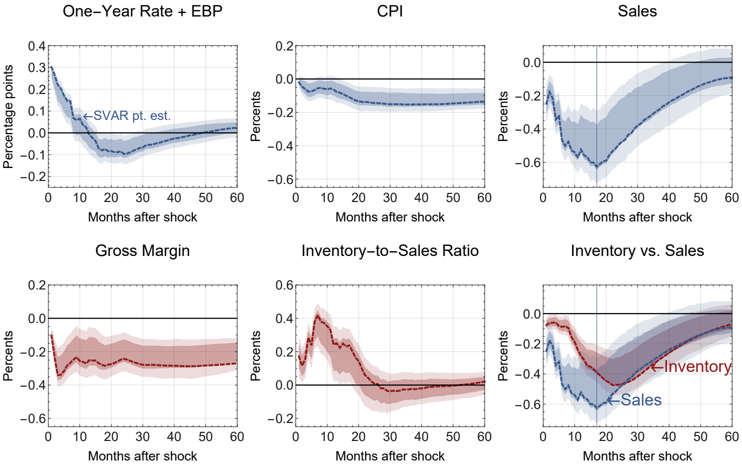

Figure: SVAR impulse responses to a monetary policy shock.

Theory

Environment

- Continuous time $t \in [0, \infty)$, rigid wage $v \equiv 1$

- Exogenous MIT shock to aggregate demand path under perfect foresight:

$$\text{'MIT' MP shock} \Rightarrow \begin{cases} \text{Aggregate demand path: } \{D_t(P_t) := D_{0t} P_t^{-\varepsilon}\}_t \\ \text{Discount rate path: } \{\rho_t\}_t \end{cases}$$

- Two-level/two-agent/two-market supply chain:

$$\underbrace{\text{Labor}}_{\text{Unit cost } = v} \Rightarrow \underbrace{\texttt{Producers} \Rightarrow \texttt{Shoppers}}_{\text{Wholesale market: Bilateral search}} - \underbrace{\texttt{Distributors} \Rightarrow D_t(P_t)}_{\text{Retail market: Walrasian}}$$

Producers

Producers: Key Mechanism / Friction

- Set up atomless *outposts* at sunk cost $\phi v > 0$

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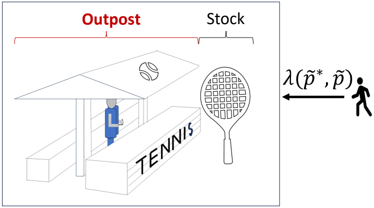
  - long lasting, fragment production and distribution
  - complementary to (inventory) stock in attracting customers

Producers: Key Friction / Mechanism

- Set up atomless *outposts* at sunk cost $\phi v > 0$
  - *Marketing* state (mass M_t)



  - *attracts* shoppers
  - does not produce

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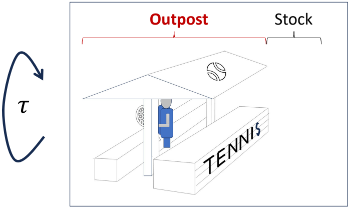

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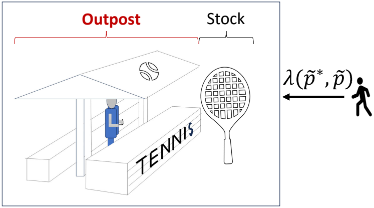

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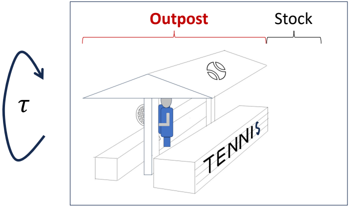


Mechanism (demand recession):

- inaction, no liquidations
- inventories flat for some time

  ↑ slower sales

  ↓ attrition of outposts

- markups *down* to boost sales

Producers: Outpost Value Maximization

- Outpost values in each state (1-marketing) evolve according to HJB:

$$\rho V_1 = -(\zeta_0 + \zeta)v + \max_{\tilde{p}} \{\lambda(\tilde{p}, \tilde{p}^*)\,(\tilde{p} + V_0 - V_1)\} - \delta V_1 + \dot{V}_1$$

$$\rho V_0 = -\zeta_0 v + \max_{0 \le \hat{\tau} \le \tau} \{\hat{\tau}\,(-v + V_1 - V_0)\} - \delta V_0 + \dot{V}_0$$

$\lambda(\tilde{p}, \tilde{p}_t^*)$ — arrival rate of shoppers given price $\tilde{p}$ & equilibrium price $\tilde{p}_t^*$

τ — bounded production rate $\Rightarrow 0 \le \hat{\tau} \le \tau$

$\zeta_0 v$ — flow cost of operating outpost & warehousing capacity

ζv — variable cost of holding inventory stock

δ — exogenous Poisson destruction rate of outposts

Producers: First Order Conditions

- Policies a function of the value of stock $X_t := V_{1t} - V_{0t}$

$$\rho X_t = -\zeta v + \max_{0 \leq \hat{\tau} \leq \tau, \tilde{p}} \{\lambda(\tilde{p}, \tilde{p}_t^*)(\tilde{p} - X_t) - \hat{\tau}(-v + X_t)\} - \delta X_t + \dot{X}_{tt}$$

  - Quoted price:

$$\tilde{p}_t = X_t - \frac{\lambda_{\tilde{p}_t}(\tilde{p}_t, \tilde{p}_t^*)}{\lambda(\tilde{p}_t, \tilde{p}_t^*)}$$

  - Production rate $\hat{\tau}$ (think $\hat{\tau} = \tau$, corner solution):

$$\hat{\tau}_t = \begin{cases} \tau & \text{if } X_t \geq v, \\ 0 & \text{otherwise.} \end{cases}$$

Assumption: Sufficiently High Inventory Holding Costs

- Define off-equilibrium HJB equations

$$\begin{aligned}\rho_t V_{1t}^{+} = & -\left(\zeta_0 + \sigma\zeta_0 + \zeta\right) v \\ & + \max_{\tilde{p}, \hat{\tau} \leq \tau} \left\{ \hat{\tau}\left(-v + V_{2t} - V_{1t}^{+}\right) + \lambda\left(\tilde{p}, \tilde{p}_t^{*}\right)\left(\tilde{p} + V_{0t} - V_{1t}^{+}\right)\right\} \\ & - \delta V_{1t}^{+} + \dot{V}_{1t}^{+}\end{aligned}$$

$$\begin{aligned}\rho_t V_{2t} = & -\left(\zeta_0 + \sigma\zeta_0 + 2\zeta\right) v \\ & + \max_{\tilde{p}} \left\{ \lambda\left(\tilde{p}, \tilde{p}_t^{*}\right)\left(\tilde{p} + V_{1t}^{+} - V_{2t}\right)\right\} - \delta V_{2t} + \dot{V}_{2t}\end{aligned}$$

Assumption

In steady state, ζ or σ such that $V_{1t}^{+,ss} \leq V_{1t}^{ss}$.

Producers: Firm Value Maximization

- Entry rate of new outposts a and liquidation rate d solve

$$\mathcal{W}_t := V_{1t}M_t + V_{0t}N_t + \max_{a\geq 0, d\geq 0}\left[\begin{array}{c} (M_t+N_t)\left(|a|(V_{0t}-\phi v)-\kappa_a a^2\right) \\ -N_t\left(|d|V_{0t}-\kappa_d d^2 N_t\right)\end{array}\right]$$

- Implied entry/liquidation policy function
  - $a_t = \delta\,\mathbf{1}_{\{d_t=0\}} + \frac{1}{2}\max\left\{V_{0t}-\phi v, 0\right\}/\kappa_a$
  - $d_t = -\frac{1}{2}\min\left\{V_{0t}, 0\right\}/\kappa_d$

(Report results for limiting case: $\kappa_d \to 0$.)

Producers: Law of Motion for Stock of Outposts

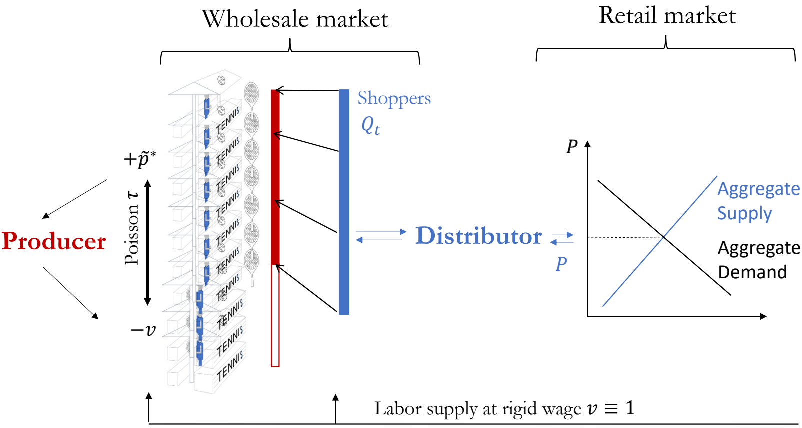
- Masses N_t and M_t follow the law of motion given by

$$\dot{M}_t = \hat{\tau} N_t - \Lambda_t M_t - \delta M_t$$

$$\dot{N}_t = \Lambda_t M_t - \hat{\tau}_t N_t - (\delta + d_t) N_t + a_t (M_t + N_t)$$

Distributors and Search

Setup Schematics



- $\phi = 0 \Rightarrow$ collapses to canonical model à la Bils and Kahn (2007)

Distributors

- Purchase intermediate goods by sending *mass* Q_t of shoppers
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  - shopper can haul only one unit per instance of time

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$$\underbrace{Q_t}_{\text{will think of } Q \text{ as aggregate demand}} (1 + \mathbb{E}_\pi \{\eta\}) = D_{0t} P_t^{-\varepsilon}$$

Distributor's Policy

- Distributor's policy: indicator function on space of $(\tilde{p}, \eta)$

Distributor's Policy

$$\underbrace{\underbrace{P_t(1+\eta)}_{\text{resell value}} - \underbrace{\tilde{p}_t}_{\text{quoted price}}}_{\text{distributor's (gross) surplus from match } (\tilde{p}, \eta)}$$

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Distributor's optimal policy represented by:

- *reservation effective price* $\bar{p}_t$ *or, equivalently,*
- *search precision* $\pi_t := Pr(p_t \geq \bar{p}_t)$.

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$$\pi = \Pr(p \geq \bar{p}) \underbrace{= \Pr(\tilde{p}^* - \eta P \geq \bar{p}) = \Pr\left(\eta \leq \frac{\tilde{p}^* - \bar{p}}{P}\right) = G\left(\frac{\tilde{p}^* - \bar{p}}{P}\right)}_{\text{in equilibrium linked by } G(\cdot)\text{: the CDF of } \eta}$$

Shopper Search

- Shoppers can draw random $(\tilde{p}, \eta) \sim p$ at cost c_0 in time $(dt)^2$
  - as $dt \to 0$: *infinite* number of draws possible in $(t, t + dt]$
  - *success* probability determined by policy: Prob.$(p \leq \bar{p})$

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- Geometric distribution of draws till *success*

$$\mathbb{E}\{\text{draws till } \textit{success}\} = \text{Prob.}(p \leq \bar{p})^{-1} = (1 - \pi)^{-1}$$

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- Surplus given by conditional distribution of η

$$s(\pi) = (1-\pi)^{-1} \int_{[0,\bar{p}(\pi)]} (P - p) \,\text{Pr}\left(dp \mid p \leq \bar{p}(\pi)\right)$$

Search Cost and Match Surplus

Lemma

Given distributor's policy $0 \leq \pi < 1$ and equilibrium quoted price $\tilde{p}^*$:

- *equilibrium search cost is*

$$c(\pi) = c_0 (1 - \pi)^{-1}$$

- *equilibrium surplus is*

$$s(\pi) = P(1 + \eta_0) - \tilde{p}^* - \eta_0 P \log(1 - \pi)$$

- *equilibrium reservation effective price is*

$$\bar{p}(\pi) = \tilde{p}^* + \eta_0 P \log(1 - \pi) .$$

Distributor Problem

- Representative distributor chooses policy π to maximize

$$y_t := \max_{0 \leq \pi < 1} \left(s_t\left(\pi\right) - c\left(\pi\right)\right)v$$

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- Gives

$$\pi = 1 - \frac{c_0}{\eta_0} \frac{v}{P_t}$$

$$\bar{p} = \tilde{p}_t^* + \eta_0 P_t \log\left(\frac{c_0}{\eta_0} \frac{v}{P_t}\right)$$

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- Assume zero profit condition given fixed cost $\chi v > 0$ of search:

$$y_t = P_t - \tilde{p}_t^* - \eta_0 P_t \log\left(\frac{c_0}{\eta_0}\frac{v}{P_t}\right) = \chi v$$

Zero Profits in Distribution

- Distributors make zero profits given fixed cost $\chi v > 0$ of search:

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Demand for Intermediate Goods

- Let the mass of "visible" producer outposts be M_t :
  - equilibrium arrival rate of purchasing shoppers to an outpost is

$$\Lambda_t = \frac{Q_t}{M_t}$$

(assuming all producers set the same price $\tilde{p}^*$)

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- Λ_t not enough to pin down prices; need "$\lambda(\tilde{p}, \tilde{p}^*)$", where we know

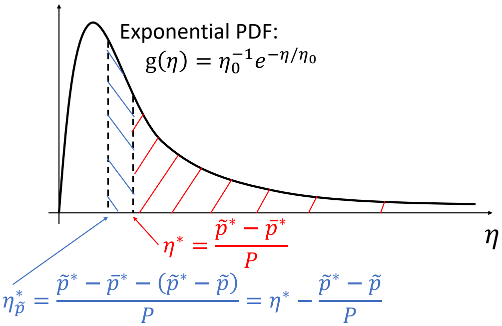
$$\lambda(\tilde{p}^*, \tilde{p}^*) = \Lambda_t$$

Demand for Intermediate Goods

Lemma

Equilibrium arrival rate to a "visible" outpost :

$$\lambda\left(\tilde{p}^{*}, \tilde{p}\right) := \Lambda_t \underbrace{\exp\left(\eta_0^{-1}\frac{\tilde{p}^{*}-\tilde{p}}{P}\right)}_{\textit{relative price impact on demand}}$$



Equilibrium

Definition

Given path of $\{\rho_t, D_{0t}\}$ *and initial condition* $\{M_0, N_0\}$*, the perfect foresight equilibrium comprises paths of:*

1. *Equilibrium prices and wholesale demand* $\{\tilde{p}_t^*, P_t, \lambda_t(\tilde{p}_t^*, \tilde{p}_t)\}$*;*
2. *Distributor's policy* $\{Q_t, \bar{p}_t, \pi_t\}$*;*
3. *Outpost values* $\{V_{0t}, V_{1t}\}$*;*
4. *Outpost policy* $\{\tilde{p}_t, \hat{\tau}\}$ *such that* $\tilde{p}_t = \tilde{p}_t^*$*;*
5. *Outpost masses* $\{M_t, N_t\}$*;*
6. *Producer's entry/exit policy* $\{d_t, a_t\}$

such that (...).

Assumption 2. $c_0 \frac{1-\theta_0}{\theta_0} < \zeta^{-1} \left(\phi(\sigma + \eta) + \delta_0 + \zeta\right) + \psi.$

Lemma 3. Steady-state equilibrium exists and is unique.

How Does It Work?

Consider Elementary Demand Shock

MP shock $\Rightarrow$ $\begin{cases} \text{Aggregate demand path: } \{D_t(P_t) := D_{0t}P_t^{-\varepsilon}\}_t \\ \text{Discount rate path: } \{\rho_t\}_t \end{cases}$

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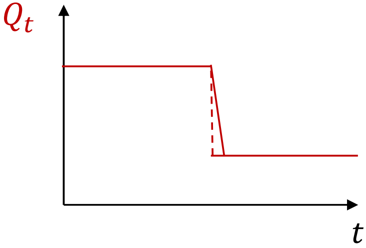

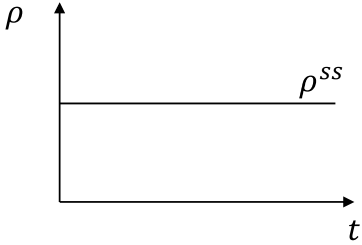

Main Result

Markup in Partial Eq. $\Rightarrow \mu_t^* := \frac{\tilde{p}_t^* - v}{v} = \frac{X_t - v}{v} + \eta_0 P_t$

Markup in General Eq. $\Rightarrow \mu_t^{*(\mathsf{GE})} \approx \frac{X_t\,(1+\Theta) - v}{v} + \Theta\Gamma$

(Θ, Γ *strictly* positive functions of model parameters.)

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Frictionless case:

$\phi = 0$ and $\kappa_a, \kappa_d \to 0$

$X := V_1 - V_0$ - constant

$\Lambda^{-1} := \frac{M}{Q}$ - constant

$\Rightarrow corr(\Lambda^{-1}, \mu^{*(\mathsf{GE})}) = 0$

Bils and Kahn (2000)

Kryvtsov and Midrigan (2013)

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Baseline case:

$\phi > 0$ and $\kappa_a, \kappa_d \to 0$

$X_t := V_{1t} - V_{0t}$ (diff. eq.)

$\Lambda^{-1} = \frac{X_t + v\Gamma}{X_t(\delta+\rho+\tau) - \dot{X}_t + v(\zeta - \tau)}\Theta$

$\Rightarrow corr(\Lambda^{-1}, \mu^{*(\text{GE})}) < 0$

Why Markups Are Variable?

- Firms "$\approx$" maximize profit flow per unit of time

  (set $\zeta = \zeta_0 = 0$, substitute out V_{0t} in HJB for V_{1t})

$$\rho V_{1t} = (\rho + \delta)^{-1} \max_{\tilde{p}} \frac{\tilde{p} - v\tau \left(\delta + \rho + \tau\right)^{-1} + T(\tilde{p}, \tilde{p}_t^*)\dot{V}_{1t} + \mathbf{T_0}\dot{V}_{0t}}{\mathbf{T_0} + T(\tilde{p}, \tilde{p}_t^*)}$$

$T(\tilde{p}, \tilde{p}_t^*) := \left(\lambda_t(\tilde{p}, \tilde{p}_t^*)\right)^{-1}$ — time to sell (get customer)

$\mathbf{T_0} := \left(\delta + \rho + \tau\right)^{-1}$ — time to restock (produce)

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- Price depends on the *level* of demand Λ_t

$$\tilde{p}_t^* = v\tau\,(\delta + \rho + \tau)^{-1} + \mathbf{T_0}\dot{X}_t + \frac{P_t}{\eta_0}(1 + \Lambda_t \mathbf{T_0})$$

Why Value of Stock Falls?

- Case 1: Liquidations do not occur

$$\Lambda_t\downarrow:\ \Lambda_t = \frac{Q_t}{M_t} \quad \text{(because } M_t \text{ decays at } \textit{slow} \text{ rate } \delta\text{)}$$

$$X_t\downarrow:\ \dot{X}_t = X_t\left(\underbrace{\rho+\tau+\delta-\Theta\Lambda_t}_{a_1>0}\right) - v\left(\underbrace{\tau-\zeta+\Theta\Gamma\Lambda_t}_{a_0>0}\right)$$

$$\mu_t^{*(\mathsf{GE})}\downarrow:\ \mu_t^{*(\mathsf{GE})} \approx \frac{X_t\left(1+\Theta\right)-v}{v} + \Theta\Gamma.$$

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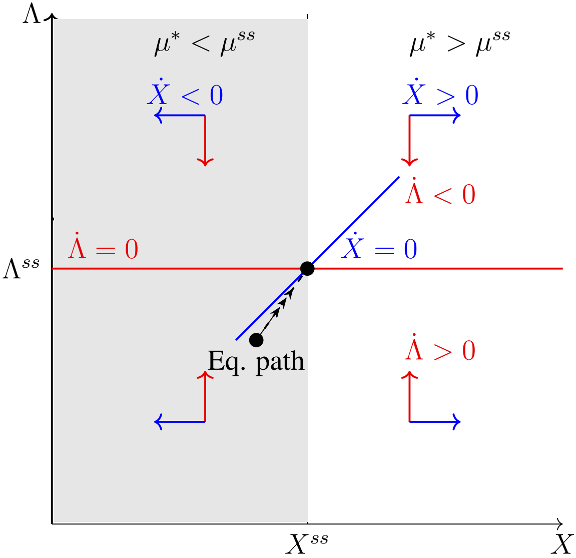
$$\mu_t^{*(\mathsf{GE})} \downarrow: \ \mu_t^{*(\mathsf{GE})} \approx \frac{X_t (1 + \Theta) - v}{v} + \Theta\Gamma.$$

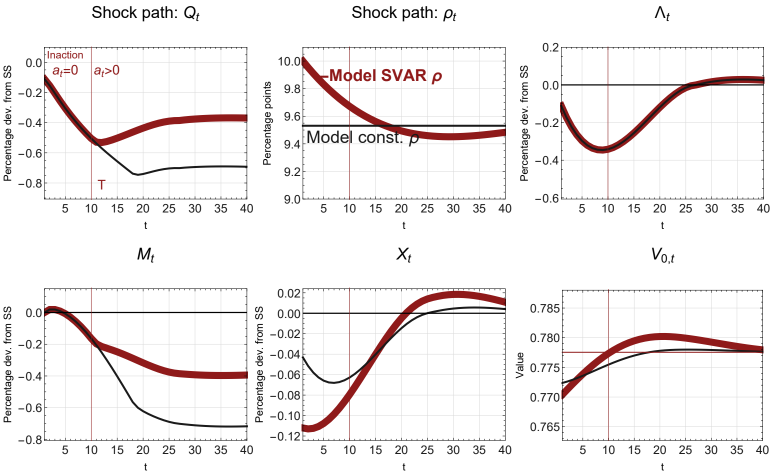
Phase Diagram



Why Value of Stock Falls?

- Case 2: Liquidations do occur (see paper)
  - if $d_t > 0$, $V_{0t} \leq 0$, and V_{0t} must follow a continuous path

Impulse Responses in Calibrated Model



Quantiative Results

Calibration: Steady State Targets

- Model calibration consistent with the following targets
  1. Weighted-average cost of capital of 10 percent $\Rightarrow$ Aswath Adamodar
  2. Average delivery delay of 60 days $\Rightarrow$ ISM & Deloitte
  3. Wholesale markup of 38% $\Rightarrow$ '97 BEA IO tables
  4. Distribution & trade margin of 42% $\Rightarrow$ '97 BEA IO tables
  5. SG&A-to-Sales Ratio of 25% $\rightarrow$ IO tables and Compustat
  6. Inventory to sales ratio of 1.5 $\Rightarrow$ BEA series, 1979-2011
  7. Three quotes before purchase, on average $\Rightarrow$ industry best practice

Calibration: Dynamic Targets and MP Shock

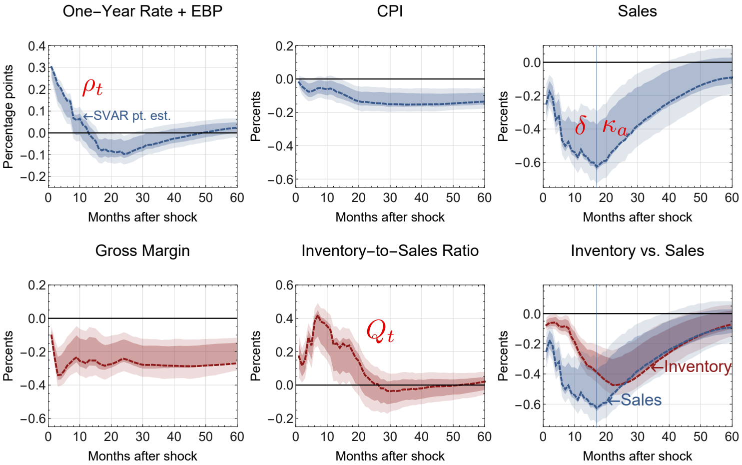


Figure: SVAR impulse responses to a monetary policy shock.

Parameter Values

Table: Parameter Values.

Parameter	Value
η_0	.088
c_0	.056
δ	4.0×10^{-4}
ϕ	.78
χ	.7
τ	.5
(ζ_0, ζ, σ)	$(.095, .0, .57)$
ρ^{ss}	7.94×10^{-3}
Γ	.87
Θ	.081
(κ_a, κ_d)	$(90.9, 0)$

Quantitative Results

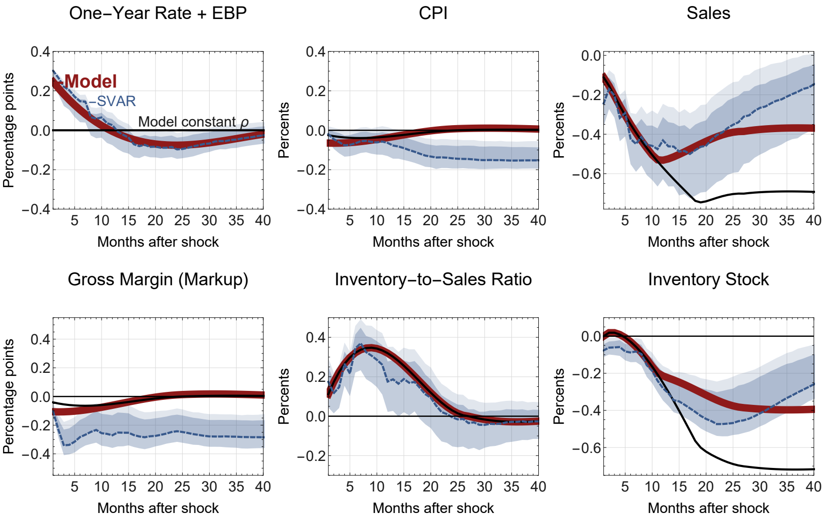


Figure: Comparison of Model to Proxy SVAR.

Quantitative Results

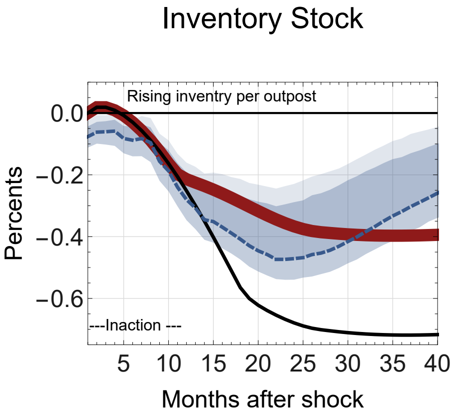


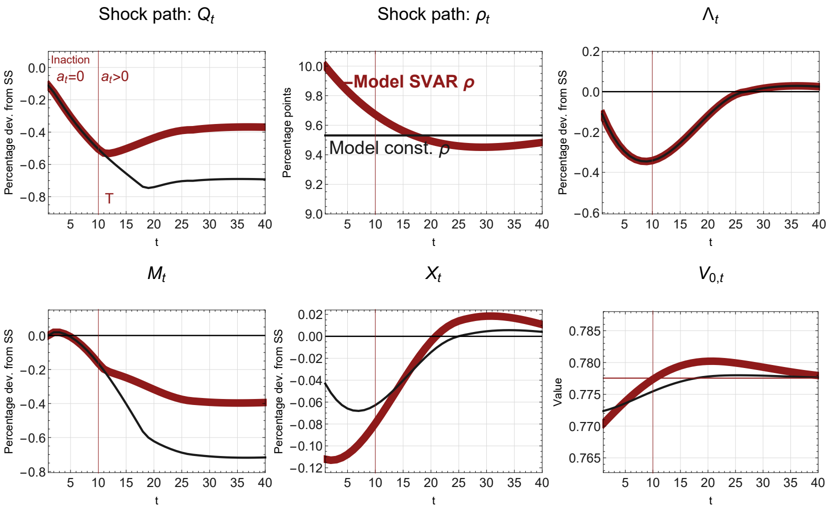
Figure: Inaction in Adjustment of M and Inventory Dynamics.

Conclusions

- Proposed theory that divorces inventory dynamics from markups
  - inventory falls in recessions but
  - markups are procyclical & inventory-to-sales ratio is countercyclical
- Derived analytic results for the key mechanism
- Documented comovement of inventories & *gross* margins after an MP shock

Extras

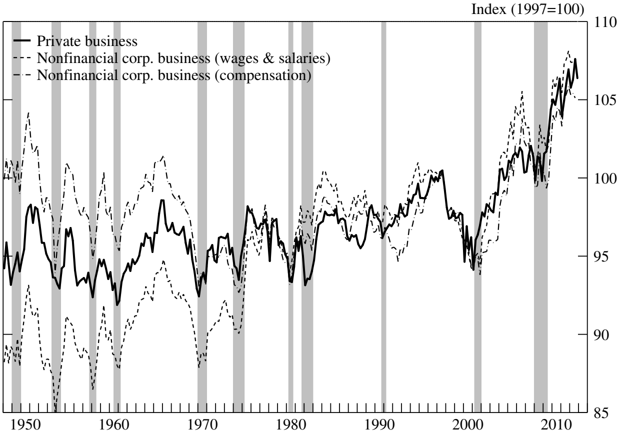
Impulse Response From the Calibrated Model



Aggregate Markup as Labor's Share Inverse

Cobb-Doulgas Aggregate Production Function $\Rightarrow$ Markup $\mu \propto$ labor share^{-1}

Figure 1. Aggregate Price-Cost Markup



Source: Nekarda and Ramey (2013)

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