The Trade-Comovement Puzzle*

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Abstract

Standard international transmission mechanism of productivity shocks predicts a weak endogenous linkage between trade and business cycle synchronization: a problem known as the *trade-comovement puzzle*. We provide the foundational analysis of the puzzle, pointing to three natural candidate resolutions: i) financial market frictions; ii) Greenwood–Hercowitz–Huffman preferences; and iii) dynamic trade elasticity that is low in the short run but high in the long run. We show the effects of each of these candidate resolutions analytically and evaluate them quantitatively. We find that, while i) and ii) fall short of the data, iii) goes a long way toward resolving the puzzle.

Keywords: trade-comovement puzzle, elasticity puzzle, trade elasticity, international comovement

JEL codes: E32, F44, F41, F32

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A central question in international macroeconomics is how shocks originating from one country are transmitted to other countries. Empirically, trade linkages have been found relevant, thereby suggesting that trade plays a crucial role in the endogenous transmission of business cycle fluctuations across countries.[1] In an influential paper, Kose and Yi (2006) have shown that this basic proposition runs counter to how productivity shocks are transmitted in the standard international macro model: a result known as the *trade-comovement puzzle.* While the failure of the standard model has been well-documented quantitatively, to date very little is known about the underlying mechanism through which trade affects business cycle transmission in the standard theory. In particular, it is not clear which features are critical for the model's failure, and hence which structural modifications may be promising in remedying the issue.

In this paper, we accomplish two goals. First, we close the gap in the literature by providing the foundational analysis of the trade-comovement puzzle; that is, we lay out the basic forces underlying the puzzle and trace these forces back to the model's structural assumptions. Second, we explore three natural candidate resolutions that this analysis points toward: i) financial market frictions, ii) Greenwood–Hercowitz–Huffman (GHH) preferences, and iii) a dynamic trade elasticity that is low in the short run but high in the long run, as implied by a trove of empirical evidence.[2] We show the effects of each of these candidate resolutions analytically and evaluate them quantitatively. We find that i) and ii) improve model's performance but still fall short of the data, while iii) goes a long way toward resolving the puzzle. Considering the fact that dynamic trade elasticity does not fundamentally change the model's transmission mechanism or its shock structure, and it is independently grounded in evidence, we conclude that the trade-comovement puzzle is best interpreted as imposing empirically viable parametric and structural restrictions on the standard transmission mechanism as opposed to

[1] An extensive empirical literature documents a tight linkage between bilateral trade intensities and business cycle comovement across countries. By running cross-country regressions, Frankel and Rose (1998), Clark and van Wincoop (2001), Calderon et al. (2002), Otto et al. (2001), Baxter and Kouparitsas (2005), Kose and Yi (2006) and Inklaar et al. (2008) all find that, among bilateral country pairs, more trade is associated with more synchronized business cycle fluctuations. In an important paper, Johnson (2014) shows that the effect of trade on output is pronounced even after controlling for the relation between measured TFP and trade. For more details, see literature review at the end of this section.

[2] Trade elasticity describes how the volume of trade responds to the price of imported goods relative to domestically produced goods. Estimates of trade elasticity focusing on the effect of persistent or permanent shocks – such as trade liberalization episodes — point to high levels of trade elasticity (see Head and Ries (2001), Eaton and Kortum (2002), Clausing (2001), Anderson and Van Wincoop (2004) or Romalis (2007)). These estimates are referred to as long-run trade elasticity estimates. In contrast, business cycle frequency estimates that utilize time-series of quantities traded and prices point to trade elasticity estimates that are low, close to one or even below one (see Reinert and Roland-Holst (1992), Blonigen and Wilson (1999)). These estimates are referred to as short-run trade elasticity estimates. Standard CES setup implies equal short- and long-run trade elasticities and cannot account for the discrepancy in measurement. Our model of dynamic trade elasticity reconciles these distinct sets of estimates as an artifact of sluggish adjustment of trade to relative prices. See Ruhl (2008) for a detailed discussion of the elasticity puzzle in international economics.

rejecting it outright.

With regard to the analysis of the puzzle, our baseline analytic setup follows closely the model by Backus et al. (1995) and allows us to explore such features as the role of risk aversion, elasticity of substitution between home and foreign goods, financial market frictions that sever risk sharing, and different functional forms for the preferences for consumption and leisure, as well as to examine the distinct roles played by capital and labor inputs in shock transmission. As we find, trade affects comovement in the standard theory via two offsetting channels. We label them as the *substitution* effect and the *income* effect channels of trade.[3]

Consider first the *substitution effect channel*. This channel is a robust source of a positive relation between the steady state level of trade between countries and the comovement of their output levels over the business cycle.

Intuitively, due to the built-in complementarity between differentiated home and foreign goods, a positive productivity shock in the foreign country increases the supply of the foreign good and lowers its price relative to the home good, thereby improving the home country's terms of trade. Since both consumption and investment in the home country involve the imported foreign good, whereas home labor and capital exclusively produce the home good, this raises the price of home labor and capital in terms of home consumption/investment good. Through the usual substitution effect between labor and leisure, and due to lower cost of investment, this raises home labor supply, investment and output. Importantly, since trade corresponds to the share of the foreign good in home consumption and investment spendings, this relative price effect is stronger the more countries trade, implying a positive relation between trade and the strength of the transmission of the foreign productivity shock into the home country's output.[4]

Consider next the *income effect channel*. With the exception of very low levels of the elasticity of substitution between home and foreign goods, the income effect channel is a potent source of a negative relation between trade and output comovement. In a usual parameterization of the standard model, this channel weakens or even reverses the positive linkage between trade and comovement that is implied by the substitution effect channel alone, crucially contributing to the trade-comovement

[3]Our naming convention departs from the one introduced by Kose and Yi (2006), who labeled these channels as *complementarity channel* and *risk-sharing channel*, respectively. Since complementarity between home and foreign goods would also be a source of risk-sharing even under financial autarky, we opt for more generic labels. Since the standard model is the backbone of much of the work in international macroeconomics, these effects apply to a broad class of models used in international economics.

[4]Trade also changes the response of terms of trade to shocks, but this turns out to be a secondary factor because it is proportional to the level of trade, which is small in the context of bilateral cross-country analysis.

puzzle.

The income effect channel is attributed to the fact that a positive productivity shock abroad has a positive income (wealth) effect on the home country owing to the appreciation of its terms of trade and a positive asset payout associated with international borrowing and lending (in the baseline model under complete markets). The additional income (wealth) raises home consumption, and due to the built-in complementarity between consumption and leisure, it reduces home labor supply and output. This effect is further reinforced by reduced investment in capital due to the built-in complementarity between capital and labor.

Crucially, the size of the income effect depends on trade, making it relevant for the trade-comovement pattern. This is because the effective transfer of resources from the foreign country to the home country that is implied by the income effect leads to an excess supply of the foreign good that is *inversely proportional to trade* – since trade determines how absorption in each country affects the demand for the individual country-specific goods and the transfer raises absorption in the home country at the expense of the foreign country that demands more of the foreign good. The excess supply of the foreign good, in turn, discourages endogenous risk sharing by making home consumption relatively more expensive in equilibrium, which means that trade encourages risk sharing and leads to *larger* transfers in equilibrium. Since transfers are a source of negative comovement, *more* trade implies *less* comovement.

Informed by the analysis of the effects of trade on comovement in the standard theory, we examine the set of modifications that can suppress the income effect channel without fundamentally changing the transmission mechanism or the shock structure. In particular, we analyze the following modifications of the baseline theory: i) financial autarky – an extreme form of a financial market friction that bans cross-border asset trade; ii) GHH preferences; and iii) dynamic elasticity that is low in the short run but high in the long run.[5] The first modification shuts down endogenous asset trade, and this way suppresses the income effect channel. The second modification eliminates the income effect on labor supply, which is at the heart of the income effect channel. The third modification fundamentally changes the workings of the income effect channel in the presence of durable capital.

Our financial autarky results demonstrate that the restrictions on asset trade are unlikely to

[5]We use a simple approach of imposing a convex adjustment cost on trade shares. This approach can be microfounded via search frictions a la Drozd and Nosal (2012). It can also be interpreted as the effect of deep habit, as in Mazzenga and Ravn (2002). We have analyzed the model by Drozd and Nosal (2012) and it implies even results effect due to variable markups that additionally help account for the trade-comovement puzzle, along the lines of work by de Soyres and Gaillard (2016).

be quantitatively promising – a finding that is in line with the observations made by Kose and Yi (2006) and others. We find that the financial autarky model accounts for only 24% of the trade-comovement relationship in the data, as compared to 19% in the baseline complete markets model. As we demonstrate, the key issue is that the bulk of risk sharing under financial autarky is driven by the income effect of terms of trade that autarky fails to eliminate – a result known at least since the work of Cole and Obstfeld (1991). The income effect of terms of trade is naturally proportional to trade because terms of trade affects the value of imports relative to the value of exports.

GHH preferences offer further improvement of the model's performance by accounting for 33% of the trade-comovement relationship in the data – which is a more than 75% improvement over the baseline model and 40% over financial autarky but still far short of the data. The additional caveat of using GHH preferences with productivity shocks is that they suppress countercycliality of the current account – which is a weak point of the standard theory to start with (Raffo, 2008). However, as we point out in the paper, GHH preferences have some potential for accounting for the trade-comovement puzzle for parameter values that raise the Frisch elasticity above our baseline setting that is slightly above one. For example, when we calibrate the GHH model to match the Frisch elasticity of two, the GHH model accounts for about 47% of the puzzle, while the standard model featuring higher labor elasticity still falls well short of the data.

In contrast, dynamic trade elasticity performs well quantitatively and largely resolves the trade-comovement puzzle. For our target value for the short-run trade elasticity of 1.17 – which is conservative in light of data estimates – the model featuring dynamic trade elasticity accounts for 61% of the trade-comovement relation in the data. For less conservative but still viable targets for short-run trade elasticity, it goes up to 76%. To top it off, through the same mechanism that helps resolve the trade-comovement puzzle, dynamic trade elasticity brings theory closer to the data by increasing the countercyclicality of the current account.

The key effect of introducing dynamic trade elasticity is that it flips the direction of the income effect channel. As a result, the two channels work in unison to drive a robustly positive relation between trade and comovement. At the same time, given that the model features a low short-run trade elasticity, the substitution effect channel works in a similar manner as in the baseline model.

Intuitively, dynamic trade elasticity affects the income effect channel because the excess supply of the foreign good associated with risk sharing transfers can be alleviated by delaying them. This is because, with enough lead time, home and foreign goods become more substitutable and the cost of the excess supply of the foreign good associated with transfers declines over time. In a model with durable capital, transfers can be delayed without sacrificing consumption smoothing by changing investment in capital, which falls in the home country to finance higher consumption in the expectation of future transfers and rises in the foreign country to finance future transfers after the shock. Trade severs this mechanism because, to achieve that for higher levels of trade, a larger adjustment of investment across countries is necessary, and this is costly for two reasons: one, the marginal productivity of capital diminishes and, two, there is a convex adjustment cost on capital. The end result is that in the dynamic elasticity model trade severs risk sharing and leads to lower rather than higher transfers. In contrast, in the standard model, delaying transfers would be merely kicking the proverbial can down the road because home and foreign goods are equally substitutable on impact as in the future.

The reported quantitative results in our paper are based on an exercise that is conceptually similar to that in Kose and Yi (2006). Specifically, we use cross-country correlation of output as our baseline measure of comovement and distinguish between trade openness and bilateral trade intensity by introducing a large third country to the model that serves as the relative rest-of-the-world.[6] We calibrate the models to match trade openness and bilateral trade intensity of a median country pair in our dataset. We then parameterize the stochastic productivity process so that the initial comovement of output, output volatility, and its persistence are consistent with the median values in our sample for individual countries and their relative rest of the world. Finally, in order to test the models, we consider an experiment of increasing trade intensity within the calibrated country pair and also raising trade openness to match the transition from the median to the 90th percentile of country pairs ranked by bilateral trade intensity. Subsequently, we evaluate the impact of such a change on the implied correlation of output vis-à-vis those implied by a regression between trade and comovement across country pairs.[7]

Related literature. Our paper is most closely related to Kose and Yi (2006), who were the first ones to document the trade-comovement puzzle and provided the first analysis of the effect of trade on comovement in the standard theory. Our paper is also related to the literature that explores the role of other forces that can potentially link trade to comovement. Most notable among these

[6] Our modeling of trade uses a more standard approach of having different preference weights as opposed to quadratic iceberg cost.

[7] Similar results are obtained by generating all 190 pairs in the model by matching trade intensity in the bilateral pair and the relative rest of the world and running analogous regression on model-generated data. Our baseline regression coefficient implies that moving from the 10th to the 90th percentile of the bilateral trade intensity distribution raises the predicted GDP correlations by 0.2, which is similar to the magnitude reported by Kose and Yi (2006) and other studies.

efforts are theories that generate endogenous TFP spillovers that are proportional to trade, from which we abstract but which are independently relevant. In this regard, Liao and Santacreu (2015) develop a theory in which trade leads to larger technology spillovers. In related work, de Soyres and Gaillard (2016) adopts a different approach and contends that it is the presence of markups that leads the measured TFP to comove more when there is more trade. While TFP and markups are features worth exploring, Johnson (2014) studies industry-level TFP correlations and input-output linkages and shows that assuming industry-level TFP correlation with trade is only a partial remedy. In particular, Johnson (2014) shows that service sectors exhibit the exact same trade-comovement pattern but measured TFP fails to be correlated with trade. Input-output linkages turn out insufficient to account for this feature of the data and the trade-comovement puzzle. His findings thus call for an endogenous mechanism that correlates comovement with trade independently of the correlation between trade and comovement of TFPs, such as the one provided in this paper.

The rest of the paper is organized as follows. Section 1 lays out our theoretical results. Section 2 discusses data, presents our quantitative model, elucidates parameterization, and discusses quantitative findings. Section 3 concludes.

1 Theory

This section provides the foundational analysis of the trade-comovement puzzle and explores three candidate resolutions of the puzzle that this analysis points toward. The analytic setup is simplified to gain tractability and illuminate the mechanisms. Relative to our later quantitative setup, we focus here on a two-country setup and abstract from trade openness. To ease the exposition, we first simplify the model by assuming full depreciation of capital and only relax this assumption later as an extension. The analysis is semi-parametric in the sense that the less pertinent parameters assume a numeric value typically used in the literature.

To streamline the exposition, we explore symmetry and present the setup from the home country's perspective. We drop time subscripts and history-dependent notation in all expressions pertaining to the concurrent date and state. Foreign analogs of home country variables are differentiated by an asterisk. A bar placed over a variable indicates the deterministic steady state value of this variable. A hat placed over a variable denotes its log-deviation from the steady state value – unless the steady state value is zero, in which case we normalize it by the steady state level of output.

1.1 Baseline model

There are two symmetric countries, referred to as home and foreign. Each country is populated by a large number of identical firms and households. Firms produce a homogeneous, country-specific tradable good using country specific technology and labor and capital supplied by home households. Households purchase and aggregate goods produced in each country into a composite final good that they use for consumption or investment in capital. All markets are Walrasian.

Firms. Technology in each country is described by a Cobb-Douglas production function of the form:

$$y = Ak^{\alpha}l^{1-\alpha}, \tag{1}$$

where k and l denote the amount of capital and labor inputs used in production, and $0 < \alpha < 1$ signifies division of income between capital and labor. A represents country-specific, stochastic total factor productivity that follows an unrestricted exogenous mean-reverting stochastic process, unless otherwise noted.

Firms maximize profits by employing labor and capital locally supplied by households at competitive prices w and r (in units of good d). Output can be sold at home or abroad with no friction, implying the law of one price in the goods market. Constant returns to scale in production imply zero profits in equilibrium; hence, in equilibrium

$$r = \alpha A(\frac{l}{k})^{1-\alpha}, \quad w = (1-\alpha)A(\frac{k}{l})^{\alpha}. \tag{2}$$

Households. Household preferences for home and foreign goods are determined by the usual constant elasticity of substitution (CES) aggregator:

$$G(d, f) := (\omega^{\frac{1}{\rho}} d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} f^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}} \tag{3}$$

Parameter $0 < \omega < 1/2$ determines the importance of each type of good in the consumption basket and $\rho > 0$ determines the elasticity of substitution between them (henceforth *Armington elasticity*). The final good obtained via aggregation is either consumed or invested in capital that depreciates within the period and comes online immediately, implying the following feasibility condition:

$$c + k = G(d, f). \tag{4}$$

Household utility function features constant relative risk aversion (CRRA) and it is Cobb-Douglas in consumption c and leisure $1-l$:

$$u(c,l) := \frac{(c^{\eta}(1-l)^{1-\eta})^{1-\sigma}}{1-\sigma}, \tag{5}$$

where $\sigma \geq 1$ denotes the coefficient of relative risk aversion and $0 < \eta < 1$ signifies the share of consumption.

Households trade a complete set of state-contingent assets (bonds) in a centralized world asset market and their choices satisfy an intertemporal budget constraint and standard condition ruling out Ponzi schemes. Let s^t denote the history of productivity shocks A, A^* up to and including period t and let the price of good d in terms of good f be denoted by p (*terms of trade* henceforth). Following any history s^t, the budget constraint of the home country's household is

$$d(s^t) + f(s^t)/p(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1}) = B(s^t) + w(s^t)l(s^t) + r(s^t)k(s^t), \tag{6}$$

where $B(s^t)$ are their bond holdings in state s^t that pay out one unit of good d per bond and which are purchased one period in advance at price $Q(s^{t+1})$.[8] For comparison, the foreign household budget constraint is

$$f^*(s^t)/p(s^t) + d^*(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1})B^*(s^{t+1}) = B^*(s^t) + (w^*(s^t)l^*(s^t) + r^*(s^t)k^*(s^t))/p(s^t). \tag{7}$$

Households choose consumption c, capital (investment) k, labor supply l, purchases of individual goods d and f, and bond holdings $B(s^{t+1})$ to maximize expected present discounted value of lifetime utility

$$\sum_t \sum_{s^t} \beta^t Prob(s^t) u(c(s^t), l(s^t)), \tag{8}$$

subject to (3), (4) and (6). First order conditions imply that i) labor supply decisions are determined by the marginal rate of substitution between consumption and leisure and the wage rate wG_d expressed in units of home consumption:[9]

$$wG_d(d,f) = -\frac{u_l(c,l)}{u_c(c,l)}; \tag{9}$$

ii) demand for individual goods is determined by the marginal rate of substitution between goods d

[8]Summation $s^{t+1}|s^t$ includes all histories s^{t+1} that are directly preceded by s^t.

[9]Supscripted notation, such as G_d or u_c, denotes partial derivatives.

and f and by the home country's terms of trade p (the price of good d in terms of good f):

$$p = \frac{G_d(d, f)}{G_f(d, f)}; \tag{10}$$

iii) capital accumulation pins down the rental price of capital rG_d in units of home consumption:

$$rG_d(d, f) = 1; \tag{11}$$

finally, iv) purchases (issuance) of state contingent bonds are determined by the marginal rate of substitution of consumption across countries and the ideal real exchange rate that measures the relative price of consumption:

$$\frac{u_c(c, l)}{u_c^*(c^*, l^*)} - \underbrace{\frac{c^* + k^*}{d^* + f^*/p} \frac{d + f/p}{c + k}}_{=:q} = 0. \tag{12}$$

This last condition is the usual *perfect risk-sharing condition* and it implies that home and foreign households act as a family that perfectly shares consumption risk. This implies that consumption is equal across the two countries only to the extent that the cost of delivering consumption is the same. For later use, we denote the ideal real exchange by q, as indicated in the formula above.

Market clearing and equilibrium. Market clearing in the goods market requires

$$\begin{aligned} d + d^* &= y, \\ f + f^* &= y^*. \end{aligned} \tag{13}$$

Asset market clearing requires $B^*(s^t) + B(s^t) = 0$. The definition of competitive equilibrium is standard and we omit it. By welfare theorems, the equilibrium allocation satisfies a static planning problem of maximizing the joint utility of home and foreign household subject to (1), (3), (4), (13), and a set of analogous conditions that are applicable to the foreign country. This planning problem is *effectively* static because there are only temporal constraints and the objective function is additively separable over time.

1.2 Preliminaries

Our goal is to characterize how trade affects shock transmission across borders. We begin by laying out the key definitions. We then decompose the international transmission mechanism to expose the key effects of trade.

1.2.1 Basic definitions

Empirical studies of the trade-comovement relation use long-term average trade between partner countries. In our model, we associate these measures with the ratio of the (deterministic) steady state value of imports to the steady state value of GDP ($A = A^* = 1$):

$$\bar{x} := \frac{\bar{f}}{\bar{y}} = \frac{\bar{f}}{\bar{f} + \bar{d}}. \tag{14}$$

Being an endogenous object, steady state trade is determined by parameter ω through the relation:

$$\bar{x} = 1 - \omega. \tag{15}$$

Correspondingly, examining the effect of trade on comovement amounts to assessing the effect of $1 - \omega$ on comovement. Other parametric approaches, such as an iceberg cost, deliver similar results and will not be considered.[10]

The standard measure of business cycle synchronization in the trade-comovement literature is the correlation coefficient between home and foreign country output. Since the correlation coefficient is less tractable, we focus here on a more direct measure of comovement that is monotonically related to the correlation coefficient:[11] the elasticity of the home country's output to foreign productivity shock,

$$\mathcal{S}(\bar{x}) := \left(\frac{\partial \log y(A, A^*)}{\partial \log A^*}\right)\left(\frac{\partial \log y(A, A^*)}{\partial \log A} + \frac{\partial \log y(A, A^*)}{\partial \log A^*}\right)^{-1}. \tag{16}$$

We normalize this elasticity by the overall response of home output to shocks to distinguish shock transmission across borders from the overall sensitivity of output to productivity shocks – which in our quantitative exercise is eliminated through calibration. Finally, we define the trade-comovement relation as

$$\mathcal{L}(\bar{x}) := \frac{d\mathcal{S}(\bar{x})}{d\bar{x}}. \tag{17}$$

$\mathcal{L}(\bar{x})$ is the marginal effect of trade on the above defined elasticity evaluated at the initial level of

[10] In the model with just labor and an iceberg cost of trade – assuming trade is defined as $\bar{x} = (1+\tau)\bar{f}/(\bar{d} + \bar{f} + \tau\bar{f})$ (*) – the results are exactly identical. The same holds true for a CES aggregator without the $1/\rho$ power applied to ω and $1 - \omega$. However, in the model with capital, the iceberg cost specification has a slight impact on the steady state level of capital and the results are qualitatively the same but the equations are not exactly identical. Defining trade in the presence of trade costs requires taking a stand on what fraction of these costs is explicitly measured in National Income and Product Accounts (NIPAs). The Bureau of Economic Analysis measures imports net of explicitly measured trade costs (insurance and freight, c.i.f.), while GDP and its components in fixed prices would include these costs. The formulation of $\bar{x}$ given by (*) assumes that trade costs are a negligible fraction of the explicitly measured trade costs, and hence the steady state price of the imported good is $(1 + \tau)$, which is what appears in the numerator of (*). The results are stronger under the assumption that trade costs are explicitly measured in NIPAs and $1 - \tau$ does not appear in the numerator of (*).

[11] This is shown in the online Appendix, Section I.

trade $\bar{x}$.

1.2.2 Decomposition of shock transmission

In order to obtain a decomposition of shock transmission in the model, we log-linearize equilibrium conditions to isolate what we refer to as the *substitution* effect and the *income* effect of shocks. To that end, we introduce an auxiliary definition of zero-sum transfers between the two countries:

$$T := \underbrace{(1 - 1/p(s^t))f(s^t)}_{=:T_p} + B(s^t) - \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1}). \tag{18}$$

T captures the zero-sum transfer payments by soaking up prices and asset payouts from the budget constraints. This is clear from (6) and (7), which after manipulations boil down to:[12]

$$d + f = y + T, \tag{19}$$

$$d^* + f^* = y^* - T. \tag{20}$$

With this definition in hand, we log-linearize the model's equilibrium conditions in two incremental steps so as to separate the substitution effect of the terms of trade p on the home country's output from the income effect of terms of trade and asset payout captured by T. We refer to these steps as *within country transmission* and *cross-country transmission*.

Within country transmission. The *first step* of our procedure assumes that both p and T are exogenous stochastic processes in order to trace back their effect on the home country's output. Since terms of trade and asset payout are the equilibrium objects that link the two countries, equilibrium conditions for each country are effectively disjoint and can be treated separately in log-linearization. Accordingly, for the home country, we log-linearize conditions comprising equations (2)-(4) and (9)-(11), as well as the budget constraint (6) replaced by (19) to introduce the auxiliary variable T to the equilibrium system. Since T is zero in the steady state, we normalize it by the steady state level of output: $\hat{T} := T/\bar{y}$. After simplifications, we obtain that the log-deviation of home output from the steady state $\hat{y}$ as a function of $\hat{A}$, $\hat{p}$ and $\hat{T}$ is given by:[13]

$$\hat{y}(\hat{A}; \hat{p}, \hat{T}) := \frac{\hat{A}}{1-\alpha} + \underbrace{\frac{1+\alpha-\eta}{1-\alpha}\bar{x}\hat{p}}_{\text{substitution effect}} - \underbrace{\frac{1-\eta}{1-\alpha}\hat{T}}_{\text{income effect}} . \tag{21}$$

[12]We used the fact that in equilibrium $rk + wl = y$.

[13]Detailed algebraic derivations are in a Mathematica notebook available online. Conditions for the foreign country follow by symmetry.

The remaining equilibrium conditions are stated in Appendix A.2 at the end.

Equation (21) decomposes the response of the home country's output to what we label as the *substitution effect* of terms of trade p (second term) and the *income effect* of bilateral transfer payment T (third term).[14] As is evident from the formula, it is only the substitution effect that directly involves trade. The effect of trade is straightforward: home labor and capital exclusively produce the home good, whereas home consumption involves the foreign good in proportion to the level of trade $\bar{x}$. Accordingly, the price of the home good relative to the final consumption/investment good – given by $\bar{x}\hat{p}$ – is naturally proportional to trade $\bar{x}$. Through the usual substitution effect, and the induced within country income effects, this relative price affects labor supply as well as the accumulation of capital, and hence output by (1). This is formally captured by equations for $\hat{k}$ and $\hat{l}$ in Appendix A.2, and can be directly gleaned from (9)-(11) and (4).

In contrast, the income effect does not directly involve trade. The income effect is associated with the direct effect of transfer payment $\hat{T}$ (for fixed $\hat{p} = 0$) on home consumption c by (19), which lowers labor supply. This effect is further reinforced by the built-in complementarity between capital and labor. Again, this is clear from equations for $\hat{k}$ and $\hat{l}$ in Appendix A.2, and can be directly seen from (2) and (4). Crucially, since all these effects work by affecting total income and within country prices, they are independent of trade.

Cross-country transmission. The *second step* of our procedure closes the equilibrium system by finding $\hat{p}$ and $\hat{T}$ so as to satisfy market clearing condition (13) and risk-sharing condition (12).

Consider first the market clearing condition (13). It is convenient to think of this condition as endogenizing the terms of trade p, as it clears the goods market. Since the first stage of the decomposition involves budget constraints (19) and (20), we only need one market clearing condition by Walras's law. To facilitate the interpretation of our results, we use the market clearing condition for good f in (13), which we conveniently express in terms of a normalized excess demand for good f: $(f + f^* - y^*)/\bar{y} = 0$. Using policy functions from the first step for both countries (see Appendix A.2.), log-linearization gives

$$2\rho(1-\bar{x})\bar{x}\hat{p} + \bar{x}(\hat{y} - \hat{y}^*) - (1-2\bar{x})\hat{T} = 0, \tag{22}$$

[14] These effects are analogous to the substitution and income effects used in consumer theory to the extent that each country is treated as an entity. Note that the substitution effect in our exercise involves induced within-country income effect.

and hence, by (21), the market clearing terms of trade level is given by:

$$\hat{p}(\hat{A}, \hat{A}^*; \hat{T}) := \frac{-\frac{\hat{A}-\hat{A}^*}{1-\alpha} + (\frac{1}{\bar{x}} + 2\frac{\alpha-\eta}{1-\alpha})\hat{T}}{2(\rho(1-\bar{x}) + \bar{x}\frac{1-\eta+\alpha}{1-\alpha})}. \tag{23}$$

The first term in the numerator of (23) relative to the denominator corresponds to the direct effect of productivity on the market clearing value of the terms of trade (assuming $\hat{T} = 0$). The link to relative productivity is implied by the relative supply term $\hat{y} - \hat{y}^*$ in (22), and the equation for output (21) at home and abroad. The second term corresponds to the effect of transfers on market clearing, which have a *direct* effect and also an *indirect* effect via output in (21). The key observation here is that trade crucially affects the effect of transfers. The *indirect* effect is of secondary importance because typical parameterizations of the standard theory imply $\alpha \approx \eta$, as we later also assume. However, the *direct effect* is particularly relevant because in the bilateral trade context the levels of trade are very small and this term is thus highly sensitive to changes in the level of trade.

The intuition for the direct effect of trade can be gleaned from equation (22). This equation shows that, first, the terms of trade affects the excess demand in proportion to trade $\bar{x}$ and, second, that the transfer payment $\hat{T}$ affects the excess demand in proportion to $1 - 2\bar{x}$ – and hence its effect is outsized for small values of $\bar{x}$. The first property simply follows from CES aggregation. The second property follows from the fact that the transfer payment $T > 0$ is redistributive and affects the excess demand for good f by raising home absorption over income $d + f - y$ by (19) at the expense of foreign absorption over income $d^* + f^* - y^*$ by (20). By homotheticity, then, demand for good f in the home country must rise in proportion to its consumption share $\bar{x}$ and the demand for good f in the foreign country must fall in proportion to its share $1 - \bar{x}$, for a total combined effect given by $1 - \bar{2}x$.

Finally, consider the risk sharing condition (12). It is convenient to think of this condition as endogenizing the transfer payment T, since risk sharing is on the margin determined by trade in assets. As above, we log-linearize this condition and plug in policy functions derived in the first step (see Appendix A.2), which implies that equilibrium transfer payment is given by:

$$\hat{T}(\hat{A}, \hat{A}^*; \hat{p}) := \frac{-(1+\eta(\sigma-1))\frac{\hat{A}-\hat{A}^*}{1-\alpha} - (1 - 2\bar{x}\frac{\eta-\alpha}{1-\alpha})\hat{p}}{2\sigma\eta/(1-\alpha)}. \tag{24}$$

Using equation (18), we additionally uncover that the contribution of the terms of trade is

$$\hat{T}_p(\hat{p}) = \bar{x}\hat{p}. \tag{25}$$

The first term in the numerator of (24) relative to the denominator captures the direct effect of productivity on equilibrium transfers. Intuitively, higher foreign productivity raises consumption in the foreign country, increasing the marginal rate of substitution u_c/u_c^* in (12), which ceteris paribus raises the transfer payment T. This follows from (19) and (20) (assuming $\hat{p} = 0$). The second term captures the effect of the terms of trade. The effect of terms of trade operates via both the marginal rate of substitution – since by (21) and (4) terms of trade affects consumption – as well as the ideal real exchange rate q in (12) given by $\hat{q} = (1 - 2\bar{x})\hat{p}$ after log-linearization. Trade enters through the second term and it is of lesser importance. This is particularly true under a typical parameterization of the standard theory that implies $\alpha \approx \eta$, in which case trade drops out from the equation. However, even when $\alpha \neq \eta$, the effect of trade is small in comparison to (23) for low levels of trade.

Equations (23) and (24) define a fixed point. For later use, we denote this fixed point by dropping interdependent notation; that is, we write $\hat{p}(\hat{A}, \hat{A}^*)$ instead of $\hat{p}(\hat{A}, \hat{A}^*; \hat{T})$, and analogously for $\hat{T}$. We use partial derivatives in reference to $\hat{p}(\hat{A}, \hat{A}^*)$ whenever we mean the partial derivative of the fixed point values of these variables. We now proceed to the analysis of the effect of trade on comovement.

1.3 Analysis of trade-comovement puzzle in baseline model

Equations (21)-(24) imply the following decomposition of the trade-comovement relation (17):

$$\mathcal{L} = \underbrace{(1 - \eta + \alpha)\left(\frac{\partial \hat{p}(\hat{A}, \hat{A}^*)}{\partial \hat{A}^*} + \bar{x}\frac{\partial^2 \hat{p}(\hat{A}, \hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}}\right)}_{\text{substitution effect channel } \mathcal{L}_S} - \underbrace{(1 - \eta)\frac{\partial^2 \hat{T}(\hat{A}, \hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}}}_{\text{income effect channel } \mathcal{L}_I}. \tag{26}$$

The first term $\mathcal{L}_{\mathcal{S}}$ captures the effect of trade operating through the substitution effect of the foreign shock. What is important here is how much the terms of trade responds to the foreign shock (first term in the bracket), and to a lesser extent the effect of trade on the size of this response – since this second term is proportional to the level of trade that is a small number in the bilateral context considered here. We label this term the *substitution effect channel* of trade. The second term $\mathcal{L}_{\mathcal{I}}$ captures the effect of trade through the income effect of the foreign shock. This term solely depends on how trade affects the equilibrium response of the risk-sharing transfer T to the foreign shock. We label this term the *income effect channel* of trade.

As we show next, for most parameter values, these two channels have offsetting effects on the model-implied trade-comovement relation, which crucially contributes to the trade-comovement puzzle. They are also driven by distinct structural assumptions.

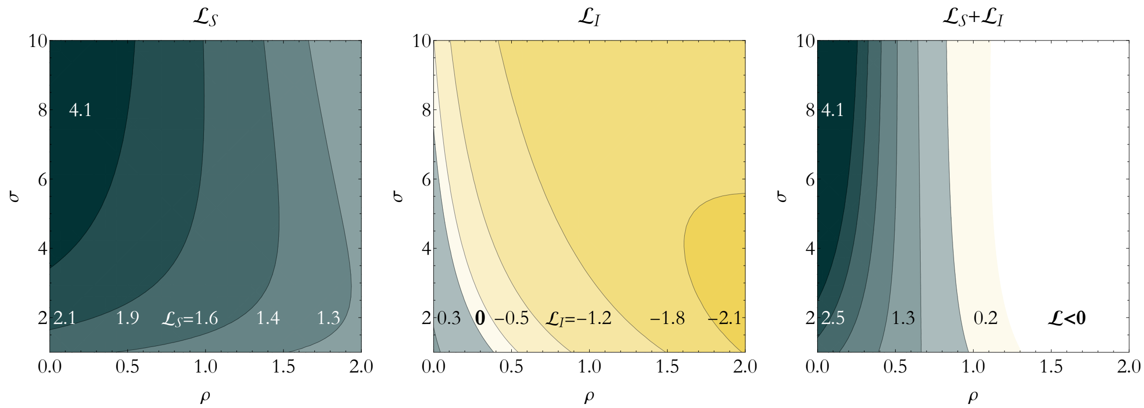


Figure 1: Decomposition of model-implied trade-comovement relation $\mathcal{L}$.

Notes: The figure illustrates the decomposition of the model-implied trade-comovement relation $\mathcal{L}$ implied by equation (26) as a function of σ (vertical axis) and ρ (horizontal axis). The leftmost panel shows the contribution of the substitution effect channel $\mathcal{L}_S$, the middle panel shows to the income effect channel $\mathcal{L}_I$, and the rightmost panel shows the combined effect $\mathcal{L} := \mathcal{L}_I + \mathcal{L}_S$. The figure assumes the following parameter values: $\bar{x} = 5\%, \alpha = \eta = 1/3$.

1.3.1 Decomposition of the effect of trade on comovement

In order to gain tractability, we further restrict the values of the parameters that are less pertinent to our analysis.[15] We use a numeric value for $\alpha = 1/3$ and $\eta = 1/3$, which correspond to a typical calibration of the standard theory. We focus on trade levels satisfying $0 < \bar{x} \leq \min\{1/(1+\sigma/2), 1/3\}$. In a bilateral trade context, which involves low values of trade, the latter restriction is innocuous except for unreasonably high values of the relative risk aversion parameter σ.[16]

Substitution effect channel. Figure 1 (left panel) illustrates the contribution of the substitution effect channel $\mathcal{L}_S$ to the trade-comovement relation $\mathcal{L}$ implied by the model for all values of the free parameters ρ and σ and for a fixed (initial) level of trade at $\bar{x} = 5\%$.[17] As the figure shows, the substitution effect channel is a robust source of positive relation between trade and comovement. We formalize this result in Proposition 1 below. All proofs are contained in Appendix A.1.

Proposition 1. *$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > 0$ and $\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} + \bar{x}\frac{\partial^2 \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} > 0$, hence $\mathcal{L}_S > 0$.*

As equation (26) shows, the substitution effect leads to a positive association between trade and comovement for two reasons. First, trade increases the sensitivity of the home country's output to terms of trade p, giving rise to the first order term $\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}$, and in equilibrium the terms of trade

[15]These values imply empirically plausible values for the labor share and the share leisure in time endowment.

[16]We prove all results by plugging in numeric values for α and η. The results qualitatively generalize to a range of values of $\eta \neq \alpha$ but they are cumbersome to handle.

[17]We choose a relatively high number – given bilateral trade levels in the data – because lower values only reinforce the results we are about to discuss.

appreciates after a positive productivity shock abroad. Second, trade affects how much terms of trade moves in response to the shock, which is captured by the second order term $\bar{x}\frac{\partial^2 \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}}$. The first effect is key because the second effect is proportional to trade and hence small in the bilateral trade context considered here (the median level of trade between country pairs in our sample is 0.85%). We now discuss the intuition for the first effect.

As discussed in Section 1.2.2, trade increases the sensitivity of the home country's output to the terms of trade because it increases the share of the foreign good in final consumption/investment spendings, which increases the effect of the terms of trade on the price of the home good relative to the home consumption/investment given by $\bar{x}\hat{p}$. Since home labor and capital exclusively produce the home good, labor supply and capital both increase in response to this relative price. Formally, this is clear from equations for $\hat{k}$ and $\hat{l}$ listed in Appendix A.2, and more directly can be gleaned from (9)-(11) and (2). The increase in labor and capital has a positive effect on home output by (1), which gives rise to the first term $(1-\eta+\alpha)$ in (26).

The terms of trade appreciates after a positive productivity shock in the foreign country because such a shock increases the supply of the foreign good and the foreign good is a normal good. Formally, this is evident from the system that pins down the equilibrium response of terms of trade and transfers, i.e., equations (23) and (24), which, under parametric restrictions imposed here, boil down to (since we focus on foreign shock, we set $\hat{A}=0$):

$$\begin{aligned} \hat{p} &= \frac{1}{3\bar{x}+2(1-\bar{x})\rho}\left(\frac{3}{2}\hat{A}^* + \frac{1}{\bar{x}}\hat{T}\right) \\ \hat{T} &= \frac{1}{\sigma}\left(\frac{2+\sigma}{2}\hat{A}^* - \hat{p}\right). \end{aligned} \tag{27}$$

As these equations show, the direct effect of the shock is that it leads to an appreciation of the terms of trade (assuming $\hat{T}=0$) and a positive transfer payment $\hat{T}>0$ (assuming $\hat{p}=0$). Since positive transfers only reinforce the appreciation of the terms of trade, the terms of trade always appreciates in equilibrium. The intuition for these effects follows from the discussion of (23) and (24) in Section 1.2.2.

Income effect channel. Figure 1 (middle panel) illustrates the contribution of the income effect channel $\mathcal{L}_I$ to the trade-comovement relation $\mathcal{L}$ implied by the model. Figure 1 (rightmost panel) shows the net effect. As we can see, with the exception of very low values of Armington elasticity ρ, the income effect channel is negative and potent enough to reverse the trade-comovement relation implied by the model. We formalize this result in Proposition 2 below.

Proposition 2. *If* $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$*, then* $\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > 0$ *and* $\frac{\partial^2 \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} > 0$*, hence* $\mathcal{L}_I < 0$*.*

As equation (26) shows, the income effect channel is negative because trade increases risk-sharing transfers after a positive productivity shock abroad, as implied by the positive cross-partial derivative $\frac{\partial^2 \hat{T}(A,A^*)}{\partial A^* \partial \bar{x}}$. This is because risk-sharing transfers are positive under the parametric restrictions we have imposed – as Proposition 2 also shows – and positive transfers are a source of negative comovement due to their negative income effect on home labor supply. Formally, this is clear from (9), since transfers raise consumption by (19) and (4) and hence increase leisure. The built-in complementarity between capital and labor only reinforces this effect, which follows from (2) and (11). Finally, the foreign country pays a transfer to the home country because the shock increases foreign consumption. This follows from the risk sharing condition (12) and the above link between transfers and consumption.

Trade enters the picture because a positive transfer paid by the foreign country leads to an *inversely proportional to trade* excess supply of the foreign good – since trade determines how absorption in each country affects the demand for the individual country-specific goods and the transfer raises absorption in the home country over output $(d + f - y)$ at the expense of that in the foreign country. This follows from equations (19) and (20) and homotheticity. By the risk sharing condition (12), the excess supply of the foreign good discourages risk sharing by making home consumption relatively more expensive than foreign consumption (the ideal real exchange rate q appreciates), which implies that trade *encourages* risk sharing and leads to larger equilibrium transfers. This is what gives rise to the term $1/\bar{x}$ in (27) and makes the cross-partial derivative $\frac{\partial^2 \hat{T}(A,A^*)}{\partial A^* \partial \bar{x}}$ positive.

Interestingly, when Armington elasticity ρ is sufficiently low, transfers change direction because the response of terms of trade is so pronounced.[18] This can be seen in Figure 1 (middle panel), which shows a reversed sign of the income effect for low values of ρ. The reason why the direction of the income effect reverses when Armington elasticity is low can be understood by applying our reasoning in reverse. Negative transfers *alleviate* rather than exacerbate the oversupply of the foreign good that arises after the positive shock abroad, and via the general equilibrium feedback effect trade amplifies this now *beneficial* effect. As a result, a higher level of trade implies that equilibrium transfers become even more negative and trade has a *positive* rather than negative effect on comovement. Corollary 1 formalizes this result:

Corollary 1. $\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} < 0$ *implies* $\frac{\partial^2 \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} < 0$ *and hence* $\mathcal{L}_I > 0$*.*

[18] This is a sufficient condition but not necessary.

1.3.2 Trade-comovement puzzle

With the exception of low values of Armington elasticity ρ, the effect of trade via the income channel offsets or significantly weakens the otherwise positive effect of trade via the substitution channel. This is the essence of the trade-comovement puzzle, since most parameterizations of the model use Armington elasticity above one. For such parameter values, the model implies a negative or weak relation between trade and comovement. As shown by Kose and Yi (2006), while low levels of elasticity do improve the model's performance, elasticity has to be very low to be able to make a quantitative difference, which we confirm in the quantitative section.

Our analysis of the puzzle implies that the key culprit is the income effect channel, and hence any modifications that suppress this channel or change how trade affects the income effect are potentially promising in resolving the puzzle. We exploit this insight in the next section, but before we do so, we relax the assumption of the baseline model that capital depreciates within a single period. This extension is essential for the discussion of dynamic trade elasticity in a later section.

1.4 Extended baseline model

We now drop the assumption of full depreciation of capital by considering a prototypical two-period economy featuring durable capital with an adjustment cost. The model assumes that productivity is i.i.d. and that shocks only occur in the first period – which is associated with the frequency at which business cycle moments are measured. Since this setup boils down to the baseline model under parameter settings that imply full depreciation of capital and no convex adjustment cost, we refer to it as the *extended baseline model*. For brevity, we exploit welfare theorems and lay out the setup by focusing on the efficient allocation.

The equilibrium allocation in the extended baseline model satisfies the following two-period planning problem:

$$\max\{u(c,l) + u(c^*,l^*) + u(c_{+1},l_{+1}) + u(c^*_{+1},l^*_{+1})\}$$

subject to

$$c + \delta\bar{k} + \Delta k + \psi\Delta k^2 = G(d,f) \tag{28}$$

$$c_{+1} + \delta\bar{k} - (1-\delta)\Delta k = G(d_{+1},f_{+1}) \tag{29}$$

$$d + d^* = A(\bar{k}+\Delta k)^\alpha l^{1-\alpha} \tag{30}$$

$$d_{+1} + d^*_{+1} = \bar{k}^\alpha l_{+1}^{1-\alpha}, \tag{31}$$

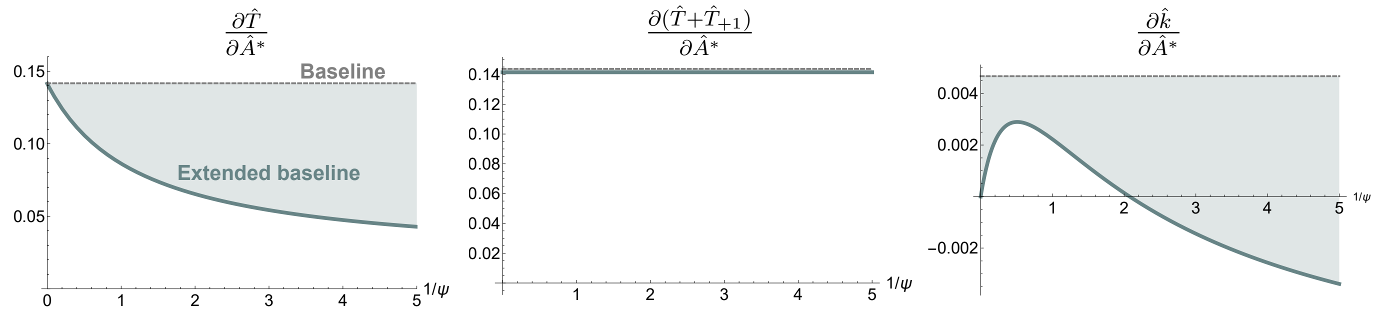


Figure 2: Risk-sharing transfers and capital accumulation in extended baseline model.

Notes: The first two panels from the left show the response of first period transfer payment $\hat{T}$ and the present value of transfers $(\hat{T} + \hat{T}_{+1})$ to a positive productivity shock in the foreign country. The rightmost panel shows the response of home country's capital stock to the shock, where we define $\hat{k} = \Delta k/\bar{k}$. The horizontal axis is the inverse of the convex adjustment cost on capital, $1/\psi$. The dotted line (labeled as baseline) shows the baseline model from Section 1.1 for reference. The figure assumes the following parameter values: $\rho = 5/4$, $\sigma = 2$, $\delta = 1/20$, $\alpha = \eta = 1/3$ and $\bar{x} = 5\%$.

and an analogous set of constraints applying to the foreign country. The subscripted variables pertain to second-period variables, δ denotes capital depreciation, and ψ parameterizes the convex cost of investing in capital (or disinvesting). Choice variables include $d, d_{+1}, f, f_{+1}, c, c_{+1}, l_{+1}, \Delta k$ in the home country in addition to an analogous set of variables pertaining to the foreign country. Other than that, the notation is as in the baseline setup.

The planner maximizes the joint utility of home and foreign households subject to resource constraints that are analogous to the baseline setup with one exception: the choice of Δk that connects the two periods. Δk pertains to the change in investment in capital relative to its initial value $\bar{k}$. Its effect on consumption and the level of capital in the economy is consistent with the usual law of motion for capital and no time to build.[19] The initial level of capital $\bar{k}$ is assumed to be stationary in the sense that $\Delta k = 0$ solves the above problem for deterministic value of $A = A^* = 1$. As a special case, the baseline setup amounts to setting $\psi = 0$ and $\delta = 1$.

One can think of this model as approximating an infinite horizon economy with capital. Relative to an infinite horizon model, the above two period framework assumes that capital returns to the steady state value after a deviation for a single period and features no discounting across periods. Since we consider a linear approximation of a response to a marginal i.i.d. shock around the steady state, the fact that capital cannot be smoothed over the subsequent periods is without much loss. Assuming a positive discount factor would only scale down the level of capital in the steady state and have a similar effect to lowering depreciation of capital δ.

[19] We maintain the assumption of no time to build in the law of motion of capital. Introducing time to build has little impact on the results discussed here.

The supporting equilibrium prices can be found from the first order conditions (2), (10) and (12), which apply without any modifications. The definition of T in (18) also applies, except that with durable capital it is not only the first period transfer T that is relevant for first period choices but also the future transfer T_{+1}. This is because households in the extended setup can reduce investment in capital and use capital stock as a buffer to smooth consumption across the two periods.

1.5 Analysis of trade-comovement puzzle in extended baseline model

Figures 2 and 3 illustrate how the responses of key variables differ in the extended model. As before, we consider a positive productivity shock in the foreign country and plot the first-period responses of each respective variable as a function of the inverse of the convex adjustment cost on investment $1/\psi$, which implies that moving along the horizontal axis amounts to relaxing the capital adjustment friction. We set $\rho = 5/4$, $\sigma = 2$ and $\delta = 1/20$.

The key difference with respect to the baseline model is that the presence of durable capital brings about a back-loaded response of transfers and capital to shocks. This is clear from the leftmost and the middle panels of Figure 2, which show a higher response of the present value of transfers $\partial(\hat{T} + \hat{T}_{+1})/\partial\hat{A}^*$ than the response of the first period transfer $\partial\hat{T}/\partial\hat{A}^*$ as the convex adjustment cost on capital is relaxed.[20] The rightmost panel shows that consumption is insulated from lower first period transfers by reduced investment in capital (negative Δk). We do not illustrate it here but investment abroad rises to similarly offset the effect of deferred transfers on foreign consumption. This dynamic raises utility because it helps take advantage of higher productivity of capital in the foreign country and facilitates consumption smoothing across periods.

The trade-comovement relation is nonetheless similar as in the baseline model. This is clear from the leftmost panel of Figure 3. The key to this result is the fact that the effect of trade on the present value of transfers is essentially identical as in the baseline model – as shown in the middle panel. Since capital can be used to smooth consumption, it is the present value of transfers that is relevant for consumption and labor supply decisions in the extended model. This is because investment in capital can be adjusted to smooth consumption, and it is consumption that affects labor supply decisions in (9). As the rightmost panel illustrates, the effect of trade on the price of the home country's good relative to the final consumption/investment good – governing the substitution effect channel – is

[20] Linearization and the fact that $T = 0$ in the steady state implies that the stochastic discount factor drops out from the linearized equation of the present value of transfers, which is $\hat{T} + \hat{T}_{+1}$, where, as before $\hat{T} = T/\bar{y}$ and similarly for $\hat{T}_{+1}$.

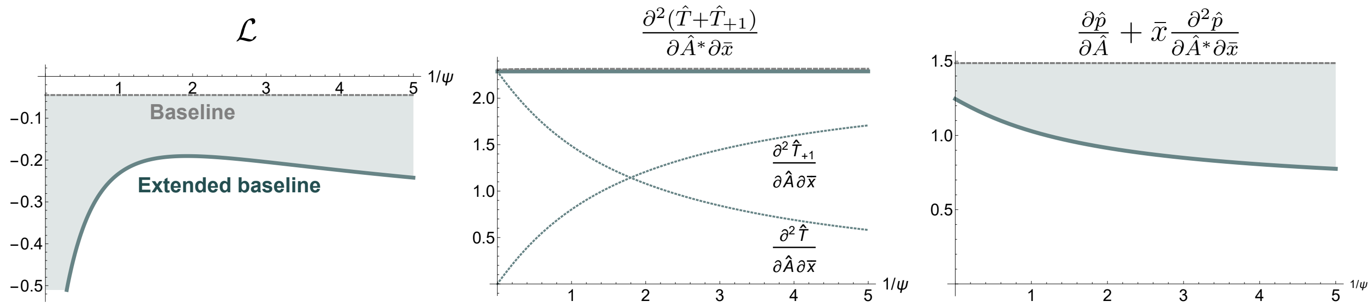


Figure 3: Decomposition of trade-comovement relation in extended baseline model.

Notes: The leftmost panel illustrates the trade-comovement relation $\mathcal{L}$ in the extended baseline model. The remaining panels illustrate the key terms underlying the income effect and the substitution effect channels. The middle panel additionally shows the decomposition of the present value of transfers to first- and second-period transfers (dotted lines). The horizontal axis is the inverse of the convex adjustment cost on capital accumulation $1/\psi$. The dotted line (labeled as baseline) shows the baseline model from Section 1.1 for reference. The figure assumes $\rho = 5/4$, $\sigma = 2$, $\delta = 1/20$, $\alpha = \eta = 1/3$ and $\bar{x} = 5\%$.

only slightly weaker relative to the baseline model.

Intuitively, trade does not interfere with the illustrated dynamic response of transfers and capital because its effects cancel out. On the one hand, higher trade implies that more of the home good is required to build up foreign capital after the shock to defer transfers. On the other hand, trade makes it easier to convert the excess foreign capital in the second period to home consumption by reducing foreign investment. The end result is an identical response of the present value of transfers as in the baseline model, and hence a similar response of consumption and labor. These offsetting effects of trade on first- and second-period transfer payments can be seen in the middle panel of Figure 3.

1.6 Candidate resolutions of trade-comovement puzzle

We now consider three natural candidate resolutions of the trade-comovement puzzle that our analysis of the puzzle points toward: i) financial autarky – an extreme form of a financial market friction that prohibits cross-border asset trade; ii) GHH preferences; and iii) dynamic trade elasticity that is low in the short run but high in the long run. These modifications are expected to improve the model's performance by attenuating the income effect channel or, in the case of dynamic trade elasticity, by fundamentally changing the impact of trade on the income effect.

We apply these modifications to both the baseline setup from Section 1.1 and the extended setup from Section 1.4. The results for the baseline setup are depicted in Figure 4 and the results for the extended baseline setup are depicted in Figure 5. In the case of extended baseline setup, we further simplify the parameterization by assuming $\sigma = 2, \delta = 1/20$ and place the inverse of the convex

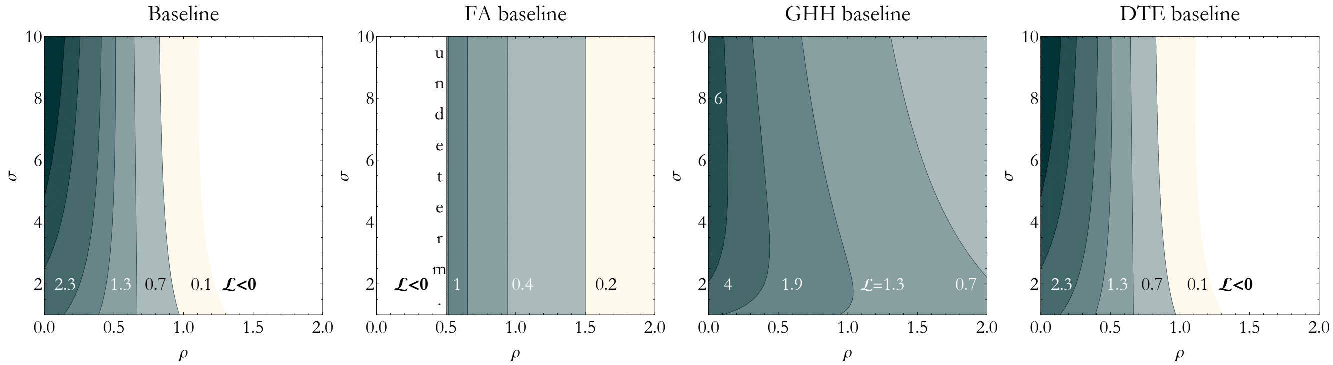


Figure 4: Trade-comovement relation across variants of baseline model.

Notes: The figure illustrates the trade-comovement relation $\mathcal{L}$ in the baseline setup from Section 1.1 (leftmost panel) and in the baseline setup featuring modifications discussed in Section 1.6 (other panels). The figure assumes the following parameter values: $\alpha = \eta = 1/3$ and $\bar{x} = 5\%$.

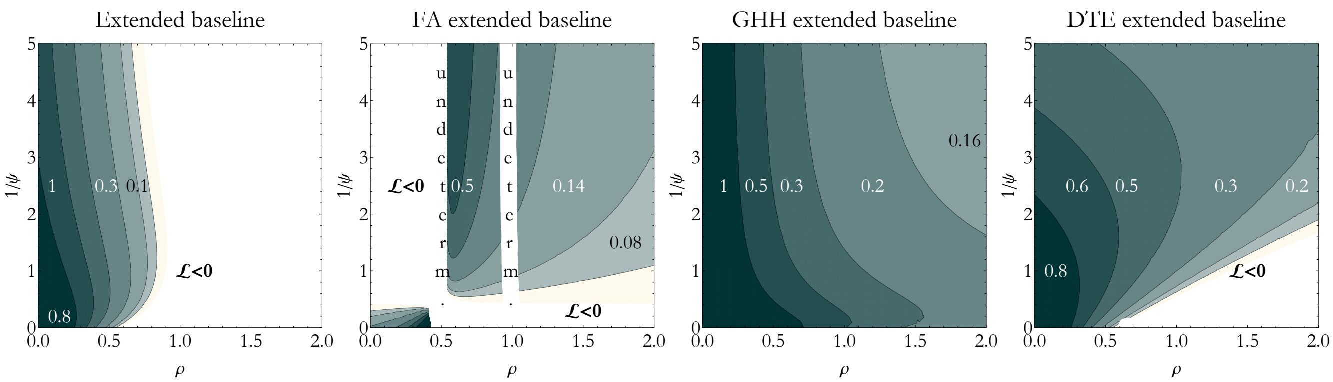


Figure 5: Trade-comovement relation across variants of extended baseline model.

Notes: The figure illustrates the trade-comovement relation $\mathcal{L}$ in the extended baseline setup from Section 1.4 (leftmost panel) and in the extended baseline setup featuring modifications discussed in Section 1.6 (other panels). In the case of the DTE model, Armington elasticity applies to the first period only and it determines the short-run trade elasticity. The long-run trade elasticity is infinite. The figure assumes $\sigma = 2$, $\delta = 1/20$, $\alpha = \eta = 1/3$, and $\bar{x} = 5\%$.

adjustment cost $1/\psi$ on the vertical axis instead of the risk aversion parameter σ.

1.6.1 Financial autarky (FA)

Financial autarky is an extreme form of asset trade restriction that bans cross-border trade and thus eliminates the income effect implied by asset payouts, i.e., the last two terms in the definition of T in (18). Formally, financial autarky imposes a market clearing condition requiring that bond holdings in each country are zero in equilibrium; that is, $B(s^t) = B^*(s^t) = 0$ for all s^t.

As is clear from Figure 4 (second panel), financial autarky in the baseline setup generates a more positive trade-comovement relation for some parameter values. However, the results fall short considerably relative to the substitution effect channel in Figure 1.

Table 1: Decomposition (21)-(25) across variants of the baseline model.

Baseline model		Autarky	GHH
$\hat{y} \propto$	$\hat{A} + \bar{x}\hat{p} - \frac{2}{3}\hat{T}$	$\hat{A} + \frac{1}{3}\bar{x}\hat{p}$	$\hat{A} + \frac{13}{8}\bar{x}\hat{p}$
$\hat{p} =$	$\frac{1}{2\rho(1-\bar{x})+3\bar{x}} \left(-\frac{3}{2}(\hat{A} - \hat{A}^*) + \frac{1}{\bar{x}}\hat{T}\right)$	same	$\frac{1}{8\rho(1-\bar{x})+13\bar{x}} \left(-\frac{21}{2}\left(\hat{A} - \hat{A}^*\right) + \frac{4(1-2\bar{x})}{\bar{x}}\hat{T}\right)$
$\hat{T} =$	$\frac{1}{\sigma}\left(-\frac{2+\sigma}{2}(\hat{A} - \hat{A}^*) - \hat{p}\right)$	$\bar{x}\hat{p}$	$-\frac{1}{2}\left(\hat{A} - \hat{A}^*\right) - \frac{4(1-2\bar{x})}{21\sigma}\hat{p}$
$\hat{T}_p =$	$\bar{x}\hat{p}$	$\bar{x}\hat{p}$	$\bar{x}\hat{p}$

Notes: The table derives (21)-(25) for listed modifications discussed in Section 1.6 embedded in the baseline model (including the baseline model).

The reason why this is the case is apparent from the equation for T_p derived in Table 1 (column two, row 4) for the baseline model assuming financial autarky. The equation shows that risk sharing transfers are in part associated with the income effect of terms of trade, as implied by (18), which is not eliminated by financial autarky, and which is proportional to trade because trade determines the exposure of the home country to the terms of trade. As a result, the income effect channel remains sizable. The results are similar in the case of the extended baseline model.[21]

1.6.2 GHH preferences

GHH preferences suppress the income effect channel by eliminating the income effect on labor supply decisions. In order to analyze the effect of these preferences, we replace (5) by

$$u(c,l) = \frac{1}{1-\sigma}\left(c - \theta\frac{l^{1+v}}{1+v}\right)^{1-\sigma}, \tag{32}$$

where v and θ match the Frisch elasticity in the baseline setup as well as the share of labor in time endowment.[22]

Table 1 confirms that GHH preferences indeed eliminate the income effect channel. As Figure 4 and Figure 5 show, the improvement over financial autarky is notable. However, when trade elasticity

[21]Financial autarky implies a locally undetermined response of comovement to trade for low values of Armington elasticity parameter ρ. This is because both the income and the substitution channels are linked to the terms of trade and, as is well known from the work of Corsetti et al. (2008), financial autarky leads to a reversal in the terms of trade dynamics – which is depreciating instead of appreciating after a positive productivity shock abroad. In these cases, home and foreign goods become Giffen goods and demand line becomes upward sloping. Around these points of the parameter space the volatility of terms of trade locally vanishes, which implies an undetermined value of our local measure of trade-comovement relation $\mathcal{L}$.

[22]The Frisch elasticity implied by preferences in (5) is $\frac{1-\bar{l}}{\bar{l}}\frac{1-\eta(1-\sigma)}{\sigma} = \frac{4}{3}$ in the neighborhood of the deterministic steady state. The share of labor in time endowment is 1/3. This amounts to setting $v = 4/3$ and $\theta = \frac{2}{\sqrt[6]{3}}$, and $\theta = \frac{2}{\sqrt[6]{3}\sqrt{\delta}}$ in the GHH model using the setup with durable capital from Section 1.4.

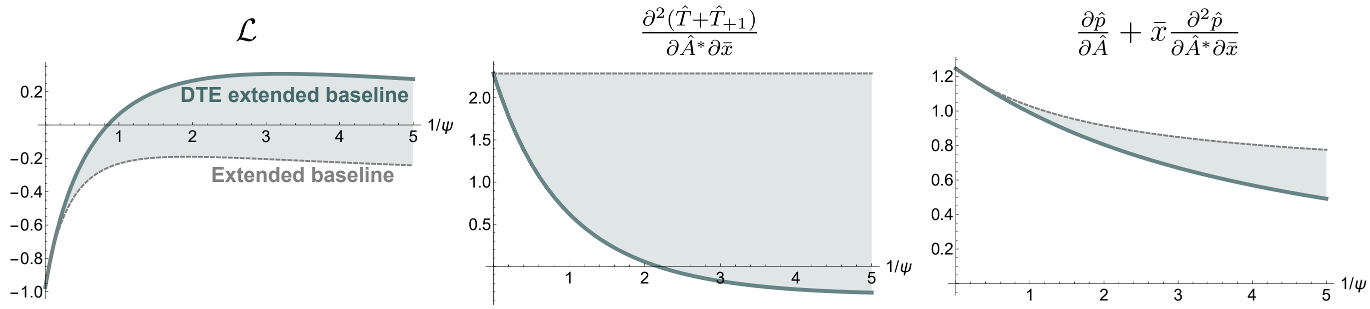


Figure 6: Effect of dynamic trade elasticity on trade-comovement in extended baseline model.

Notes: The leftmost panel shows the model-implied trade-comovement relation $\mathcal{L}$ in the extended baseline setup featuring dynamic trade elasticity (as described in Section 1.6.3). The middle panel shows the effect of trade on the response of the present value of transfers across the two models. The rightmost panel focuses on the effect of trade on the response of the price of home country final consumption good relative to the home country's good. The dotted line (labelled extended baseline) shows the extended baseline setup from Section 1.4. The figure assumes $\rho = 5/4$, $\sigma = 2$, $\delta = 1/20$, $\alpha = \eta = 1/3$, and $\bar{x} = 5\%$.

is above one, dynamic trade elasticity model implies trade-comovement relation that is at least 30 percent stronger. We find an even larger difference in the quantitative model.

1.6.3 Dynamic trade elasticity (DTE)

Dynamic elasticity affects the income effect channel by suppressing the countervailing effect of trade on back-loading of transfers seen in the extended baseline setup – which, recall, is beneficial to smooth consumption across periods and takes advantage of higher relative productivity of capital in the foreign country. As we show, this can flip the sign of the income effect channel.

To illustrate this effect of dynamic trade elasticity, we use a stylized setup based on the extended baseline model. Specifically, we assume that the Armington elasticity between home and foreign goods ρ becomes infinite in the second period, implying that at that point home and foreign goods become perfectly substitutable. The increase in elasticity is modeled under perfect foresight. Formally, we replace (29) in the setup from Section 1.4 by

$$c_{+1} + \delta\bar{k} + (1-\delta)\Delta k = \lim_{\rho\to\infty} G(d_{+1}, f_{+1}) = d_{+1} + f_{+1}.$$

As a result, the Armington elasticity pertains to the first period only and it describes the short-run response of trade to relative prices. In our later quantitative model, we endogenize dynamic elasticity by assuming a high value of the Armington elasticity and introducing a convex adjustment cost applied to the trade share f/d in each country. For a microfounded interpretation of such a convex adjustment cost in a search model of trade, refer to Drozd and Nosal (2012).

As shown in Figure 4, dynamic trade elasticity has no bite in the baseline model ($\delta = 1, \psi = 0$).

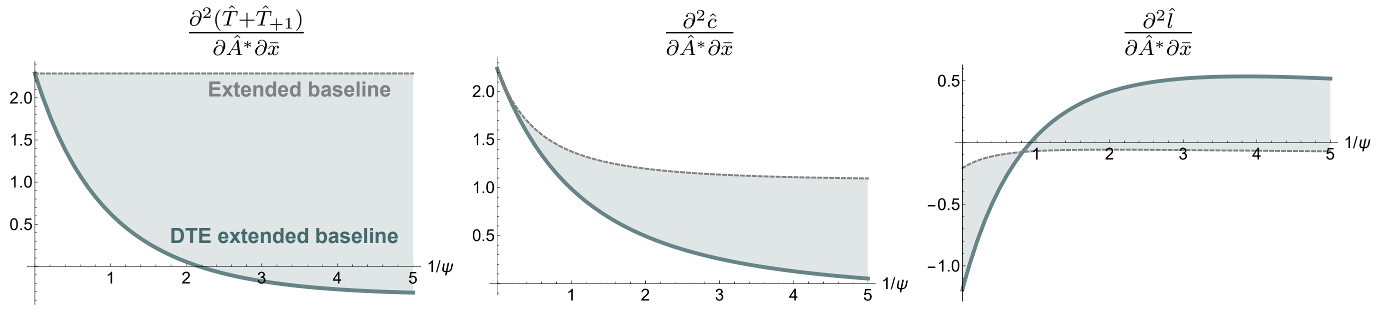


Figure 7: Effect of dynamic trade elasticity on income effect channel in extended baseline model.

Notes: The leftmost panel shows the effect of trade on the response of the present value of transfer payments to the foreign productivity shock as a function of the inverse of the convex adjustment cost on capital $1/\psi$ in the extended baseline model featuring dynamic trade elasticity. The right panel shows the effect of trade on the response of consumption and labor. The dotted line (labelled extended baseline) shows the extended baseline setup from Section 1.4. The figure assumes $\rho = 5/4$, $\sigma = 2$, $\alpha = \eta = 1/3$, $\delta = 1/20$ and $\bar{x} = 5\%$.

This is understandable. The baseline economy does not have the requisite technology to move resources across the periods and the resulting responses are similarly determined by the static trade-off in the first period. Formally, this follows from the fact that the planning problem becomes separable across the two periods when $\delta = 1$.

However, as Figure 5 shows, introducing dynamic trade elasticity in the presence of durable capital and convex adjustment cost on capital leads to a significant improvement. Figure 6 illustrates the core mechanism. As the middle panel shows, the key difference is that in the dynamic trade elasticity model with durable capital the effect of trade on the response of the present value of transfers to the foreign shock is smaller than in the extended baseline setup. The difference is substantial as the convex adjustment cost is relaxed along the horizontal axis, with the effect of trade eventually becoming negative. The rightmost panel shows that the response of the price of home consumption is similar across the two models; that is, the response of $\bar{x}p$ to the foreign shock is similarly affected by trade. Accordingly, the key effect of the dynamic trade elasticity is that it weakens and even flips the sign of the income effect channel.

Figure 7 confirms that this is the case. As the middle panel shows, trade affects the response of consumption in the first period less in the dynamic trade elasticity model than in the baseline model. In a similar vein, by the built-in complementarity between consumption and leisure in preferences, trade affects the response of home labor in the first period more because the adverse effect of consumption on labor supply is smaller. This follows from (9), which applies to the extended baseline model without changes.

Intuitively, dynamic trade elasticity affects the income effect channel because the excess supply of the foreign good associated with risk sharing transfers can be alleviated by simply delaying them – which is what the responses in the leftmost and the middle panels of Figure 8 show. This is because, with enough lead time, home and foreign goods become more substitutable and the cost of the excess supply implied by transfers declines over time. In a model with durable capital, transfers can be delayed without sacrificing consumption smoothing by changing investment in capital, which falls in the home country to finance higher consumption in the expectation of second period transfer T_{+1} and rises in the foreign country to finance the second period transfer after the shock dies out. This effect is illustrated for the home country in the rightmost panel of Figure 8. Crucially, trade severs this mechanism because, to achieve that, a larger adjustment of investment across countries is necessary, and this is costly for two reasons. One, the marginal productivity of capital diminishes and, two, there is convex adjustment cost on investment. The end result is that in the dynamic elasticity model trade severs risk sharing and leads to lower risk-sharing transfers (for some parameter values). This is illustrated in the middle panel of Figure 6. In contrast, in the standard model, delaying transfers would be merely kicking the proverbial can down the road because home and foreign goods are equally substitutable on impact as in the future – which explains the flat line in the middle panel of Figure 6 for the baseline model.[23]

The same mechanism implies a more pronounced back-loading of transfers in the dynamic elasticity model to start with. This is clear from Figure 8. As we later show in the quantitative section, this is beneficial for the model's quantitative performance because it makes the current account more countercyclical – a weak point of the standard theory.

2 Quantitative analysis

We now put all the models through a rigorous quantitative test in the spirit of the one in Kose and Yi (2006). Before we present our results, we discuss how we generalize the model so that it can be taken to the data. We then discuss the quantitative goal we set for the theory to account for and describe the calibration of model parameters.

2.1 Baseline quantitative model

Our quantitative model generalizes the setup along the following four key dimensions:

[23] In the online Appendix, we consider a simple example with storage to illustrate how storage is used to smooth consumption and formally show that the variance of investment is increasing in $\bar{x}$.

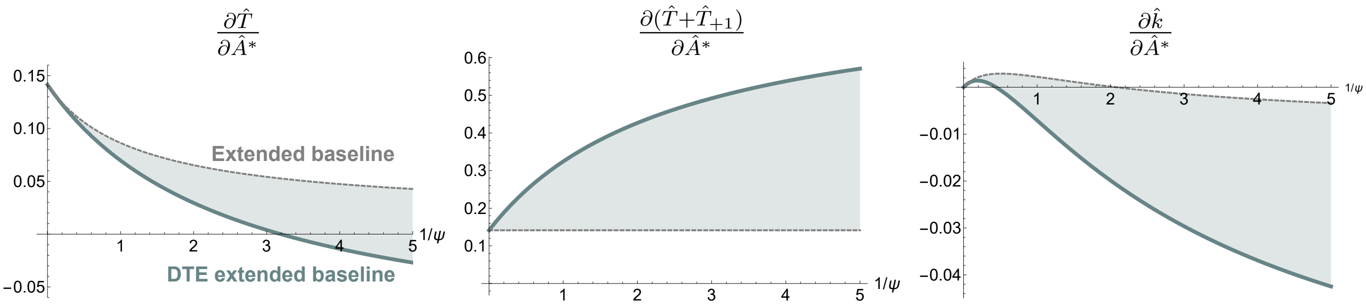


Figure 8: Effect of dynamic trade elasticity on risk-sharing transfers in the extended baseline setup.

Notes: The left panel shows the response of first period transfer payment $\hat{T}$ to the foreign productivity shock as a function of the inverse of the convex adjustment cost on capital $1/\psi$. The middle panel shows the present value of transfer payments $(\hat{T}+\hat{T}_{+1})$ and the rightmost panel shows the response of home capital. The dotted line corresponds to the encompassing extended baseline model laid out in Section 1.4. The shaded area shows the difference between the two models. The figure assumes $\rho = 5/4$, $\sigma = 2$, $\delta = 1/20$, $\alpha = \eta = 1/3$, and $\bar{x} = 5\%$.

1. We distinguish between trade openness and bilateral trade intensity by considering a three-country system: a large country, referred to as *rest of the world* (Country 3), and a symmetric bilateral pair of two smaller countries: *home* and *foreign* (Countries 1 and 2, respectively). Consistent with this assumption, the composite final good in each country is a weighted CES of three goods:
$$G(d,f,g) = \left(\omega_d d^{\frac{\rho-1}{\rho}} + \omega_f f^{\frac{\rho-1}{\rho}} + \omega_g g^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}. \tag{33}$$
We model country sizes by assuming that the measure of households is L times higher in the large country than in each of the small countries.

2. We endogenize *dynamic trade elasticity* by adding a convex adjustment cost on trade shares. The convex adjustment cost is paid in final consumption by households in each country and for the home country is given by:
$$\Phi(d,f,g) = \frac{\phi}{2}\left\{\left(\frac{f/d}{f_{-1}/d_{-1}} - 1\right)^2 + \left(\frac{g/d}{d_{-1}/g_{-1}} - 1\right)^2\right\}. \tag{34}$$

3. As in the extended baseline setup, we assume that capital is durable and depreciates at a constant rate $\delta < 1$ per model period. Accordingly,
$$c + i + \Phi(d,f,g) = G(d,f,g), \tag{35}$$
and the law of motion for capital is given by:
$$k = (1-\delta)k_{-1} + i - \psi(\frac{\delta k_{-1}}{i} - 1)^2, \tag{36}$$
where k is the capital stock, i is investment, δ is depreciation rate of capital and ψ parameterizes

the convex adjustment cost. However, since shocks are persistent, we adhere to the standard formulation that assumes one-period time to build.

4. We assume that the country-specific productivity shock $A(s^t)$ follows an AR(1) process with no cross-country spillovers:

$$\log(A_i(s^t)) = \zeta_i \log(A_i(s^{t-1})) + \varepsilon_i(s_t),$$

where $i = 1, 2, 3$, and the residuals ε_i are assumed to be normally distributed with zero mean, standard deviation σ_i^2, and correlation matrix μ_{ij}. The parameters are specified symmetrically for the two small countries, which results in identical processes, and separately for the large country.

Other than that, the structure of the model is as described in Section 1.1.

2.2 Quantification of trade-comovement relation in the data

To set the quantitative target for the theory, we use data for 20 industrialized countries over the period 1980Q1-2011Q4 and then back out the implied trade-comovement relation using a cross-country regression of the form:[24]

$$corr(GDP_i, GDP_j) = \alpha + \beta_x trade_{ij} + X_i + X_j + E_{ij} + \varepsilon_{ij}, \tag{37}$$

where $corr(GDP_i, GDP_j)$ is the correlation between countries i and j of the logged and HP-filtered series of real GDP. X_i and X_j are country dummies, and E_{ij} is the European dummy, which takes the value of 1 if both countries in the pair are in the European Union. The variable $trade_{ij}$ is a symmetric measure of bilateral trade intensity of countries i and j, measured at the beginning of the sample (in 1980), and given by the log of

$$\max\{\frac{IM_{ij}}{GDP_i}, \frac{IM_{ji}}{GDP_j}\}, \tag{38}$$

where IM_{ij} are nominal imports (in US dollars) by country i from country j and GDP_i is the nominal GDP (in US dollars) of country i, both measured in 1980.[25] The specifics of the data sources and series used in the paper are described in the Online Appendix.

[24] Countries in our sample constitute about 59% of world GDP and 53% of world trade (as of 2011). For a complete list of data sources, see the online Appendix. The country list includes: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Ireland, Japan, Korea, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

[25] All of the results are robust to picking other years as the base year for the bilateral trade measure.

The measure of trade in (38) varies in our sample from 0.03% (Korea with Portugal) to 27% (Ireland with the United Kingdom). Notably, it is symmetric and robust to having trade partners of very different sizes. For example, if the U.S. is an important trading partner for Canada, but Canada does not assume as much significance for the U.S., equation (38) still returns a high number.[26]

Table 2 presents the results. We include OLS results as well as results from an IV regression in which the instruments are common border, common language, and distance. Both OLS and IV regressions give highly significant positive coefficients, which suggests a strong effect of bilateral trade on comovement of GDP. These estimates imply that moving from the 10th to the 90th percentile of the bilateral trade spectrum increases the GDP correlations by 0.21 (IV) or 0.11 (OLS). Relative to median GDP correlation of 0.52 in our sample, this is an economically significant effect.[27]

Table 2: Trade-comovement relation in cross-country data.

	Dependent Variable: GDP correlation	
	OLS	IV
$trade_{ij}$	0.034**	0.065***
	(0.016)	(0.024)
E_{ij}	0.060	-0.028
	(0.093)	(0.106)
Country FE	yes	yes
R-squared	0.694	0.684

Notes: The table shows estimated cross-country pair regression between bilateral GDP correlation, bilateral trade intensity (trade$_{ij}$), EU membership dummy (E$_{ij}$), and country fixed effects. **,*** denote significance at the 5% and the 1% levels. Numbers in parentheses are standard errors.

2.3 Calibration of model parameters and model variants

We calibrate model parameters to match moments characterizing the median country in our sample along with its relative rest of the world. To set up the exercise of varying trade in the model, we add several moment conditions so as to ensure that the levels of trade and output correlations are

[26] This contrasts with measures expressed as averages, such as $\frac{IM_{ij}+IM_{ji}}{GDP_i+GDP_j}$. Such asymmetric measures give small numbers when trade partners have asymmetric sizes, i.e., small countries trade with big countries. To illustrate the difference, our measure is 8 times higher than $\frac{IM_{ij}+IM_{ji}}{GDP_i+GDP_j}$ for the Germany-Austria pair, 6 times higher for the US-Canada pair, and 15 times higher for the UK-Ireland pair. The results do not crucially depend on how we measure trade intensity but conceptually only a symmetric measure is consistent with the fact that correlation coefficient is symmetric.

[27] Since the above results have been confirmed by other studies, and also for a variety of econometric specifications, we do not consider any other specifications here. The reader is referred to the empirical literature that approaches this problem from a variety of angles. See, for example, Kose and Yi (2006), Baxter and Kouparitsas (2005) or Clark and van Wincoop (2001), among others.

consistent with those characterizing the median country *pair* ranked by bilateral trade intensity. The baseline period length is one quarter. Parameter values for each model variant are summarized in Tables 3, 4 and 5.

Baseline. The baseline model corresponds to the three-country quantitative setup outlined above and assumes $\phi = 0$, utility function given by (5), and complete markets. This model adheres closely to that used by Kose and Yi (2006) and it is used as a benchmark to define the trade-comovement puzzle.

Table 3: Parameter values.

Parameter		Value
	Baseline	
ρ	elasticity of substitution	1.17
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.8, 0.01665, 0.18335
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	symmetric to country 1
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.18335, 0.18335, 0.6333
η	leisure weight in utility	0.332
σ	risk aversion	2
β	time discount factor	0.99
α	capital share	0.36
δ	depreciation of physical capital	0.025
ϕ	cost of adjustment of goods ratio	0
ψ	capital adjustment cost	2.35
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	0.82, 0.82, 0.90
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.5, 0.65,0.65
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.0088,0.0088,0.00607
	FA (if different than baseline)	
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	0.83, 0.83, 0.905
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.49, 0.49,0.59
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.0076,0.0076,0.0053
ψ	capital adjustment cost	0

Notes: The table shows calibrated parameter values for each model variant in Section 2.3. Country 1 and 2 are two symmetric small countries and country 3 is relative rest of the world.

In terms of preference and technology parameters, we follow Backus et al. (1995). This includes: the risk aversion parameter $\sigma = 2$, discount factor $\beta = 0.99$, capital share in production $\alpha = 0.36$, consumption share in utility $\eta = 0.33$, and the depreciation rate of capital $\delta = 2.5\%$ per quarter.[28]

[28]The discount factor implies that the average real interest rate in the model is 4 percent (return on capital), capital share implies that labor share of income is 64 percent, and depreciation of capital implies that investment to GDP

In terms of country sizes, we assume that the population size of the rest of the world is 20 times that in each of the two small countries. This roughly matches the population difference between a median country in our sample and the size of the rest of the world for the median country.

We choose the parameters governing the stochastic process in (37) ($\zeta_i, \mu_{ij}, \sigma_i$), the share of consumption in the utility function η , Armington elasticity ρ, and the adjustment cost in the law of motion for capital ψ to jointly account for the median behavior of real GDPs in our sample: the median volatility of investment relative to real GDP of 3.04, and the median upper-bound measure of the short-run trade elasticity as volatility ratio of 1.17, in consonance with the approach and measurement used by Drozd and Nosal (2012).[29]

We set the parameters of the stochastic productivity process to be symmetric across the bilateral pair in the model, set to match the median autocorrelation and standard deviation of bilateral real GDPs in our sample (0.83 and 1.41%, respectively). We also match the autocorrelation and the standard deviation of real GDP for the median country's relative rest of the world (0.89 and 1.05%, respectively).

In order to be consistent with the correlation of output within the median country pair by trade intensity – which will be the starting point of our exercise of varying trade intensity in the model – we assume that the correlation of shocks μ_{ij} matches the correlation of real GDPs within the median pair along with the correlation of their total GDP with their relative rest of the world. In particular, we target the bilateral correlation of GDPs of 0.52 and the correlation of GDP of the country pair with their relative rest of the world of 0.66.

Financial autarky (FA). This model variant modifies the asset structure of the baseline model by assuming financial autarky. We set the other parameter values analogously as in the baseline model.

ratio hovers at around 25 percent, which is higher than the most recent OECD average but consistent with historic values during the sample period.

[29]The volatility ratio measures the volatility of the ratio of quantity indices of imports to domestic absorption relative to the volatility of their respective price deflator. It is an upper bound in the sense that any regression-based measure of elasticity between quantities in the numerator and relative price in the denominator would have to be scaled down by the correlation coefficient, which is always less than one. To construct the *volatility ratio*, we use constant and current price values of imports and domestic absorption DA, given by $DA = (C + G) + I - IM$. The prices are taken to be their corresponding price deflators. Denoting the deflator price of domestic absorption by P_{DA} and the deflator price of imports by P_{IM}, the *volatility ratio* is then defined as $\sigma(\frac{IM}{DA})/\sigma(\frac{P_{DA}}{P_{IM}})$, where σ refers to the standard deviation of the logged and Hodrick-Prescott filtered quarterly time series. The estimated values for individual countries in our sample are in the online Appendix.

GHH. This model variant u is given by (32). This utility function ensures that the Frisch elasticity of labor supply as well as the share of labor in time endowment is as in the baseline model. We set the other parameter values as in the baseline model.

Table 4: Parameter values continued.

Parameter		Value
	GHH (whenever different from baseline)	
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.775, 0.016, 0.209
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	symmetric to country 1
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.209, 0.209, 0.582
η	inverse Frisch elasticity	3/4
θ	multiplier on disutility from labor	0.544
ψ	capital adjustment cost	7.3
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.8, 0.8, 0.875$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.47, 0.55,0.55
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.00755,0.00755,0.00515
	GHH high Frisch elasticity (whenever different from DTE)	
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.772, 0.016, 0.212
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	symmetric to country 1
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.212, 0.212, 0.576
η	inverse Frisch elasticity	0.5
θ	multiplier on disutility from labor	0.659
ψ	capital adjustment cost	7.7
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.79, 0.79, 0.875$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.455, 0.515,0.515
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.007,0.007,0.0047

Notes: As in Table 3.

Dynamic trade elasticity (DTE). This model variant modifies the baseline by assuming $\phi > 0$, which is set to account for the difference between the short- and long-run trade elasticity estimates. In order to calibrate ρ and ϕ, we observe that the long-run response of trade x to the relative price of the domestic good versus the foreign good is solely determined by Armington elasticity ρ. Intuitively, in the long run, the adjustment cost has no bite, which is why the response of trade to permanent or very persistent price changes is governed by Armington elasticity ρ. Since this matches the approach of estimating the so called long-run trade elasticity, we set ρ equal to 15 – close to the upper limit of the values reported in the trade literature.[30] We then calibrate the convex adjustment parameter

[30]See the discussion of trade elasticity measured in footnote 2 and the references therein.

Table 5: Parameter values continued.

Parameter		Value
	DTE (if different than baseline)	
ρ	elasticity of substitution	15
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.3943, 0.2912, 0.3144
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	symmetric to country 1
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.3144, 0.3144, 0.3711
ϕ	cost of adjustment of goods ratio	4.932
ψ	capital adjustment cost	5.15
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.82, 0.82, 0.90$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.495, 0.65,0.65
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.0088,0.0088,0.00607
L_D, L_F, L_W	population sizes	$1, 1, 20$
	DTE low SRE target (if different than above)	
ϕ	cost of adjustment of goods ratio	18.1
ψ	capital adjustment cost	5.415
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.825, 0.825, 0.90$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.47, 0.624,0.624
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.00895,0.00895,0.00615

Notes: As in Table 3.

ϕ so as to match the same 1.17 target for the volatility ratio as in the other model variants. The remaining parameter values are calibrated analogously as in the baseline model.

DTE low SRE target. This is an identical model to the DTE model variant above except that it targets a lower value the volatility ratio of 0.5.

GHH high Frisch elasticity. This model variant assumes GHH preferences featuring a Frisch elasticity equal to 2. Other than that, the targets are as in the GHH model.

2.4 Findings

This section presents our quantitative findings. Before we discuss the results for the trade-comovement relation, we validate the models by showing that they have reasonable business cycle predictions vis-à-vis the baseline model.

Business cycle properties. Table 6 evaluates each model's performance vis-à-vis the standard set of business cycle statistics. As is clear from the comparison to the data (column one), the DTE model improves upon the baseline and other models in an important respect: it more closely matches the volatility and countercyclicality of net exports, which is a known shortcoming of the baseline model.[31] Specifically, in the DTE model the volatility of net exports relative to output is 27% versus 59% in the data and 10% in the baseline model. Unsurprisingly, financial autarky performs poorly in this respect.

The table shows that the main weakness of GHH preferences in comparison to dynamic elasticity is also the behavior of net exports. The GHH model implies a very low volatility of net exports relative to GDP, only 3%. It also implies a counterfactual positive correlation of net exports with terms of trade and real GDP.[32]

Trade-comovement relation. In order to obtain each model's prediction for the trade-comovement relation, we increase trade among the two symmetric small countries from the median value in our sample (0.85%) to the 90th percentile (3.85%). In addition, we adjust trade with rest of the world (third country) to match the higher level of total imports to GDP ratio of the 90th percentile pair (22%) versus the 50th percentile pair (20%). We do all that by varying utility weights ω_{ij}. We then compute the implied slope of the trade-comovement relationship by dividing the change in the correlation of real GDPs within the bilateral pair by the change in their log of imports to GDP ratio – which corresponds to our measure of trade intensity in the empirical exercise.

Table 7 reports the fraction of the data slope explained by each of the models, where the data slope corresponds to the OLS coefficient in Table 2.[33] The first three lines of Table 7 show that the baseline, the financial autarky and the GHH model variants all fall short of accounting for the data.

Interestingly, the GHH model featuring a higher target for the Frisch elasticity (GHH high Frisch in the table) performs better and accounts for about half of the trade-comovement relation in the data. Increasing the Frisch elasticity in the baseline setup does not come near this result, which underscores the potential of GHH preferences to address the trade-comovement puzzle to the extent that a higher value of the Frisch elasticity would be acceptable.[34]

[31] See the discussion in Raffo (2008).

[32] To be precise, given the extremely low variance of the net exports, these statistics are essentially undefined.

[33] We obtain virtually identical conclusions from an exercise of generating 190 trade pairs, as in our data, and running a cross-sectional regression on model generated data, identical to our data regression. For clarity of exposition, we focus attention on the implied slope using the 50th and 90th percentiles.

[34] In the online Appendix, we look at the baseline model assuming standard separable preferences that allow us to explore the effect of raising labor supply elasticity, and hence the implied Frisch elasticity. We find that the model

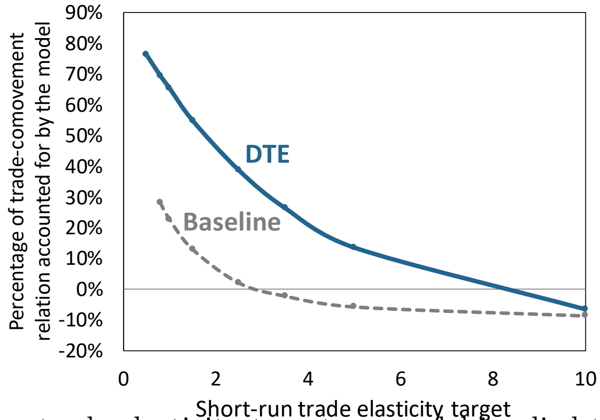


Figure 9: Effect of short-run trade elasticity target on model-implied trade-comovement relation.

Notes: The figure shows the difference between the quantitative baseline model and the dynamic trade elasticity model consistent with a range of target values for the implied short-run trade elasticity, after recalibration of all parameters to satisfy the stated targets.

The DTE model variant performs best in comparison, by accounting for 61% of the trade-comovement relationship in the data for a conservative setting of short-run trade elasticity of 1.17. For a more aggressive – but still empirically supported – setting of short-run elasticity of 0.5, the model accounts for 76% of the trade-comovement relationship. Figure 9 additionally compares the effect of lowering targeted short-run elasticity in the quantitative baseline model and the DTE model after recalibrating all parameter values to satisfy the calibration targets. The gap between the two models is substantial for all reported values of the targeted short-run trade elasticity.[35]

Validation of the DTE model's mechanism. Finally, we show that the mechanism of DTE in the three-country model is consistent with the analysis of the prototypical DTE model considered in Section 1.6.3. Analogously to our two-country setup from Section 1.6.3, we back out the implied transfers in the three-country setup. To that end, we distinguish transfers between the two small countries T from the transfer paid by the large country T^w. The home and foreign countries' analogs of budget constraints in (19)-(20) that define the transfers are as follows:

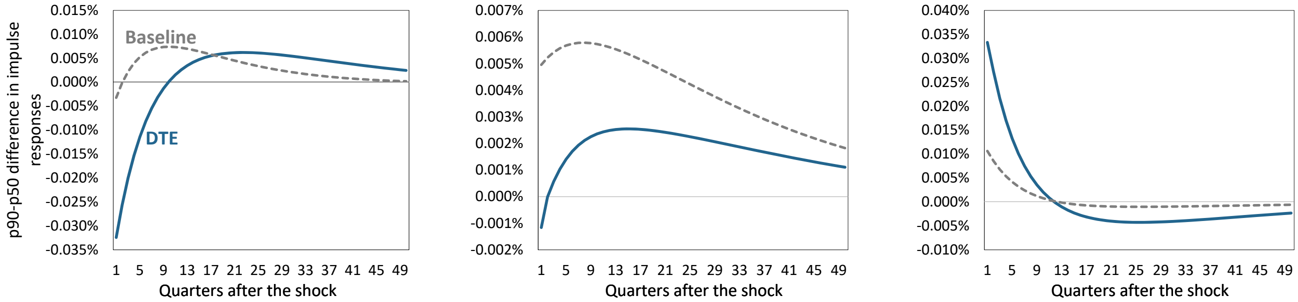
$$\begin{aligned} d + \bar{p}_f f + \bar{p}_g g &= wl + rk + T + T^w \\ d^* + \bar{p}_f f^* + \bar{p}_g g &= (w^* l^* + r^* k^*)\bar{p}_f - T + T^w, \end{aligned}$$

where $\bar{p}$ with subscripts pertains to steady state prices.

Figure 10 plots the difference in impulse responses between the median and the 90th percentile

with a separable utility function accounts for 9.5% of the data, which is lower than the non-separable case reported here. Doubling elasticity from one to two implies that the model accounts for 13% of the data coefficient, which is still lower than the 19% resulted reported for the baseline model in Table 7.

[35]We have also analyzed a dynamic trade elasticity under GHH preferences. The results are very similar as for the baseline dynamic trade elasticity model and we do not report them.



(a) Bilateral transfer T (b) Home consumption c (c) Home labor supply l

Figure 10: Difference in impulse responses to a one percent foreign productivity shock after increasing trade from the median country pair value to the 90th percentile country pair.

Notes: This figure shows the difference in impulse responses to a one percent foreign productivity shock after increasing trade from the median country pair value to the 90th percentile country pair, as implied by the quantitative exercise reported in Table 6.

of country pairs ranked by bilateral trade intensity, which corresponds to the quantitative exercise reported in Table 7. The leftmost panel shows that the key difference is that trade changes the present value of transfer payments T in the DTE model *less* than in the baseline model. The remaining panels of Figure 10 show that the response of consumption is less affected by trade in the DTE model and consequently that the response of labor is more pronounced. These results mirror the effects shown in Figure 7 for the analytic DTE model discussed in Section 1.6.3.

3 Conclusions

We characterized the effect of trade on output comovement in the standard theory and explored three candidate resolutions of the trade-comovement puzzle suggested by this analysis, two of which have not been explored in the literature thus far. We demonstrated that dynamic trade elasticity largely resolves the puzzle. Considering the fact that dynamic trade elasticity is neutral for the model's business cycle performance, and it is firmly grounded in the empirical measurement of trade elasticity, we come to the conclusion that the trade-comovement puzzle is best interpreted as imposing empirically viable parametric and structural restrictions on the standard transmission mechanism as opposed to rejecting it outright.

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Appendix

A.1. Omitted proofs

Detailed derivations can be found in the Mathematica notebook posted online and are omitted from the appendix.

Proof of Propositions 1 and 2 from text

Plugging in $\alpha = \eta = 1/3$ to (23)-(24), we obtain the following identities that we can implicitly differentiate:

$$\hat{p}(\hat{A}, \hat{A}^*) \equiv \frac{1}{2\rho(1-\bar{x}) + 3\bar{x}} \left(-\frac{3}{2}(\hat{A} - \hat{A}^*) + \frac{1}{\bar{x}} \hat{T}(\hat{A}, \hat{A}^*) \right), \tag{39}$$

$$T(\hat{A}, \hat{A}^*) \equiv -\frac{2+\sigma}{2\sigma} \left(\hat{A} - \hat{A}^* \right) - \frac{\hat{p}(\hat{A}, \hat{A}^*)}{\sigma}. \tag{40}$$

For later use, we denote the functions that solve the above fixed point problem by $\hat{p}(\hat{A}, \hat{A}^*), T(\hat{A}, \hat{A}^*)$. The proofs of Propositions 1 and 2 now follow from the sequence of lemmas established below and equation (26) in text.

Lemma 1. $\frac{\partial \hat{p}(\hat{A}, \hat{A}^*)}{\partial \hat{A}^*} > 0$.

Proof. Differentiating (39), we obtain:

$$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} \equiv \frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial \hat{A}^*} + \frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial T}\left(\frac{\partial \hat{T}(\hat{A},\hat{A}^*,\hat{p})}{\partial \hat{A}^*} + \frac{\partial \hat{T}(\hat{A},\hat{A}^*,\hat{p})}{\partial \hat{p}}\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}\right),$$

and hence

$$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}(1 - \frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial \hat{T}}\frac{\partial \hat{T}(\hat{A},\hat{A}^*,\hat{p})}{\partial \hat{p}}) \equiv \frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial \hat{A}^*} + \frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial \hat{T}}\frac{\partial \hat{T}(\hat{A},\hat{A}^*,\hat{p})}{\partial \hat{A}^*}.$$

Using (39)-(40), we can sign this expression as follows:

$$(\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}) \times (1 - (+) \times (-)) \equiv (+) + ((+) \times (+)).$$

Accordingly,

$$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}) > 0.$$

□

Lemma 2. *If* $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$ *and* $0 < \bar{x} \leq 1/3$, *then* $\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > 0$.

Proof. By way of contradiction, suppose

$$\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} < 0. \tag{41}$$

Differentiating (39), we obtain

$$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} \equiv \frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial \hat{A}^*} + \frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial \hat{A}^*}\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}.$$

The first term on the right-hand side is positive, since

$$\frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial \hat{A}^*} = \frac{3}{6\bar{x} + 4(1-\bar{x})\rho} > 0,$$

and from Lemma 1 and equation (41) we conclude that

$$0 < \frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} < \frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial \hat{A}^*}.$$

Next, differentiating (40), we obtain

$$\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} \equiv \frac{\partial \hat{T}(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{A}^*} + \frac{\partial \hat{T}(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{p}}\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*},$$

which, given the fact that

$$\frac{\partial \hat{T}(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{p}} = -\frac{1}{\sigma} < 0,$$

and given Lemma 1, gives

$$\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} \equiv \frac{\partial \hat{T}(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{A}^*} + \frac{\partial \hat{T}(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{p}}\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > \frac{\partial \hat{T}(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{A}^*} + \frac{\partial \hat{T}(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{p}}\frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial \hat{A}^*}.$$

Accordingly, if

$$\frac{\partial \hat{T}(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{A}^*} + \frac{\partial \hat{T}(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{p}} \frac{\partial \hat{p}(\hat{A},\hat{A}^*;\hat{T})}{\partial \hat{A}^*} \geq 0, \tag{42}$$

we must have

$$\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > 0,$$

which would be a contradiction. To complete the proof, we must show that (42) is satisfied under the premise of the lemma. Plugging in (39) and (40) to (42), the necessary and sufficient condition for (42) is

$$\rho \ \geq \frac{3}{2}\frac{1-\bar{x}(2+\sigma)}{(1-\bar{x})(2+\sigma)}. \tag{43}$$

It is easy to verify that the derivative of the expression on the right-hand side of (43) with respect to $\bar{x}$ is negative, and hence the expression is decreasing in $\bar{x}$. Accordingly, the sufficient condition for (43) can be obtained by setting $\bar{x}=0$, which boils down to the premise of the lemma: $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$. □

Lemma 3. *If $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$ and $0<\bar{x}\leq 1/3$, then $\frac{\partial^2 p(\hat{A},\hat{A}^*)}{\partial \hat{A}^*\partial \bar{x}} < 0$ and $\frac{\partial^2 \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*\partial \bar{x}} > 0$.*

Proof. It is clear that (40) indirectly depends on trade via $\hat{p}(.)$. In particular, differentiating (40), we obtain:

$$\frac{\partial^2 \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*\partial \bar{x}} = \frac{\partial \hat{T}(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{p}}\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*\partial \bar{x}} = -\frac{1}{\sigma}\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*\partial \bar{x}},$$

which implies

$$sign\left(\frac{\partial^2 \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*\partial \bar{x}}\right) = -sign\left(\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*\partial \bar{x}}\right).$$

Differentiating (39) and (40), we derive

$$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} = \frac{1}{3\bar{x}+2(1-\bar{x})\rho}\left(\frac{3}{2}+\frac{1}{\bar{x}}\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}\right),$$

and

$$\frac{\partial \hat{T}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} = \frac{2+\sigma}{2\sigma} - \frac{1}{\sigma}\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}.$$

Combining the above equations, we obtain

$$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} = \frac{3\bar{x}\sigma + \ 2+\sigma}{\bar{x}\sigma(6\bar{x}+4(1-\bar{x})\rho)+2}. \tag{44}$$

Differentiating the resulting expression with respect to $\bar{x}$ gives

$$\frac{\partial^2 \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*\partial \bar{x}} = \sigma\frac{3(2\rho-3)\sigma\bar{x}^2+2(2\rho-3)(\sigma+2)\bar{x}-2\rho(\sigma+2)+3}{2\left(\sigma\bar{x}\left((3-2\rho)\bar{x}+2\rho\right)+1\right)^2} \tag{45}$$

and hence

$$sign\frac{\partial^2 \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*\partial \bar{x}} = sign\left(3(2\rho-3)\sigma\bar{x}^2+2(2\rho-3)(\sigma+2)\bar{x}-2\rho(\sigma+2)+3\right).$$

To complete the proof, the expression

$$E(\bar{x},\rho) := 3(2\rho-3)\sigma\bar{x}^2+2(2\rho-3)(\sigma+2)\bar{x}-2\rho(\sigma+2)+3$$

must be negative, which follows from the following observations: 1) $E(\bar{x},\rho)$ is a decreasing function of ρ for all $0 \leq \bar{x} \leq 1/3$. This can be verified by evaluating the partial derivative of E with respect to ρ, which is given by

$$\frac{\partial E(\bar{x},\rho)}{\partial \rho} = 6\bar{x}^2\sigma + 4\bar{x}(2+\sigma) - 2(2+\sigma),$$

and showing that it is a quadratic function of $\bar{x}$ with upward pointing ends and roots given by:

$$\begin{aligned} \bar{x}_1 &= \frac{-2-\sigma-\sqrt{4+10\sigma+4\sigma^2}}{3\sigma} < 0, \\ \bar{x}_2 &= \frac{-2-\sigma+\sqrt{4+10\sigma+4\sigma^2}}{3\sigma} > \frac{-2-\sigma+\sqrt{4+8\sigma+4\sigma^2}}{3\sigma} = \frac{1}{3}. \end{aligned}$$

Accordingly, $E(\bar{x},\rho)$ is a decreasing function of $\bar{x}$ for all $0 \leq \bar{x} \leq \frac{1}{3}$ and any $\sigma > 0$. 2) The critical value $\tilde{\rho}(\bar{x})$ defined by $E(\bar{x},\tilde{\rho}(\bar{x})) = 0$ is

$$\tilde{\rho}(\bar{x}) := \frac{9\bar{x}^2\sigma + 6\bar{x}(2+\sigma) - 3}{6\bar{x}^2\sigma - 2(2+\sigma) + 4\bar{x}(2+\sigma)}, \tag{46}$$

and it is a decreasing function of $\bar{x}$. This follows from its derivative with respect to $\bar{x}$:

$$-\frac{3(\sigma+1)\left(3\sigma\bar{x}+\sigma+2\right)}{\left(2+\sigma-\bar{x}\left(\sigma\left(3\bar{x}+2\right)+4\right)\right)^2} < 0.$$

Consequently, the sufficient condition for this condition to apply to all admissible values of ρ is $\tilde{\rho}(\bar{x}=0) \leq 0$, but this boils down to the premise of the lemma: $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$. □

Lemma 4. *If* $\bar{x} \leq \frac{1}{1+\sigma/2}$, *then* $\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} + x\frac{\partial^2 \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} > 0$.

Proof. Using (44) and (45), and simplifying, we obtain

$$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} + x\frac{\partial^2 \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} = \frac{\sigma\bar{x}\left(\rho(8\sigma+4)\bar{x} - 3(\sigma+2)\bar{x} + 6\right) + \sigma + 2}{2\left(\sigma\bar{x}\left((3-2\rho)\bar{x}+2\rho\right)+1\right)^2}.$$

The sign of this expression depends on the sign of

$$Z(\bar{x},\rho) := \sigma\bar{x}\left(\rho(8\sigma+4)\bar{x} - 3(\sigma+2)\bar{x} + 6\right) + \sigma + 2.$$

This expression is globally increasing in ρ. Accordingly, if we show that the critical value $\tilde{\rho}(\bar{x})$ that satisfies $Z(\bar{x},\tilde{\rho}(\bar{x})) = 0$ is strictly negative, we have finished the proof because $\rho \geq 0$ by assumption. The critical value is given by

$$\tilde{\rho}(\bar{x}) = \frac{-\sigma\left(3\bar{x}\left(2-(\sigma+2)\bar{x}\right)+1\right) - 2}{4\sigma(2\sigma+1)\bar{x}^2}.$$

The expression is negative if and only if the expression in the numerator, given by

$$N(\bar{x}) := -\sigma\left(3\bar{x}\left(2-(\sigma+2)\bar{x}\right)+1\right) - 2,$$

is negative under the premise of the lemma. To see this is indeed the case, note that $N(\bar{x})$ is a quadratic function of $\bar{x}$ with upward pointing ends. The larger root is positive and can be bounded from below as follows:

$$\bar{x}_2 = \frac{3\sigma + \sqrt{3}\sqrt{\sigma(4+7\sigma+\sigma^2)}}{6\sigma+3\sigma^2} \geq \frac{2}{2+\sigma} = \frac{1}{1+\sigma/2}.$$

Accordingly, by the premise of the lemma, it is clear that $\bar{x} \leq \bar{x}_2$. The smaller root $\bar{x}_1 < \bar{x}_2$ is negative because $N(\bar{x}=0) = -2-2\sigma$, which trivially ensures $\bar{x} > \bar{x}_1$. □

Proof of Corollary 1 from text

The proof directly follows from that of Lemma 2 and 3 above, since the cutoff value for ρ given by equation (46) is always above the cutoff value for ρ given by (43). Note that these cutoffs are a necessary and sufficient condition for the stated effects.

A.2. Omitted conditions from Section 1.2.2

The complete system implied by the first stage of the decomposition in Section 1.2.2 is:

$$\hat{y} = \frac{\hat{A}}{1-\alpha} + \bar{x}\frac{1-\eta+\alpha}{1-\alpha}\hat{p} - \frac{1-\eta}{1-\alpha}\hat{T} \tag{47}$$

$$\hat{k} = \bar{x}\hat{p} + \hat{y} \tag{48}$$

$$\hat{l} = \frac{1-\eta}{1-\alpha}\bar{x}\hat{p} - \frac{1-\eta}{1-\alpha}\hat{T} \tag{49}$$

$$\hat{d} = \rho\bar{x}\hat{p} + \hat{y} + \hat{T} \tag{50}$$

$$\hat{f} = (1-\bar{x})\rho\hat{p} + \hat{y} + \hat{T} \tag{51}$$

Foreign analogs of the above conditions can be obtained by symmetry; that is, replacing $\hat{p}$ by $-\hat{p}$, $\hat{T}$ by $-\hat{T}$, $\hat{d}$ by $\hat{f}$, $\hat{f}$ by $\hat{d}$, and placing asterisks on all variables except for $\hat{p}$ and $\hat{T}$. For a complete derivation of these conditions refer to the Mathematica notebook posted online.

Table 6: Business cycle statistics: Data versus models[a].

Statistic	Data [b] (median)	DTE	Baseline	FA	GHH	GHH high Frisch	DTE low SRE target
A. Correlation							
domestic with foreign							
TFP (measured)	0.49	0.50	0.51	0.49	0.47	0.45	0.47
GDP	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Consumption	0.41	0.55	0.56	0.63	0.87	0.90	0.48
Employment	0.38	0.64	0.56	0.59	0.62	0.62	0.73
Investment	0.43	0.37	0.48	0.57	0.58	0.58	0.38
GDP with							
Consumption	0.71	0.99	0.97	0.91	0.82	0.79	0.99
Employment	0.60	0.99	0.99	0.99	0.99	0.99	0.97
Investment	0.74	0.98	0.99	0.99	0.99	0.99	0.98
Net exports	-0.20	-0.62	-0.68	0.04	0.52	0.58	-0.66
Terms of trade with							
Net exports	-0.32	-0.92	-0.83	0.04	0.80	0.88	-0.97
B. Volatility relative to GDP							
Consumption	0.79	0.46	0.34	0.25	0.21	0.21	0.48
Investment	3.04	3.04	3.04	2.97	3.04	3.04	3.04
Employment	0.71	0.34	0.44	0.49	0.52	0.61	0.32
Net exports	0.59	0.27	0.10	0.003	0.03	0.03	0.30

Notes: The table reports business cycle statistics for each model variant described in Section 2.3 relative to the values in the data.
[a]Statistics based on logged and Hodrick-Prescott filtered time series with a smoothing parameter $\lambda = 1600$.
[b]Unless otherwise noted, data column refers to the median in our sample of countries for the period 1980Q1-2011Q4.

Table 7: Slope of trade-comovement relation: Fraction explained relative to data estimate.

Model variant	Model-implied slope relative to data estimate
Baseline	19%
FA	24%
GHH	33%
DTE	61%
DTE low SRE target	76%
GHH high Frisch	47%

Notes: The table reports the implied slope between trade and comovement (output correlation) by each model variant considered in Section 2.3 relative to the corresponding value for the data. Data value is derived from the OLS regression reported in Table 2. The slope value for the models has been calculated by increasing bilateral trade intensity from the median value to the 90th percentile and accordingly adjusting trade openness with the rest of the world.