Online Appendix

Pricing-to-Market in Business Cycle Models*

Lukasz A. Drozd
Federal Reserve Bank of Philadelphia

Marcin Kolasa
International Monetary Fund
SGH Warsaw School of Economics

Jaromir B. Nosal
Boston College

Abstract

This document contains the Online Appendix for the paper "**Pricing-to-Market in Business Cycle Models**."

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Contents

1 Omitted Analytic Details from Section 3

This appendix provides detailed derivations of the expressions used in Section 3 of the paper.

The key defining object is the partial differentiation operator $\partial_{\log x}\left(.|_{ss}\right)$ used in the definitions of TE and PT. This operator focuses on the direct impact of a (one-time) i.i.d. exchange rate shock and treats general equilibrium prices as constant. As stated in the text: 1) The pass-through coefficient describes the log-linear coefficient on the reaction function of an importer who assumes that all other importers respond to the shock but none of the aggregate local or foreign prices/marginal costs change. 2) Trade elasticity considers the reaction function of a representative (atomless) final good producer who assumes that all importers from whom it sources imported goods raise prices symmetrically, and none of the local producers change their price. The goal is to preserve the decision rule of the optimizing price setter, while assuming away all general equilibrium channels that may affect decisions indirectly.

We start by deriving the demand system for the CES baseline model. This part is standard, but the derivation here is helpful in the context of the next setup: the KA model.

1.1 Frictionless Baseline (FB)

Preliminaries: Since the final sector is consolidated into a single representative competitive firm, the core of the problem is the cost minimization problem of "producing" A units of a homogeneous final good (consumption/investment good):

$$P\left(s^t\right) A\left(s^t\right) = \min_{d_i, f_i} \int_0^1 \left[p_d\left(i,s^t\right) d\left(i,s^t\right) + p_f\left(i,s^t\right) f\left(i,s^t\right)\right] \tag{1}$$

subject to

$$A\left(s^t\right) = \underbrace{\left[\omega^{\frac{1}{\gamma}} \underbrace{\left(\left[\int_0^1 d\left(i,s^t\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}\right)^{\frac{\gamma-1}{\gamma}}}_{d(s^t)} + (1-\omega)^{\frac{1}{\gamma}} \underbrace{\left(\left[\int_0^1 f\left(i,s^t\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}\right)^{\frac{\gamma-1}{\gamma}}}_{f(s^t)}\right]^{\frac{\gamma}{\gamma-1}}}_{G(d(s^t),f(s^t))}. \tag{2}$$

Given constant returns to scale aggregation, it is convenient to rewrite the problem in terms of shares

$$s_d\left(i,s^t\right) = \frac{d\left(i,s^t\right)}{\omega A\left(s^t\right)}, \quad s_f\left(i,s^t\right) = \frac{f\left(i,s^t\right)}{(1-\omega) A\left(s^t\right)}, \tag{3}$$

which gives the following:

$$1 \equiv P\left(s^t\right) = \min_{s_{fi}, s_{di}} \int_0^1 \left[p_d\left(i,s^t\right) \omega s_d\left(i,s^t\right) + p_f\left(i,s^t\right)(1-\omega) s_f\left(i,s^t\right)\right]. \tag{4}$$

subject to

$$\left[\omega\left(\left[\int_0^1 s_d\left(i,s^t\right)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}\right)^{\frac{\gamma-1}{\gamma}}+(1-\omega)\left(\left[\int_0^1 s_f\left(i,s^t\right)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}=1. \tag{5}$$

where $\lambda\left(s^t\right)$ is the Lagrange multiplier imposed on the above constraint and $P\left(s^t\right)\equiv 1$ by the numeraire normalization introduced in the text. (Throughout, we use "$\equiv$" to indicate an identity.)

To avoid the confusion with differentiating integrals, we consider a perturbation from the optimum towards some arbitrary perturbation function h: $s_d\left(i,s^t\right)+\varepsilon_d\left(i,s^t\right)h_d\left(i,s^t\right)$, where $s_d\left(i,s^t\right)$ is the optimum and $\varepsilon_d\left(i,s^t\right)\in\mathbb{R}$ is the perturbation scale parameter (same for good f). Note that optimality implies that there is no perturbation function that lowers the value of the program, which means that there is no gradient of improvement at $\varepsilon=0$. To derive the first order conditions, we differentiate the following:

$$\min_{\varepsilon_{fi},\varepsilon_{di}}\int_0^1\left[p_d\left(i,s^t\right)\omega\left(s_d\left(i,s^t\right)+\varepsilon_{di}h_d\left(i,s^t\right)\right)+p_f...\right]. \tag{6}$$

s.t.

$$(\lambda(s^t))\qquad\left[\omega\left(\left[\int_0^1\left(s_d\left(i,s^t\right)+\varepsilon_{di}h_d\left(i,s^t\right)\right)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}\right)^{\frac{\gamma-1}{\gamma}}+...\right]^{\frac{\gamma}{\gamma-1}}=1. \tag{7}$$

Taking the derivative with respect to $\varepsilon_{di}\equiv\varepsilon_d\left(i,s^t\right)\in\mathbb{R}$ and evaluating it around at $\varepsilon_{di}=0$ gives:

$$\begin{aligned}\varepsilon_{di}:&\int_0^1\omega p_d\left(i,s^t\right)\omega h_d\left(i,s^t\right)di\\&=\lambda(s^t)\omega\left[\int_0^1\left(s_d\left(.\right)\right)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}-1}\left(\left[\int_0^1\left(s_d\left(.\right)\right)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}\right)^{\frac{-1}{\gamma}}\int_0^1 s_d\left(.\right)^{\frac{-1}{\theta}}h_d\left(i,s^t\right)di.\end{aligned}$$

Let $h_d\left(i,s^t\right)$ be a Dirac-delta function that spikes at an arbitrary point. We drop the last integral, divide by $h\left(.\right)$ on both sides, and obtain the first-order condition:

$$\begin{aligned}\varepsilon_{di}:\omega p_d\left(i,s^t\right)&=\\&=\lambda(s^t)\omega s_d\left(i,s^t\right)^{\frac{-1}{\theta}}\left[\int_0^1\left(s_d\left(i,s^t\right)\right)^{\frac{\theta-1}{\theta}}di\right]^{\frac{1}{\theta-1}}\left(\left[\int_0^1\left(s_d\left(i,s^t\right)\right)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}\right)^{\frac{-1}{\gamma}}\end{aligned}$$

An analogous condition applies to good f and the remaining first-order condition is the constraint in (5).

Plugging in for s_d from (3) and using the fact that, by definition, we know

$$d\left(s^t\right):=\left(\int_0^1\left(d\left(i,s^t\right)\right)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}},$$

and hence

$$p_d\left(i,s^t\right)=\lambda(s^t)s_d\left(i,s^t\right)^{\frac{-1}{\theta}}d\left(s^t\right)^{\frac{1}{\theta}-\frac{1}{\gamma}}\omega^{\frac{1}{\gamma}-\frac{1}{\theta}}A\left(s^t\right)^{\frac{1}{\gamma}-\frac{1}{\theta}},$$

and

$$p_d\left(i,s^t\right)=\omega^{\frac{1}{\gamma}}\lambda(s^t)d\left(i,s^t\right)^{\frac{-1}{\theta}}d\left(s^t\right)^{\frac{1}{\theta}-\frac{1}{\gamma}}A\left(s^t\right)^{\frac{1}{\gamma}}.$$

The above equations imply the following state-contingent demand function (note: notation $d(p_d|i,s^t)$ indicates that the stated object is a i,s^t-specific function of variable $p_d\in\mathbb{R}$):

$$d\left(p_d|i,s^t\right)=\omega^{\frac{\theta}{\gamma}}\left(\frac{p_d\left(i,s^t\right)}{\lambda\left(s^t\right)}\right)^{-\theta}d\left(s^t\right)^{1-\frac{\theta}{\gamma}}A\left(s^t\right)^{\frac{\theta}{\gamma}}, \tag{8}$$

In terms of the shares, we express demand as follows:

$$s_d\left(p_d|i,s^t\right):=\frac{d\left(p_d|i,s^t\right)}{\omega A\left(s^t\right)}=\left(\frac{p_d\left(i,s^t\right)}{\lambda\left(s^t\right)}\right)^{-\theta}\omega^{\frac{\theta}{\gamma}-1}d\left(s^t\right)^{1-\frac{\theta}{\gamma}}A\left(s^t\right)^{\frac{\theta}{\gamma}-1},$$

(Analogous conditions apply to good f but with $1-\omega$ replacing ω.)
It is easy to verify by plugging in

$$G_d\left(d,f\right)=d^{\frac{-1}{\gamma}}\left[\omega^{\frac{1}{\gamma}}d^{\frac{\gamma-1}{\gamma}}+\left(1-\omega\right)^{\frac{1}{\gamma}}f^{\frac{\gamma-1}{\gamma}}\right]^{\frac{1}{\gamma-1}}$$

that the demand expression stated in the text is identical to the one derived above, since

$$\begin{aligned}\left(\frac{p_d\left(i,s^t\right)}{G_d\left(s^t\right)}\right)^{-\theta}d(s^t)&=\omega^{\frac{\theta}{\gamma}}p_d\left(i,s^t\right)^{-\theta}d\left(s^t\right)^{1-\frac{\theta}{\gamma}}\left[\omega^{\frac{1}{\gamma}}d\left(s^t\right)^{\frac{\gamma-1}{\gamma}}+\left(1-\omega\right)^{\frac{1}{\gamma}}f\left(s^t\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\theta}{\gamma-1}}\\&=\omega^{\frac{\theta}{\gamma}}p_d\left(i,s^t\right)^{-\theta}d\left(s^t\right)^{1-\frac{\theta}{\gamma}}A\left(s^t\right)^{\frac{\gamma-1}{\gamma}}.\end{aligned}$$

To close the system, we add the price index equation. Since

$$\begin{aligned}d\left(s^t\right)&:=\left(\int_0^1\left(d\left(i,s^t\right)\right)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}}=\left(\int_0^1\left(\omega^{\frac{\theta}{\gamma}}\left(\frac{p_d\left(i,s^t\right)}{\lambda\left(s^t\right)}\right)^{-\theta}A\left(s^t\right)^{\frac{\theta}{\gamma}}\right)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\gamma}{\theta-1}}\\&=\omega\lambda\left(s^t\right)^{\gamma}A\left(s^t\right)\left(\int_0^1p_d\left(i,s^t\right)^{1-\theta}di\right)^{\frac{\gamma}{\theta-1}}\end{aligned}$$

it is clear that

$$\frac{d\left(s^t\right)}{A\left(s^t\right)}=\omega\lambda\left(s^t\right)^{\gamma}\left(\int_0^1p_d\left(i,s^t\right)^{1-\theta}di\right)^{\frac{\gamma}{\theta-1}},$$

and that (5) gives

$$\left[\omega^{\frac{1}{\gamma}}\left(\frac{d\left(s^t\right)}{A\left(s^t\right)}\right)^{\frac{\gamma-1}{\gamma}}+\left(1-\omega\right)^{\frac{1}{\gamma}}\left(\frac{f\left(s^t\right)}{A\left(s^t\right)}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}=1.$$

Accordingly,

$$\lambda\left(s^t\right)=\left[\omega\left(\int_0^1 p_d\left(i,s^t\right)^{1-\theta}di\right)^{\frac{1-\gamma}{1-\theta}}+(1-\omega)\left(\int_0^1 p_f\left(i,s^t\right)^{1-\theta}di\right)^{\frac{\gamma-1}{\theta-1}}\right]^{\frac{1}{1-\gamma}}. \tag{9}$$

This completes the characterization of this system, since we know that, by constant returns to scale and the envelope theorem, $\lambda\left(s^t\right)\equiv P\left(s^t\right)\equiv 1$. However, since this step will be needed in the next model, we derive it explicitly. To that end, we consider the objective function in the cost minimization problem above and plug in for $s_d\left(|i,s^t\right), s_f\left(|i,s^t\right)$ from the first order conditions to derive

$$1\equiv P\left(s^t\right)=\int_0^1\left[\omega^{\frac{\theta}{\gamma}}p_d\left(i,s^t\right)^{1-\theta}\lambda\left(s^t\right)^{\theta}\left(\frac{d\left(s^t\right)}{A\left(s^t\right)}\right)^{1-\frac{\theta}{\gamma}}+(1-\omega)^{\frac{\theta}{\gamma}}p_f\left(i,s^t\right)^{1-\theta}\lambda\left(s^t\right)^{\theta}\left(\frac{f\left(s^t\right)}{A\left(s^t\right)}\right)^{1-\frac{\theta}{\gamma}}\right].$$

Plugging in for $d\left(s^t\right)/A\left(s^t\right), f\left(s^t\right)/A\left(s^t\right)$ as above, we have

$$1\equiv\lambda\left(s^t\right)^{\theta}\int_0^1\left[\omega^{\frac{\theta}{\gamma}}p_d\left(i,s^t\right)^{1-\theta}\left(\frac{d\left(s^t\right)}{A\left(s^t\right)}\right)^{1-\frac{\theta}{\gamma}}+(1-\omega)^{\frac{\theta}{\gamma}}p_f\left(i,s^t\right)^{1-\theta}\left(\frac{f\left(s^t\right)}{A\left(s^t\right)}\right)^{1-\frac{\theta}{\gamma}}\right],$$

and hence

$$1\equiv\lambda\left(s^t\right)^{\gamma}\left[\omega\left(\int_0^1 p_d\left(i,s^t\right)^{1-\theta}di\right)^{\frac{\gamma-\theta}{\theta-1}}\int_0^1 p_d\left(i,s^t\right)^{1-\theta}+...\right].$$

By (9), we obtain

$$1\equiv\lambda\left(s^t\right)^{\gamma}\left[\omega\left(\int_0^1 p_d\left(i,s^t\right)^{1-\theta}di\right)^{\frac{\gamma-1}{\theta-1}}+...\right]=\lambda\left(s^t\right)^{\gamma}\lambda\left(s^t\right)^{1-\gamma}=\lambda\left(s^t\right). \tag{10}$$

In the deterministic symmetric steady state, note that $p_d^{ss}=p_f^{ss}=1$ and $d^{ss}=f^{ss}=A^{ss}$. The pricing equation implies that the gross profit margin and the markup in this model are given by

$$\mu^{ss}=\frac{\theta}{\theta-1}-1. \tag{11}$$

We next derive PT and TE as defined in the text.

PT: Consider the formula for the import price derived above. Under the stated assumptions for the partial differentiation operator $\partial_{\log x}\left(.|_{ss}\right)$ (discussed above), we need to differentiate the expression:

$$p_f\left(x|i,s^t\right)|_{ss}=\frac{\theta}{\theta-1}\frac{x\left(s^t\right)}{v^*\left(s^t\right)}|_{ss},$$

where $v^*\left(s^t\right) = v^*$ is treated as constant under $\partial_{\log x}\left(.|_{ss}\right)$. We obtain the expected result:

$$PT := \partial_{\log x} \log p_f\left(x|i, s^t\right)|_{ss} = x\frac{\partial_x p_f\left(x|i, s^t\right)}{p_f\left(x|i, s^t\right)}|_{ss} = 1. \tag{12}$$

TE: The definition of trade elasticity in the text is

$$TE := \frac{\partial_{\log x} \log \frac{d\left(s^t\right)}{f\left(s^t\right)}|_{ss}}{PT}, \tag{13}$$

where $d\left(s^t\right), f\left(s^t\right)$ are quantities chosen by the representative (small) final good producer who faces a symmetric shock (recall that all importers from whom this producer buys goods raise the price, none of the domestic producers do and marginal cost is assumed constant under the linearization operator). We assume the import price that the importer faces is $p_f\left(x|i, s^t\right)$ and the home price is $p_d\left(x, s^t\right) \equiv p_d$. The relevant equation characterizing the pricing policy of the final goods producer is thus given by

$$\frac{G_d\left(d\left(p_d|i, s^t\right), f\left(p_f\left(x, s^t\right)|i, s^t\right)\right)}{G_f\left(d\left(p_d|i, s^t\right), f\left(p_f\left(x, s^t\right)|i, s^t\right)\right)}\left(\frac{d\left(p_d|i, s^t\right)}{d\left(s^t\right)}\frac{f\left(p_f\left(x|s^t\right)|i, s^t\right)}{f\left(s^t\right)}\right)^{-\frac{1}{\theta}} = \frac{p_d}{p_f\left(x, s^t\right)}$$

where $d\left(p_d|i, s^t\right)$,$f\left(p_f\left(x, s^t\right)|i, s^t\right)$ are the demand functions derived above. Plugging for G from (5) and using symmetry $p_f\left(x|s^t\right) \equiv p_f\left(x|i, s^t\right)$, we obtain

$$\left(\frac{d\left(p_d|s^t\right)}{f\left(p_f\left(x|s^t\right)|s^t\right)}\right)^{-\frac{1}{\gamma}} = \frac{p_d}{p_f\left(x|s^t\right)} \Leftrightarrow \frac{d\left(s^t\right)}{f\left(s^t\right)} = \left(\frac{p_d}{p_f\left(x|s^t\right)}\right)^{-\gamma},$$

and hence, as claimed in text,

$$TE := \frac{\partial_{\log x} \log \frac{d\left(s^t\right)}{f\left(s^t\right)}|_{ss}}{PT} = \frac{\partial_{\log x} \log\left(\frac{p_d}{p_f\left(x|s^t\right)}\right)^{-\gamma}|_{ss}}{PT} = \frac{\gamma\partial_{\log x}\log\left(p_f\left(x|i, s^t\right)\right)|_{ss}}{PT} = \gamma. \tag{14}$$

1.2 KA Model

Preliminaries: We begin by deriving the KA demand system, following the same steps as above. We similarly rewrite the problem in terms of shares:

$$\begin{aligned} P\left(s^t\right) &= \min_{s_{fi}, s_{di}} \int_0^1 \left[p_d\left(i, s^t\right)\omega s_d\left(i, s^t\right) + p_f\left(i, s^t\right)(1-\omega)\, s_f\left(i, s^t\right)\right]. \qquad (15) \\ \text{s.t.} & \\ & \int_0^1 \left[\omega g\left(s_d\left(i, s^t\right)\right) + (1-\omega)\, g\left(s_f\left(i, s^t\right)\right)\right] di = 1, \quad (\lambda(s^t)), \qquad (16) \end{aligned}$$

where $\lambda\left(s^t\right)$ is the Lagrange multiplier associated with the constraint and shares s_d, s_f which we defined in (3). However, unlike in the CES model, the function g is not homogeneous of degree one, and it is no longer true that $\lambda\left(s^t\right)$ is $P\left(s^t\right) \equiv 1$. To preserve the problem of the optimizing entity we need to consider how this object is affected by the exchange rate. This is the key complication relative to the baseline model.

It is easy to verify that the first-order conditions give

$$\begin{aligned}
s_d \quad &: \quad p_d\left(i,s^t\right)=\lambda\left(s^t\right)g'\left(s_d\left(i,s^t\right)\right) && (17)\\
&\rightarrow \quad s_d\left(i,s^t\right)=h\left(\frac{p_d\left(i,s^t\right)}{\lambda\left(s^t\right)}\right) && (18)\\
s_f \quad &: \quad p_f\left(i,s^t\right)=\lambda\left(s^t\right)g'\left(s_f\left(i,s^t\right)\right) && (19)\\
&\rightarrow \quad s_f\left(i,s^t\right)=h\left(\frac{p_f\left(i,s^t\right)}{\lambda\left(s^t\right)}\right) && (20)\\
\lambda \quad &: \quad 1=\int_0^1\left[\omega g\left(s_d\left(i,s^t\right)\right)+(1-\omega)\,g\left(s_f\left(i,s^t\right)\right)\right]di && (21)
\end{aligned}$$

where, as in the text, we define $h\left(.\right):=g'^{-1}\left(.\right)$. For later use, note that the inverse function theorem implies

$$h'\left(x\right)|_{x=1}=\left(\left(g'\right)^{-1}\right)'\left(x\right)|_{x=1}=\left(g''\left(\left(g'\right)^{-1}\left(x\right)\right)\right)^{-1}|_{x=1}=\frac{1}{g''\left(1\right)}, \tag{22}$$

and so in a linearized system $g''\left(1\right)$ is the key parameter that summarizes the curvature of the demand function (around the deterministic steady state).

Plugging in for shares, the KA model's demand system is given by

$$d\left(p_d|i,s^t\right)=\omega h\left(\frac{p_d}{\lambda\left(s^t\right)}\right)A\left(s^t\right), \tag{23}$$

$$f\left(p_f|i,s^t\right)=\left(1-\omega\right)h\left(\frac{p_f}{\lambda\left(s^t\right)}\right)A\left(s^t\right). \tag{24}$$

As in the baseline model, the Lagrange multiplier is implicitly defined by the constraint, which gives

$$1=\int_0^1\left[\omega g\left(h\left(\frac{p_d\left(i,s^t\right)}{\lambda\left(s^t\right)}\right)\right)+\left(1-\omega\right)g\left(h\left(\frac{p_f\left(i,s^t\right)}{\lambda\left(s^t\right)}\right)\right)\right]di. \tag{25}$$

Furthermore, plugging into the objective function, we know

$$1\equiv P\left(s^t\right)=\int_0^1\left[p_d\left(i,s^t\right)\omega h\left(\frac{p_d\left(i,s^t\right)}{\lambda\left(s^t\right)}\right)+p_f\left(i,s^t\right)\left(1-\omega\right)h\left(\frac{p_f\left(i,s^t\right)}{\lambda\left(s^t\right)}\right)\right]. \tag{26}$$

Together, the above two equations define $\lambda\left(s^t\right)$. In the symmetric steady state, $p_d^{ss}\left(i\right)=p_f^{ss}\left(i\right)=p^{ss}$ and normalization in the text implies $g\left(1\right)=g'\left(1\right)=1$ and $h\left(1\right)=1$; hence, $\lambda^{ss}=P^{ss}=1$, $p^{ss}/\lambda^{ss}=1$. Furthermore, since $f^{ss}=\left(1-\omega\right)h\left(1\right)A^{ss}=\left(1-\omega\right)A^{ss}$, the share of the foreign good in expenditures is $1-\omega$ in A^{ss}.

For later use, it is convenient to define the elasticity of demand for an individual good i:

$$\gamma\left(\frac{p_d}{\lambda}\right):=-\frac{\partial\log d\left(\frac{p_d}{\lambda}|i,s^t\right)}{\partial\log p_d}=-\frac{\partial\left(d\left(\frac{p_d}{\lambda}|i,s^t\right)\right)}{\partial p_d}\frac{p_d}{d\left(p_d|i,s^t\right)}, \tag{27}$$

which, by (23), in the steady state implies

$$\gamma\left(1\right) = \gamma\left(\frac{p_d}{\lambda}\right)|_{ss} = -h'\left(\frac{p_d}{\lambda\left(s^t\right)}\right)\frac{1}{\lambda}\frac{p_d}{h\left(\frac{p_d}{\lambda}\right)}|_{ss} = \frac{h'\left(1\right)}{h\left(1\right)} = \frac{1}{g''\left(1\right)}. \tag{28}$$

Accordingly, as noted above and in text, $g''\left(1\right)$ is the key parameter of the log-linearized equilibrium system. It determines the (local) curvature of the demand function and hence varying demand elasticity. The standard monopolistic pricing formula implies that

$$p_f\left(i, s^t\right) = x\left(s^t\right)v^*\left(s^t\right)\frac{\gamma\left(\frac{p_f\left(i,s^t\right)}{\lambda\left(s^t\right)}\right)}{\gamma\left(\frac{p_f\left(x,s^t\right)}{\lambda\left(s^t\right)}\right)-1}. \tag{29}$$

We omit the explicit derivation of this pricing formula as it immediately follows from the producer problem.

PT: As noted, the partial differentiation operator $\partial_{\log x}\left(.|_{ss}\right)$ assumes invariance of the price index and home variables with respect to the exchange rate. However, as also noted above, $\lambda\left(x|s^t\right)$ is endogenous and hence depends on x. This is consistent with the idea that we want to capture the policy function of the affected price setter.

The relevant pricing formula for the operator is equation (29):

$$p_f\left(x|i, s^t\right) = xv^*\left(s^t\right)\frac{\gamma\left(\frac{p_f\left(x|i,s^t\right)}{\lambda\left(x,s^t\right)}\right)}{\gamma\left(\frac{p_f\left(x|i,s^t\right))}{\lambda\left(x|s^t\right)}\right)-1},$$

where $v^*\left(s^t\right)$ is assumed independent of x under the operator $\partial_{\log x}\left(.|_{ss}\right)$. Under symmetry

$$\left(p_f\left(x|i, s^t\right) \equiv p_f\left(x|s^t\right)\right),$$

and so the Kimball aggregator after plugging in for prices gives

$$1 \equiv \int_0^1\left[\omega g\left(h\left(\frac{p_d}{\lambda\left(x,s^t\right)}\right)\right) + \left(1-\omega\right)g\left(h\left(\frac{p_f\left(x|s^t\right)}{\lambda\left(x,s^t\right)}\right)\right)\right]di,$$

Differentiating the above expression under the rules of the operator $\partial_{\log x}\left(.|_{ss}\right)$ yields

$$\begin{aligned}0 \equiv& \int_0^1\left[\omega g'\left(h\left(\frac{p_d}{\lambda\left(x|s^t\right)}\right)\right)h'\left(\frac{p_d}{\lambda\left(x|s^t\right)}\right)\frac{p_d}{\lambda\left(x|s^t\right)^2}\partial_{\log x}\lambda\left(x|s^t\right)\right]di\\ &-\int_0^1\left[\left(1-\omega\right)g'\left(h\left(\frac{p_f\left(x|s^t\right)}{\lambda\left(x|s^t\right)}\right)\right)h'\left(\frac{p_f\left(x|s^t\right)}{\lambda\left(x|s^t\right)}\right)\frac{\lambda\left(x,s^t\right)\partial_{\log x}p_f\left(x|s^t\right)-p_f\left(x|s^t\right)\partial_{\log x}\lambda\left(x|s^t\right)}{\lambda\left(x|s^t\right)^2}\right]di,\end{aligned}$$

and hence

$$\begin{aligned}0 \equiv& \int_0^1\left[\omega g'\left(h\left(\frac{p_d}{\lambda\left(x|s^t\right)}\right)\right)h'\left(\frac{p_d}{\lambda\left(x|s^t\right)}\right)\frac{p_d}{\lambda\left(x|s^t\right)}\partial_{\log x}\log\lambda\left(x|s^t\right)\right]di\\ &-\int_0^1\left[\left(1-\omega\right)g'\left(h\left(\frac{p_f\left(x|s^t\right)}{\lambda\left(x|s^t\right)}\right)\right)h'\left(\frac{p_f\left(x|s^t\right)}{\lambda\left(x|s^t\right)}\right)\frac{p_f\left(x|s^t\right)}{\lambda\left(x|s^t\right)}\left(\partial_{\log x}\log p_f\left(x|s^t\right)-\partial_{\log x}\log\lambda\left(x|s^t\right)\right)\right]di.\end{aligned}$$

Evaluating the above expression around the steady state implies

$$\begin{aligned} 0 \equiv & \int_0^1 \left[\omega g''\left(1\right)\partial_{\log x}\lambda\left(x|s^t\right)|_{ss}\right] di \\ & -\int_0^1 \left[\left(1-\omega\right) g''\left(1\right)\left(\partial_{\log x}\log p_f\left(x|s^t\right)|_{ss}-\partial_{\log x}\log\lambda\left(x|s^t\right)|_{ss}\right)\right] di. \end{aligned}$$

Since the expression under the integral does not depend on i, we obtain

$$\omega\partial_{\log x}\log\lambda\left(x|s^t\right)|_{ss} \equiv \left(1-\omega\right)\left(\partial_{\log x}\log p_f\left(x|s^t\right)|_{ss}-\partial_{\log x}\log\lambda\left(x|s^t\right)|_{ss}\right)$$

and thus

$$\partial_{\log x}\log\lambda\left(x|s^t\right)|_{ss} \equiv \left(1-\omega\right)\partial_{\log x}\log p_f\left(x|s^t\right)|_{ss}.$$

Accordingly, PT coefficient is

$$PT = \partial_{\log x}\log p_f\left(x|i,s^t\right)\Big|_{ss} = \partial_{\log x}\log\left(xv^*\left(s^t\right)\frac{\gamma\left(\frac{p_f\left(x|i,s^t\right)}{\lambda\left(x|s^t\right)}\right)}{\gamma\left(\frac{p_f\left(x|i,s^t\right)}{\lambda\left(x|s^t\right)}\right)-1}\right)$$

which simplifies to

$$PT = \partial_{\log x}\log p_f\left(x|i,s^t\right) = \frac{\left(\partial_x p_f\left(x|i,s^t\right)\right)}{dx}\frac{x}{p_f\left(x|i,s^t\right)}+1 = \partial_{\log x}\log\frac{\gamma\left(\frac{p_f\left(x|i,s^t\right)}{\lambda\left(x|s^t\right)}\right)}{\gamma\left(\frac{p_f\left(x|i,s^t\right)}{\lambda\left(x|s^t\right)}\right)-1}+1 =$$

$$= 1+\frac{\gamma'\left(.\right)\left(\gamma\left(.\right)-1\right)-\gamma\left(.\right)\gamma'\left(.\right)}{\left(\gamma\left(.\right)-1\right)^2}\frac{\left(\gamma\left(.\right)-1\right)x}{\gamma\left(.\right)}\frac{\lambda(.)\partial_{\log x}p_f(.)-p_f(\cdot)\partial_{\log x}\lambda(\cdot)}{\lambda(\cdot)^2}.$$

To simplify it further, we use the fact that $\partial_{\log x}\left(\lambda\left(.\right)|_{ss}\right) \equiv \left(1-\omega\right)\partial_{\log x}p_f\left(.\right)$ (derived above) and obtain

$$\begin{aligned} \partial_{\log x}\log p_f\left(x|i,s^t\right) = 1 &+ \left(\frac{p_f\left(.\right)}{\lambda\left(.\right)}\left(\partial_{\log x}\log p_f\left(.\right)-\partial_{\log x}\log\lambda\left(.\right)\right)\right)|_{ss} \\ &\times\frac{\gamma'\left(1\right)\left(\gamma\left(1\right)-1\right)-\gamma\left(1\right)\gamma'\left(1\right)}{\left(\gamma\left(1\right)-1\right)^2}\frac{\left(\gamma\left(1\right)-1\right)}{\gamma\left(1\right)} \end{aligned}$$

hence

$$\partial_{\log x}\log p_f\left(x|i,s^t\right)|_{ss} = 1+\frac{\gamma'\left(1\right)\left(\gamma\left(1\right)-1\right)-\gamma\left(1\right)\gamma'\left(1\right)}{\left(\gamma\left(1\right)-1\right)\gamma\left(1\right)}\omega\partial_{\log x}\log p_f\left(x|i,s^t\right)|_{ss},$$

$$1 = \frac{1}{\partial_{\log x}\log p_f\left(x|i,s^t\right)|_{ss}}+\frac{\gamma'\left(1\right)\left(\gamma\left(1\right)-1\right)-\gamma\left(1\right)\gamma'\left(1\right)}{\left(\gamma\left(1\right)-1\right)\gamma\left(1\right)}\omega,$$

and thus

$$\frac{1}{\partial_{\log x}\log p_f\left(x|i,s^t\right)|_{ss}} = \frac{\left(\gamma\left(1\right)-1\right)\gamma\left(1\right)-\gamma'\left(1\right)\left(\gamma\left(1\right)-1\right)\omega+\gamma\left(1\right)\gamma'\left(1\right)\omega}{\left(\gamma\left(1\right)-1\right)\gamma\left(1\right)}.$$

To conclude, we have shown that

$$PT = \partial_{\log x} \log p_f \left(x|i, s^t \right)|_{ss} = \frac{\left(\gamma\left(1\right)-1\right)\gamma\left(1\right)}{\gamma'\left(1\right)\omega + \left(\gamma\left(1\right)-1\right)\gamma\left(1\right)}. \tag{30}$$

TE: Partial differentiation using $\partial_{\log x}\left(.|_{ss}\right)$ treats the home price as a constant. Using symmetry and plugging in $p_f\left(x|i, s^t\right) \equiv p_f\left(x|s^t\right)$ for the import price, we thus need to calculate

$$TE := \frac{\partial_{\log x(s^t)} \log \frac{d\left(s^t\right)}{f\left(s^t\right)}\Big|_{ss}}{PT} = \frac{\partial_{\log x} \log \frac{(1-\omega) h\left(\frac{p_f\left(x|s^t\right)}{\lambda\left(x, s^t\right)}\right)}{\omega h\left(\frac{p_d}{\lambda\left(x, s^t\right)}\right)}\Big|_{ss}}{PT},$$

where, recall, $h'\left(x\right)|_{x=1} = g''\left(1\right), h\left(1\right) = 1$, and hence

$$\gamma\left(1\right) = \frac{h'\left(1\right)}{h\left(1\right)} = \frac{1}{g''\left(1\right)}.$$

The numerator of the expression for TE gives

$$\partial_{\log x} \log \frac{\left(1-\omega\right) h'\left(\frac{p_f\left(x|s^t\right)}{\lambda\left(x|s^t\right)}\right)}{\omega h\left(\frac{p_d}{\lambda\left(x|s^t\right)}\right)}\Big|_{ss} = \partial_{\log x} \log h\left(\frac{p_f\left(x|s^t\right)}{\lambda\left(x|s^t\right)}\right) - \partial_{\log x} \log h\left(\frac{p_d}{\lambda\left(x|s^t\right)}\right).$$

Using the fact that $\partial_{\log x}\lambda\left(x|s^t\right)|_{ss} \equiv \left(1-\omega\right)\partial_{\log x} p_f\left(x|s^t\right)$ and $h'\left(x\right)|_{x=1} = g''\left(1\right)^{-1}$, the two terms on the right-hand side imply

$$\partial_{\log x} \log h\left(\frac{p_f\left(x|s^t\right)}{\lambda\left(x|s^t\right)}\right)|_{ss} = \frac{h'\left(\frac{p_f(.)}{\lambda(.)}\right)}{h\left(\frac{p_f(.)}{\lambda(.)}\right)} \frac{\partial_{\log x} \log p_f\left(.\right) - \partial_{\log x} \log \lambda\left(.\right)}{\lambda\left(.\right)}\Big|_{ss} = \omega g''\left(1\right)\partial_{\log x} \log p_f\left(x|s^t\right)$$

which gives

$$\partial_{\log x} \log h\left(\frac{p_d}{\lambda\left(x|s^t\right)}\right)|_{ss} = \frac{h'\left(\frac{p_d}{\lambda(.)}\right)}{h\left(\frac{p_d}{\lambda(.)}\right)} \frac{-\partial_{\log x} \log \lambda\left(.\right)}{\lambda\left(.\right)}\Big|_{ss} = -\left(1-\omega\right)\partial_{\log x} \log p_f\left(x|s^t\right).$$

Accordingly, as claimed in text, we have now shown that

$$TE = \frac{\omega g''\left(1\right)^{-1}\partial_{\log x} \log p_f\left(x|i, s^t\right) + \left(1-\omega\right) g''\left(1\right)^{-1}\partial_{\log x} \log p_f\left(x|i, s^t\right)}{PT} = g''\left(1\right)^{-1} = \gamma\left(1\right). \tag{31}$$

1.3 CD Model

Preliminaries: The pricing equations are derived analogously to the baseline model.

PT: As in the baseline model, we differentiate the import price under the assumption of invariant foreign marginal cost v^*, which gives

$$\partial_{\log x} \log p_f \left(x|i,s^t\right)|_{ss} = \partial_x p_f\left(.\right) \frac{x}{p_f\left(.\right)}|_{ss} = \frac{\theta}{\theta-1} v^* \frac{x}{\frac{\theta}{\theta-1} x v^* + \frac{\xi}{\theta-1} v}|_{ss} = 1 - \frac{\frac{\xi}{\theta-1}}{\frac{\theta}{\theta-1} + \frac{\xi}{\theta-1}}.$$

Simplifying, we obtain

$$PT = \frac{\frac{\theta}{\theta-1}}{\frac{\theta}{\theta-1} + \frac{\xi}{\theta-1}} = \frac{1 + \mu^{ss} - \frac{\xi}{\theta-1}}{1 + \mu^{ss}} = 1 - \frac{\frac{\xi}{\theta-1}}{1 + \mu^{ss}}, \tag{32}$$

where, using the pricing formulas stated in the text, we know that the markup is given by

$$\mu^{ss} := \frac{p_d^{ss}}{v} - 1 = \frac{\theta}{\theta-1} + \frac{\xi}{\theta-1} - 1. \tag{33}$$

Accordingly, as stated in text, we derive

$$PT = 1 - \frac{\mu^{ss} - \frac{1}{\theta-1}}{1 + \mu^{ss}}, \tag{34}$$

since

$$\frac{\xi}{\theta-1} = \mu^{ss} - \frac{\theta}{\theta-1} + 1 = \mu^{ss} - \frac{1}{\theta-1}. \tag{35}$$

by the above.

TE: The final aggregating firm problem is the same as in the baseline model, hence $TE = \gamma$.

1.4 PD Model

Preliminaries: To solve for the key pricing formula, we need to solve the system of equations comprising the conditions discussed in text. To that end, note the following: 1) The distribution $F\left(p\right) := Pr\left(P \leq p\right)$ of quoted prices by foreign importers operating in the home country is defined on the interval $[P_l, P_h]$ and satisfies

$$\left(p - v^* x\right)\left(q + 2\left(1-q\right)\left(1 - F\left(p\right)\right)\right) = \left(P_h - v^* x\right) q, \text{ all } p \in [P_l, P_h],$$

where $v^* x$ is the marginal cost of producing the good; hence

$$F\left(p\right) = 1 - q \frac{P_h - p}{2\left(1-q\right)\left(p - v^* x\right)}. \tag{36}$$

2) The lower bound of the distribution satisfies $F\left(P_l\right) = 0$, hence

$$F\left(P_l\right) = 1 - q \frac{P_h - P_l}{2\left(1-q\right)\left(P_l - v^* x\right)} = 0 \rightarrow P_l = \frac{q P_h + 2\left(1-q\right) v^* x}{2 - q}.$$

3) The upper bound P_h satisfies that the (local) searching entity is indifferent between instructing reps who receives a single highest quote P_h to abort their purchase and replace them by new reps who purchase at the expected price $\bar{p}$, implying

$$\theta v = P_h - \bar{p}, \tag{37}$$

where, here, θv is the home country search cost for a unit of output (recall that each rep brings θ^{-1} units). As explained in text, the average price $\bar{p}$ solves

$$\bar{p} = \int_{P_l}^{P_h} p \frac{dH(p)}{dp} dp \tag{38}$$

where

$$\begin{aligned} H(p) &= qF(p) + (1-q)\left(1 - (1-F(p))^2\right) \\ &= \frac{(p(2-q) + P_h q - 2v^* x)(p(2-q) - 2(1-q)v^* x - P_h q)}{4(1-q)(p - v^* x)^2}. \end{aligned}$$

Integrating the above expression by parts gives the average price

$$\bar{p} = v^* x + q(P_h - v^* x) \tag{39}$$

and hence the upper bound

$$\theta v = P_h - \bar{p} \rightarrow P_h = v^* x + \frac{\theta v}{1-q}. \tag{40}$$

The above conditions imply that the import price—the average purchase price $p_f := \bar{p}$ by a rep of the imported good f implied by (38)—is

$$p_f = v + q(P_h - v) = v^* x + q\frac{\theta v}{1-q}. \tag{41}$$

PT: As in the baseline model and CD model above, we differentiate the import price under the assumption of invariant foreign marginal cost v^*:

$$\partial_{\log x} \log p_f\left(x|i, s^t\right)|_{ss} = \partial_x p_f(.)\frac{x}{p_f(.)}|_{ss} = v^* \frac{x}{xv^* + \frac{\theta q}{1-q}v} = 1 - \frac{\frac{\theta q}{1-q}}{1 + \frac{\theta q}{1-q}}.$$

The steady markup is

$$\mu^{ss} := \frac{p_d^{ss}}{v} - 1 = \frac{\theta q}{1-q}, \tag{42}$$

and so

$$PT = 1 - \frac{\mu^{ss}}{1 + \mu^{ss}} \tag{43}$$

TE: As for TE, divide the Armington-demand first-order conditions for goods d and f to obtain $\log(p_f/p_d) = \log\left((1-\omega)^{1/\gamma}/\omega^{1/\gamma}\right) - \gamma^{-1}\log(f/d)$ and note that the definition of TE implies $TE = \gamma$. Notes: Additional derivations and automated log-linearization of the above expressions can be found in the replication package *Mathematica* notebook TheoryResultsSection3_CS.nb.

1.5 NCES Model

Preliminaries: The key here is that a nonatomistic monopolistic producer sells the good to the final goods producer who aggregates goods according to a nested CES: the outer CES aggregates over a continuum of sectors indexed by i_1 and the inner CES aggregates over individual nonatomistic firms within the sector, indexed by i_2. The aggregator and nonatomistic structure imply that the demand function faced by the producer involves both the sectoral elasticity γ and the within-sector elasticity θ, where we assume $\theta > \gamma, \theta > 1$.

Let $\boldsymbol{i} = (i_1, i_2)$ and assume type-identical allocation / policy (or importers and all domestic producers within a sector are symmetric and make symmetric choices). The profit maximization of an importer is

$$
\begin{aligned}
\Pi_f(\boldsymbol{i}, s^t) \quad &= \max_{f(\boldsymbol{i},s^t), p_f(\boldsymbol{i},s^t)} \left[p_f(\boldsymbol{i}, s^t) - (1+\tau) v^*\left(s^t\right) x\left(s^t\right)\right] f(\boldsymbol{i}, s^t), \\
\text{s.t.} & \\
& \left(\frac{y(\boldsymbol{i}, s^t)}{Y\left(s^t\right)}\right)^{-\frac{1}{\gamma}} = \frac{P_f(\boldsymbol{i}, s^t)}{P(s^t)} \\
& \left(\frac{f(\boldsymbol{i}, s^t)}{y(\boldsymbol{i}, s^t)}\right)^{-\frac{1}{\theta}} = \frac{p_f(\boldsymbol{i}, s^t)}{P_f(\boldsymbol{i}, s^t)}, \\
& y(\boldsymbol{i}, s^t) = \left(\sum_{i_2=1}^{N} d(\boldsymbol{i}, s^t)^{\frac{\theta-1}{\theta}} + \sum_{i_2=1}^{N_X} f(\boldsymbol{i}, s^t)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}.
\end{aligned}
$$

where $\tau \geq 0$ is iceberg cost, $p_f(\boldsymbol{i}, s^t)$ is the price set by the within-sector producer, $P_f(\boldsymbol{i}, s^t)$ is the sectoral price, $y(\boldsymbol{i}, s^t)$ is sectoral output, $P(s^t) \equiv 1$ is the numéraire of final consumption, and $Y\left(s^t\right)$ is the final consumption/investment $(C+I)$ demand by the final producer.

To derive the first-order conditions, we substitute for prices into the objective function, which gives

$$
\left[\left(\frac{f(\boldsymbol{i}, s^t)}{y(\boldsymbol{i}, s^t)}\right)^{-\frac{1}{\rho}} \left(\frac{y(\boldsymbol{i}, s^t)}{Y\left(s^t\right)}\right)^{-\frac{1}{\gamma}} - (1+\tau) v^*\left(s^t\right) x\left(s^t\right)\right] f(\boldsymbol{i}, s^t),
$$

and plug in the last constraint to obtain unconstrained maximization:

$$
\left[f(\boldsymbol{i}, s^t)^{-\frac{1}{\rho}} \left(\sum_{i_2=1}^{N} d(\boldsymbol{i}, s^t)^{\frac{\theta-1}{\theta}} + \sum_{i_2=1}^{N_X} f(\boldsymbol{i}, s^t)^{\frac{\theta-1}{\theta}}\right)^{\frac{\left(\frac{1}{\theta}-\frac{1}{\gamma}\right)\rho}{\theta-1}} Y\left(s^t\right)^{\frac{1}{\gamma}} - (1+\tau)\frac{v^*\left(s^t\right)}{x\left(s^t\right)}\right] f(\boldsymbol{i}, s^t).
$$

The first-order condition with respect to $f(\boldsymbol{i}, s^t)$ is

$$
\left(-\frac{1}{\rho}+1\right) f(\boldsymbol{i}, s^t)^{-\frac{1}{\rho}} \left(\sum_{i_2=1}^{N} d(\boldsymbol{i}, s^t)^{\frac{\theta-1}{\theta}} + \sum_{i_2=1}^{N_X} f(\boldsymbol{i}, s^t)^{\frac{\theta-1}{\theta}}\right)^{\frac{1-\frac{\theta}{\gamma}}{\rho-1}} Y\left(s^t\right)^{\frac{1}{\gamma}} =
$$

$$
-f(\boldsymbol{i}, s^t)^{-\frac{1}{\theta}+1} \frac{1-\frac{\theta}{\gamma}}{\theta} \left(\sum_{i_2=1}^{N} d(\boldsymbol{i}, s^t)^{\frac{\theta-1}{\theta}} + \sum_{i_2=1}^{N_X} f(\boldsymbol{i}, s^t)^{\frac{\theta-1}{\theta}}\right)^{\frac{1-\frac{\theta}{\gamma}}{\theta-1}-1} f(\boldsymbol{i}, s^t)^{\frac{-1}{\theta}} Y\left(s^t\right)^{\frac{1}{\gamma}} + (1+\tau) v^*\left(s^t\right) x\left(s^t\right)
$$

and hence

$$
\left(-\frac{1}{\theta}+1\right) p_f(\boldsymbol{i}, s^t) =
$$

$$
-S_f(\boldsymbol{i}, s^t) \frac{1-\frac{\theta}{\gamma}}{\theta} p_f(\boldsymbol{i}, s^t) + (1+\tau) v^*\left(s^t\right) x\left(s^t\right)
$$

where

$$S_f(\boldsymbol{i}, s^t) := \frac{p_f(\boldsymbol{i}, s^t) f(\boldsymbol{i}, s^t)}{P_f(\boldsymbol{i}, s^t) y(\boldsymbol{i}, s^t)}$$

is the importer's market share. Simplifying, we obtain the key pricing equation of the model

$$\left(1 - \frac{1}{\theta}\left[1 - S_f(\boldsymbol{i}, s^t)\right] - \frac{1}{\gamma} S_f(\boldsymbol{i}, s^t)\right) p_f(\boldsymbol{i}, s^t) = (1+\tau) v^*\left(s^t\right) x\left(s^t\right),$$

and hence

$$p_f(\boldsymbol{i}, s^t) = \frac{\varepsilon_f(\boldsymbol{i}, s^t)}{\varepsilon_f(\boldsymbol{i}, s^t) - 1}(1+\tau) v^*\left(s^t\right) x\left(s^t\right),$$

where

$$\varepsilon_f(\boldsymbol{i}, s^t) := \left[\frac{1}{\theta}\left(1 - S_f\left(\boldsymbol{i}, s^t\right)\right) + \frac{1}{\gamma} S_f\left(\boldsymbol{i}, s^t\right)\right]^{-1}$$

is demand elasticity. In what follows, we assume there is N symmetric home firms within each sector and a smaller $N_X < N$ symmetric importers. We set $\tau = 0$ (analytic section only).

PT: We log-linearize the pricing equation given by

$$p_f\left(x|\boldsymbol{i}, s^t\right) = \frac{1}{1 - \left(\frac{1}{\theta}\left(1 - S_f\left(x|\boldsymbol{i}, s^t\right)\right) + \frac{1}{\gamma} S_f\left(x|\boldsymbol{i}, s^t\right)\right)} v^*\left(s^t\right) x, \tag{44}$$

which is equivalent to the one stated in the text after multiplying the numerator and the denominator by $\frac{1}{\theta}\left(1 - S_f\right) + \frac{1}{\gamma} S_f$. Log-linearization of this expression under the rules of the operator $\partial_{\log x}\left(.|_{ss}\right)$ gives

$$PT := \partial_{\log x} \log p_f\left(x|\boldsymbol{i}, s^t\right)|_{ss} = 1 - \frac{(\theta - 1)\mu_f^{ss} - 1}{\theta}\left(-\partial_{\log x} \log S_f\left(x|\boldsymbol{i}, s^t\right)|_{ss}\right). \tag{45}$$

(The operator $\partial_{\log x}\left(.|_{ss}\right)$ assumes that $p_d\left(\boldsymbol{i}, s^t\right)$ is unaffected by the exchange rate and so $d\left(\boldsymbol{i}, s^t\right)$ is treated as a constant.)

Using representativeness and symmetry, $p_f\left(x|\boldsymbol{i}, s^t\right) \equiv p_f\left(x|s^t\right)$ (all $\boldsymbol{i}$ from the foreign country), and so

$$S_f\left(x|\boldsymbol{i}, s^t\right) := \frac{\frac{p_f\left(x|\boldsymbol{i}, s^t\right) f\left(x|\boldsymbol{i}, s^t\right)}{p_d d\left(x|\boldsymbol{i}, s^t\right)}}{N + N_X \frac{p_f\left(x|\boldsymbol{i}, s^t\right) f\left(x|\boldsymbol{i}, s^t\right)}{p_d d\left(x|\boldsymbol{i}, s^t\right)}}.$$

Taking logs and applying the operator, we thus obtain

$$\begin{aligned} -\partial_{\log x}\left(\log S_f\left(x|s^t\right)\big|_{ss}\right) = {} & -\partial_{\log x} \log\left(\frac{p_f\left(x|s^t\right)}{p_d}\right)\Big|_{ss} + \left(-\partial_{\log x} \log\left(\frac{f(x|s^t)}{d(x|s^t)}\right)\Big|_{ss}\right) \\ & + \partial_{\log x} \log\left(N + N_X \frac{p_f\left(x|s^t\right) f(x|s^t)}{p_d d(x|s^t)}\right)\Big|_{ss}, \end{aligned} \tag{46}$$

where $f(x|s^t)$ denotes the relevant quantity consistent with the producer problem under the assumptions of the operator $\partial_{\log x}\left(|_{ss}\right)$ and symmetry. Dividing the demand equations side by side,

$$d = \left(\frac{p_d}{P_d(\boldsymbol{i}, s^t)}\right)^{-\theta} y\left(\boldsymbol{i}, s^t\right),$$
$$f(x|s^t) = \left(\frac{p_f\left(x|s^t\right)}{P_f(\boldsymbol{i}, s^t)}\right)^{-\theta} y\left(\boldsymbol{i}, s^t\right),$$

we note that

$$\frac{f(x|s^t)}{d(x|s^t)} = \left(\frac{p_f\left(x|s^t\right)}{p_d}\right)^{-\theta}, \tag{47}$$

and hence

$$\frac{p_f\left(x|s^t\right) f(x, s^t)}{p_d d(x|s^t)} = \left(\frac{p_f\left(x|s^t\right)}{p_d}\right)^{1-\theta}.$$

Accordingly,

$$\partial_{\log x}\frac{p_f\left(x|s^t\right) f(x|s^t)}{p_d d(x|s^t)}|_{ss} = \partial_{\log x}\left(\frac{p_f\left(x|s^t\right)}{p_d}\right)^{1-\theta}|_{ss} = (1-\theta)\left(\frac{p_f^{ss}}{p_d^{ss}}\right)^{1-\theta} PT.$$

Eliminating the constants and plugging in for f/d from (47), we obtain

$$\begin{aligned} -\partial_{\log x}\log S_f\left(x|s^t\right)\big|_{ss} = {} & -\partial_{\log x}\log\left(p_f\left(x|s^t\right)\right)\big|_{ss} + \left(\theta\partial_{\log x}\log\left(p_f\left(x|s^t\right)\right)\big|_{ss}\right) \\ & + \partial_{\log x}\log\left(1 + \frac{N_X}{N}\left(\frac{p_f\left(x|s^t\right)}{p_d}\right)^{1-\theta}\right)\big|_{ss}. \end{aligned} \tag{48}$$

Using the known approximation for logs, $d\log\left(1+x\right)/dx \leq 1$ (for all $z \geq 0$), and given $\theta > 1$, we obtain

$$-\partial_{\log x}\log S_f\left(x|s^t\right)\big|_{ss} \leq (\theta-1)\left(PT - \frac{N_X}{N}\left(\frac{p_f^{ss}}{p_d^{ss}}\right)^{1-\theta}\right) \leq (\theta-1)\, PT.$$

Plugging in to (45), we derive

$$PT \geq 1 - \frac{(\theta-1)\mu_f^{ss} - 1}{\theta}\left(\theta-1\right) PT$$

and assuming $(\theta-1)\mu_f^{ss} - 1 > 0$, we obtain

$$PT \geq \left(1 + \frac{(\theta-1)\mu_f^{ss} - 1}{\theta}\left(\theta-1\right)\right)^{-1}. \tag{49}$$

As for $(\theta-1)\mu_f^{ss}-1>0$, note that, since $\theta>\gamma>0$ and $S_f^{ss}>0$, (44) implies

$$\mu_f^{ss}=\frac{\frac{1}{\theta}\left(1-S_f^{ss}\right)+\frac{1}{\gamma}S_f^{ss}}{1-\left(\frac{1}{\theta}(1-S_f^{ss}+\frac{1}{\gamma}S_f^{ss})\right)} \tag{50}$$

$$>\frac{\frac{1}{\theta}\left(1-S_f^{ss}\right)+\frac{1}{\theta}S_f^{ss}}{1-\left(\frac{1}{\theta}\left(1-S_f^{ss}\right)+\frac{1}{\theta}S_f^{ss}\right)} \tag{51}$$

$$=\frac{\frac{1}{\theta}}{1-\frac{1}{\theta}}=\frac{1}{\theta-1}, \tag{52}$$

hence $(\theta-1)\mu_f^{ss}-1>1-1>0$.

TE: The formula for TE follows from the first-order condition of the producer:

$$\frac{f(x|s^t)}{d}=\left(\frac{p_f\left(x|s^t\right)}{p_d}\right)^{-\theta}. \tag{53}$$

Plugging in to the formula for TE, we obtain

$$TE=\frac{\partial_{\log x}\log\frac{d\left(s^t\right)}{f\left(s^t\right)}\big|_{ss}}{PT}=\theta\frac{\partial_{\log x}\log\left(\frac{p_f\left(x|s^t\right)}{p_d}\right)}{PT}=\theta\frac{\partial_{\log x}\log\left(p_f\left(x|s^t\right)\right)}{PT}=\theta. \tag{54}$$

Notes: Additional derivations and automated log-linearization of the above expressions can be found in the replication package *Mathematica* notebook TheoryResultsSection3_NCES.nb.

1.6 DH Model

Preliminaries: The importer's profit maximization is

$$\Pi_f\left(i,s^t\right) = \sum_{t=0}^{\infty}\sum_{s^t}\pi\left(s^t\right)\beta^t u_c^*\left(s^t\right)\left[(p_f^*(i,s^t)-v^*(s^t))f^*(i,s^t)+(p_f(i,s^t)/x(s^t)-v^*(s^t))f(i,s^t)\right].$$

s.t.

$$f(i,s^t)=\left(\frac{p_f(i,s^t)}{P_f(s^t)}\right)^{-\theta}h_f(i,s^{t-1})^{\zeta(\theta-1)}f(s^t),\quad (\psi_f(i,s^t)\beta^t u_c^*\left(s^t\right)\pi\left(s^t\right))$$

$$f^*(i,s^t)=\left(\frac{p_f^*(i,s^t)}{P_f^*(s^t)}\right)^{-\theta}h_f^*(i,s^{t-1})^{\zeta(\theta-1)}f^*(s^t),\quad (\psi_f^*(i,s^t)\beta^t u_c^*\left(s^t\right)\pi\left(s^t\right)) \tag{55}$$

$$h_f(i,s^t)=\rho h_f(i,s^{t-1})+(1-\rho)f(i,s^t),\quad (\Delta_f(i,s^t)\beta^t u_c^*\left(s^t\right)\pi\left(s^t\right)) \tag{56}$$

$$h_f^*(i,s^t)=\rho h_f^*(i,s^{t-1})+(1-\rho)f^*(i,s^t),\quad (\Delta_f^*(i,s^t)\beta^t u_c^*\left(s^t\right)\pi\left(s^t\right)), \tag{57}$$

where Lagrange multipliers are defined as noted in brackets next to the constraints.

The first-order conditions are

$$\begin{array}{rcl}
f^h & : & p_f(i,s^t)/x\left(s^t\right) - v^*(s^t) = \psi_f(i,s^t) - (1-\rho)\Delta_f(i,s^t) \\
f^* & : & p_f^*(i,s^t) - v^*(s^t) = \psi_f^*(i,s^t) - (1-\rho)\Delta_f^*(i,s^t) \\
p_f & : & f(i,s^t) = \theta \dfrac{f(i,s^t)}{p_f(i,s^t)/x\left(s^t\right)} \psi_f(i,s^t) \\
 & \to & p_f(i,s^t)/x(s^t)\dfrac{1}{\theta} = \psi_f(i,s^t) \\
p_f^* & : & f^*(i,s^t) = \theta \dfrac{f^*(i,s^t)}{p_f^*(s^t)} \psi_f^*(i,s^t) \\
 & \to & p_f^*(i,s^t)\dfrac{1}{\theta} = \psi_f^*(i,s^t) \\
h_f(i,s^t) & : & \Delta_f(i,s^t) = \beta \mathbb{E}_{s^t} \dfrac{u_c^*\left(s^{t+1}\right)}{u_c^*\left(s^t\right)} \left[\rho \Delta_f(i,s^{t+1}) + \psi_f(i,s^{t+1}) f(i,s^{t+1}) \dfrac{\zeta(\theta-1)}{h_f(i,s^t)}\right] \\
h_f^*(i,s^t) & : & \Delta_f^*(i,s^t) = \beta \mathbb{E}_{s^t} \dfrac{u_c^*\left(s^{t+1}\right)}{u_c^*\left(s^t\right)} \left[\rho \Delta_f^*(i,s^{t+1}) + \psi_f^*(i,s^{t+1}) f^*(i,s^{t+1}) \dfrac{\zeta(\theta-1)}{h_f^*(i,s^t)}\right],
\end{array}$$

where $\mathbb{E}_{s^t}$ denotes the conditional expectation on state s^t. Substituting out for the multipliers, we derive

$$p_f(i,s^t)/x\left(s^t\right) = \frac{\theta}{\theta-1}\left[v^*(s^t) - (1-\rho)\Delta_f(i,s^t)\right] \tag{58}$$

$$\Delta_f(i,s^t) = \beta \mathbb{E}_{s^t} \frac{u_c^*\left(s^{t+1}\right)}{u_c^*\left(s^t\right)} \left[\rho \Delta_f(i,s^{t+1}) + \frac{\zeta(\theta-1)}{\theta} \frac{p_f(i,s^{t+1})}{x\left(s^{t+1}\right)} \frac{f(i,s^{t+1})}{h_f(i,s^t)}\right] \tag{59}$$

$$x(s^t) p_f^*(i,s^t) = \frac{\theta}{\theta-1}\left[v(s^t) - (1-\rho)\Delta_f^*(i,s^t)\right] \tag{60}$$

$$\Delta_f^*(i,s^t) = \beta \mathbb{E}_{s^t} \frac{u_c\left(s^{t+1}\right)}{u_c\left(s^t\right)} \left[\rho \Delta_f^*(i,s^{t+1}) + \frac{\zeta(\theta-1)}{\theta} p_f^*(i,s^{t+1}) \frac{d^*(i,s^{t+1})}{h_f^*(i,s^t)}\right]. \tag{61}$$

As noted in text, the demand function faced by the foreign producer (one of the constraints in the optimization above) is

$$f\left(p_f, h_f | i, s^{t+1}\right) := \left(\frac{p_f}{P_f(s^t)}\right)^{-\theta} h_f^{\zeta(\theta-1)} f\left(s^{t+1}\right).$$

and so

$$\underbrace{-\frac{\partial_{h_f} f(\cdot)}{\partial_{p_f} f(\cdot)}}_{MRT} \frac{f(i,s^{t+1})}{x(s^{t+1})} = \frac{\zeta(\theta-1)}{\theta} \frac{p_f(i,s^{t+1})}{x(s^{t+1})} \frac{f(i,s^{t+1})}{h_f(i,s^t)}$$

Accordingly, the law of motion for the value of habit can be equivalently expressed as expression in text)

$$\Delta_f\left(i,s^t\right)=\beta\mathbb{E}_{s^t}\frac{u^*_{c^*}\left(s^{t+1}\right)}{u^*_{c^*}\left(s^t\right)}\left[\rho\Delta_f\left(i,s^{t+1}\right)+-\underbrace{\frac{\partial_{h_f}f\left(p_f,h_f|i,s^{t+1}\right)}{\partial_{p_f}f\left(p_f,h_f|i,s^{t+1}\right)}}_{MRT}\frac{f\left(p_f,h_f|i,s^{t+1}\right)}{x\left(s^{t+1}\right)}\right], \tag{62}$$

PT: Define

$$g_h(s^{t+1}):=\frac{h_f(i,s^{t+1})}{h_f(i,s^t)},$$

which, given the habit equation,

$$h_f\left(i,s^{t+1}\right)=\rho h_f\left(i,s^t\right)+(1-\rho)f\left(i,s^{t+1}\right), \tag{63}$$

implies

$$\frac{f(i,s^{t+1})}{h_f(i,s^t)}=\frac{g_h(s^{t+1})-\rho}{1-\rho}., \tag{64}$$

Given the pricing equation,

$$\frac{p_f(i,s^t)}{x\left(s^t\right)}=\frac{\theta}{\theta-1}\left[v^*\left(s^t\right)-(1-\rho)\Delta_f\left(i,s^t\right)\right],$$

we can express the value of habit as

$$\Delta_f\left(i,s^t\right)=-\frac{\frac{p_f(i,s^t)}{x(s^t)}\frac{\theta-1}{\theta}-v^*\left(s^t\right)}{1-\rho}.$$

We next plug in to the law of motion for Δ_f above and obtain

$$\begin{aligned}-\frac{\frac{p_f(i,s^t)}{x(s^t)}\frac{\theta-1}{\theta}-v^*\left(s^t\right)}{1-\rho}&=\beta\mathbb{E}_{s^t}\frac{u_{c^*}\left(s^{t+1}\right)}{u_{c^*}\left(s^t\right)}\left[\rho\left(-\frac{\frac{p_f(i,s^{t+1})}{x(s^{t+1})}\frac{\theta-1}{\theta}-v^*\left(s^{t+1}\right)}{1-\rho}\right)\right]\\&\quad+\beta\mathbb{E}_{s^t}\frac{u_{c^*}\left(s^{t+1}\right)}{u_{c^*}\left(s^t\right)}\left[\frac{\zeta(\theta-1)}{\theta}\frac{p_f(i,s^{t+1})}{x(s^{t+1})}\frac{g_h(s^{t+1})-\rho}{1-\rho}\right],\end{aligned}$$

hence

$$\begin{aligned}\frac{p_f(i,s^t)}{x\left(s^t\right)}&=\frac{\theta}{\theta-1}\left(v^*(s^t)-\beta\mathbb{E}_{s^t}\frac{u^*_{c^*}\left(s^{t+1}\right)}{u^*_{c^*}\left(s^t\right)}\rho v^*(s^{t+1})\right)+\beta\mathbb{E}_{s^t}\frac{u^*_{c^*}\left(s^{t+1}\right)}{u^*_{c^*}\left(s^t\right)}\left[\rho\frac{p_f(i,s^{t+1})}{x(s^{t+1})}\right]\\&\quad-\beta\mathbb{E}_{s^t}\frac{u^*_{c^*}\left(s^{t+1}\right)}{u^*_{c^*}\left(s^t\right)}\left[\zeta\frac{p_f(i,s^{t+1})}{x(s^{t+1})}\left(g_h(s^{t+1})-\rho\right)\right],\end{aligned}$$

and

$$\frac{p_f(i,s^t)}{x(s^t)} = \frac{\theta}{\theta-1}\left(v^*(s^t) - \beta\mathbb{E}_{s^t}\frac{u_{c^*}\left(s^{t+1}\right)}{u_{c^*}\left(s^t\right)}\rho v^*(s^{t+1})\right) \tag{65}$$

$$+\beta\mathbb{E}_{s^t}\frac{u^*_{c^*}\left(s^{t+1}\right)}{u^*_{c^*}\left(s^t\right)}\left[\frac{p_f(i,s^{t+1})}{x(s^{t+1})}\left(\rho-\zeta g_h(s^{t+1})+\zeta\rho\right)\right].$$

We use the fact that $p^*_f(i,s^t) := \frac{p_f(i,s^t)}{x(s^t)}$, plug in to the above and derive the following difference equation

$$p^*_f(x\left(s^t\right)|s^t) = \frac{\theta}{\theta-1}\left(v^*(s^t)-\beta\mathbb{E}_{s^t}...\right) + \beta\mathbb{E}_{s^t}\frac{u_{c^*}\left(s^{t+1}\right)}{u_{c^*}\left(s^t\right)}\left[p^*_f(x\left(s^{t+1}\right)|s^{t+1})\left(\rho-\zeta g_h(s^{t+1})+\zeta\rho\right)\right].$$

We log-linearize this equation with respect to $(p^*_f(i,s^t), p^*_f(i,s^{t+1}), g_h\left(s^{t+1}\right))$ around the deterministic steady state and obtain

$$d\log p^*_f(i,s^t) = -\beta\zeta\mathbb{E}_{s^t}d\log g_h\left(s^{t+1}\right) + \beta\left(\rho-\zeta\left(1-\rho\right)\right)\mathbb{E}_{s^t}d\log p^*_f(i,s^{t+1})$$

We solve the implied difference equation forward to obtain

$$d\log p^*_f(i,s^t) = -\beta\zeta\sum_{t=0}^{\infty}\beta^t\left(\rho-\zeta\left(1-\rho\right)\right)^t\mathbb{E}_{s^t}d\left(\log g_h\left(x|s^{t+1}\right)|_{ss}\right)$$

Finally, using the fact that

$$d\log p^*_f(i,s^t) = d\log p_f(i,s^t) - d\log x,$$

we divide both sides by $d\log x$ and derive

$$PT \equiv \partial_{\log x}\left(\log p_f\left(x|s^t\right)\right) = \frac{\partial\log p_f\left(x|s^t\right)}{\partial\log x} = 1-\beta\zeta\sum_{t=0}^{\infty}\beta^t\left(\rho-\zeta\left(1-\rho\right)\right)^t\mathbb{E}_{s^t}\partial_{\log x}\left(\log g_h\left(x|s^{t+1}\right)|_{ss}\right).$$

Note that $0\zeta < 1$ and $0 < \rho < 1$, and so $\rho - \zeta\left(1-\rho\right) > 0$.

TE: TE derivation is derived analogously as in the baseline model: $TE = \gamma$. We omit the details.

Notes: Additional derivations and automated log-linearization of the above expressions can be found in the replication package *Mathematica* notebook TheoryResultsSection3_DH.nb.

1.7 CC Model

Preliminaries: Consider the simplified "analytic" setup from Section 3. The key equations are: 1) the bargaining equation

$$p_f(x|i,s^t) = \eta\partial_f G\left(\frac{d\left(s^t\right)}{f\left(x|s^t\right)},1\right) + (1-\eta)xv^*, \tag{66}$$

where $G(d,f)$ corresponds to (5), 2) the endogenous market-share determination equation

$$\frac{f\left(x|s^t\right)}{d\left(s^t\right)} = \frac{m_f\left(x|s^t\right)}{\bar{m}_d\left(s^t\right)}, \tag{67}$$

3) and the first-order condition associated with the analytic maximization stated in Section 3 of the paper,

$$\max_{a_f(i,s^t)} \left(p_f(s^t)/x(s^t) - v(s^t)\right) h_d(s^t) \frac{m_f(i,s^t)}{\bar{m}_f(s^t) + \bar{m}_d(s^t)} - v^*(s^t) a_f(i,s^t), \tag{68}$$

subject to

$$m_f(i,s^t) = a_f(i,s^t) - \psi a_f^{ss} \left(\frac{a_f(i,s^t)}{a_f^{ss}} - 1\right)^2. \tag{69}$$

which, after plugging in the steady state value for $v^* = v^{ss} = \eta/(\eta + \mu^{ss})$—consistent with the partial differentiation to obtain PT—gives

$$\left(\frac{p_f(s^t)}{x(s^t)} - \frac{\eta}{\eta + \mu^{ss}}\right) \left(1 - 2\Psi \frac{a_f(s^t) - a_f^{ss}}{a_f^{ss}}\right) = (\bar{m}_d + \bar{m}_f) \frac{\eta}{\eta + \mu^{ss}} \tag{70}$$

where $a_f^{ss} = m_f^{ss}$ is the steady-state value associated with markup μ^{ss} that solves $p_f^{ss} = v^{ss}(1 + \mu^{ss})$. In the steady state, we assume $d^{ss}/f^{ss} = \omega/(1-\omega)$, which gives

$$a_f^{ss} = m_f^{ss} = \mu^{ss}(1-\omega), \qquad a_d^{ss} = m_d^{ss} = \mu^{ss}\omega.$$

The term μ^{ss} ensures that the convex adjustment cost does not bind in the steady state. While the model is static, this emulates the long-run equilibrium in the full model.

PT: We start from the bargaining equations, which, in log-linear form around the symmetric steady state and under the rules of the operator $\partial_{\log x}(\log .|_{ss})$, is

$$\gamma \left(\eta + (1 + \mu^{ss}) \underbrace{\partial_{\log x}\left(\log p_f(x|i,s^t)|_{ss}\right)}_{PT} - 1\right) = \frac{\omega}{1-\omega}(\eta + \mu^{ss}) \left(\underbrace{-\partial_{\log x} \log\left(\frac{f(x|s^t)}{d(s^t)}|_{ss}\right)}_{TE \times PT}\right),$$

where, by definition,

$$TE := (PT)^{-1} \underbrace{-\partial_{\log x} \log\left(\frac{f(x|s^t)}{d(s^t)}|_{ss}\right)}_{TE \times PT}. \tag{71}$$

It is now easy to calculate that the above equations give the expression in text:

$$PT = \frac{1-\eta}{1 + \mu^{ss} - \frac{TE}{\gamma}(\eta + \mu^{ss})\omega}. \tag{72}$$

TE: We log-linearize the constraint (69), the first-order condition in (70), the market-share equation in (67) and the bargaining equation in (66), all around the steady state under the rules of the operator $\partial_x(.|_{ss})$. The resulting log-linearized equations, in the same sequence, are:

$$\partial_{\log x} \log\left(\bar{m}_f(x|i,s^t)|_{ss}\right) = \partial_{\log x} \log\left(a_f(x|i,s^t)|_{ss}\right)$$

$$(1+\mu^{ss})\left(\underbrace{\partial_{\log x}\left(\log p_f(x|i,s^t)|_{ss}\right)}_{PT}-1\right)=\partial_{\log x}\log\left(a_f(x|i,s^t)|_{ss}\right)(1+2\psi-\omega),$$

$$\underbrace{-\partial_{\log x}\log\left(\frac{f\left(x|s^t\right)}{d\left(s^t\right)}|_{ss}\right)}_{TE\times PT}=\partial_{\log x}\log\left(\bar{m}_f(x|i,s^t)|_{ss}\right),$$

and bargaining equation

$$\gamma\left((1+\mu^{ss})\underbrace{\partial_{\log x}\left(\log p_f(x|i,s^t)|_{ss}\right)}_{PT}-(1-\eta)\right)=(\eta+\mu^{ss})\,\omega\underbrace{\left(-\partial_{\log x}\log\left(\frac{f\left(x|s^t\right)}{d\left(s^t\right)}|_{ss}\right)\right)}_{TE\times PT}.$$

Solving the above system yields the expression from text:

$$TE=\frac{\gamma(1+\mu^{ss})(\eta+\mu^{ss})}{\gamma(1-\eta)\mu^{ss}(1-\omega+2\psi)+\omega(1+\mu^{ss})(\eta+\mu^{ss})}.$$

Notes: Additional derivations and automated log-linearization of the above expressions can be found in the replication package *Mathematica* notebook TheoryResultsSection3_CC.nb.[1]

2 Log-linearized First-Order Conditions

Unless indicated otherwise, all variables showing up in the linearized equations are defined as proportional deviations from the steady state: i.e., for any variable $X\left(s^t\right)$, we define $X_t \equiv \mathrm{d}X\left(s^t\right)/X^{ss}$, where X^{ss} is the steady state value of $X\left(s^t\right)$. We make exceptions for variables that can take non-positive values (e.g., net foreign assets), which we scale by steady state final goods production: i.e., for any such variable $Y\left(s^t\right)$, we define $Y_t \equiv \mathrm{d}Y\left(s^t\right)/y^{ss}$, where $y^{ss} \equiv d^{ss}+d^{*,ss}$. We also define the steady state share of hours worked in time endowment as $\mathcal{L} \equiv l^{ss}/\bar{l}$.

2.1 Frictionless Baseline (FB)

Marginal utility

$$u_{c,t}=\left(\nu(1-\sigma)-1\right)c_t-(1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l_t \tag{73}$$

$$u^*_{c,t}=\left(\nu(1-\sigma)-1\right)c^*_t-(1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l^*_t \tag{74}$$

Capital accumulation

$$k_t=(1-\delta)k_{t-1}+\delta i_t \tag{75}$$

$$k^*_t=(1-\delta)k^*_{t-1}+\delta i^*_t \tag{76}$$

Demand for final goods

$$\gamma P_{d,t}+d_t=\omega d_t+(1-\omega)f_t \tag{77}$$

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[1] Note: in the file, m corresponds to a above, θ corresponds to η and ϕ corresponds to ψ.

$$\gamma P^*_{d,t} + d^*_t = \omega f^*_t + (1-\omega) d^*_t \tag{78}$$

$$\gamma P_{f,t} + f_t = \omega d_t + (1-\omega) f_t \tag{79}$$

$$\gamma P^*_{f,t} + f^*_t = \omega f^*_t + (1-\omega) d^*_t \tag{80}$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1-\mathcal{L}} l_t + c_t \tag{81}$$

$$w^*_t = \frac{\mathcal{L}}{1-\mathcal{L}} l^*_t + c^*_t \tag{82}$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \tag{83}$$

$$u^*_{c,t} = R^*_t + u^*_{c,t|t+1} \tag{84}$$

Optimal investment

$$u_{c,t} + \phi\delta\left(i_t - k_{t-1}\right) = u_{c,t|t+1} + \left[1-\beta(1-\delta)\right] r_{t|t+1} + \beta\phi\delta\left(i_{t|t+1} - k_t\right) \tag{85}$$

$$u^*_{c,t} + \phi\delta\left(i^*_t - k^*_{t-1}\right) = u^*_{c,t|t+1} + \left[1-\beta(1-\delta)\right] r^*_{t|t+1} + \beta\phi\delta\left(i^*_{t|t+1} - k^*_t\right) \tag{86}$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \tag{87}$$

$$k^*_{t-1} - l^*_t = w^*_t - r^*_t \tag{88}$$

Marginal cost

$$v_t = (1-\alpha) w_t + \alpha r_t - z_t \tag{89}$$

$$v^*_t = (1-\alpha) w^*_t + \alpha r^*_t - z^*_t \tag{90}$$

Price setting

$$P_{d,t} = v_t \tag{91}$$

$$P_{f,t} = x_t + v^*_t \tag{92}$$

$$P^*_{d,t} = v_t - x_t \tag{93}$$

$$P^*_{f,t} = v^*_t \tag{94}$$

Resource constraint

$$\omega d_t + (1-\omega) d^*_t = z_t + \alpha k_{t-1} + (1-\alpha) l_t \tag{95}$$

$$(1-\omega) f_t + \omega f^*_t = z^*_t + \alpha k^*_{t-1} + (1-\alpha) l^*_t \tag{96}$$

Final goods market clearing, where $\frac{i^{ss}}{y^{ss}} = \frac{\theta-1}{\theta} \frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t + \frac{i^{ss}}{y^{ss}} i_t = \omega d_t + (1-\omega) f_t \tag{97}$$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c^*_t + \frac{i^{ss}}{y^{ss}} i^*_t = \omega f^*_t + (1-\omega) d^*_t \tag{98}$$

UIP condition

$$R^*_t - R_t + x_{t|t+1} - x_t = -\Gamma\left(b_t + n_t\right) \tag{99}$$

Net foreign assets

$$b_t = \frac{1}{\beta} b_{t-1} + (1-\omega)\left(d_t^* - f_t + x_t + P_{d,t}^* - P_{f,t}\right) \tag{100}$$

2.2 Kimball Aggregation (KA)

Marginal utility

$$u_{c,t} = \left(\nu(1-\sigma) - 1\right) c_t - (1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}} l_t \tag{101}$$

$$u_{c,t}^* = \left(\nu(1-\sigma) - 1\right) c_t^* - (1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}} l_t^* \tag{102}$$

Capital accumulation

$$k_t = (1-\delta)k_{t-1} + \delta i_t \tag{103}$$

$$k_t^* = (1-\delta)k_{t-1}^* + \delta i_t^* \tag{104}$$

Demand for final goods, where $\gamma = \gamma(1)$

$$\gamma P_{d,t} + d_t = \omega d_t + (1-\omega) f_t \tag{105}$$

$$\gamma P_{d,t}^* + d_t^* = \omega f_t^* + (1-\omega) d_t^* \tag{106}$$

$$\gamma P_{f,t} + f_t = \omega d_t + (1-\omega) f_t \tag{107}$$

$$\gamma P_{f,t}^* + f_t^* = \omega f_t^* + (1-\omega) d_t^* \tag{108}$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1-\mathcal{L}} l_t + c_t \tag{109}$$

$$w_t^* = \frac{\mathcal{L}}{1-\mathcal{L}} l_t^* + c_t^* \tag{110}$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \tag{111}$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \tag{112}$$

Optimal investment

$$u_{c,t} + \phi\delta\left(i_t - k_{t-1}\right) = u_{c,t|t+1} + \left[1-\beta(1-\delta)\right] r_{t|t+1} + \beta\phi\delta\left(i_{t|t+1} - k_t\right) \tag{113}$$

$$u_{c,t}^* + \phi\delta\left(i_t^* - k_{t-1}^*\right) = u_{c,t|t+1}^* + \left[1-\beta(1-\delta)\right] r_{t|t+1}^* + \beta\phi\delta\left(i_{t|t+1}^* - k_t^*\right) \tag{114}$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \tag{115}$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \tag{116}$$

Marginal cost

$$v_t = (1-\alpha) w_t + \alpha r_t - z_t \tag{117}$$

$$v_t^* = (1-\alpha) w_t^* + \alpha r_t^* - z_t^* \tag{118}$$

Price setting, where $\psi = \frac{1+\gamma-h"(1)/\gamma}{2\gamma-h"(1)/\gamma}$

$$P_{d,t} = (1-\psi)v_t \tag{119}$$

$$P_{f,t} = (1-\psi)\left(x_t + v_t^*\right) \tag{120}$$

$$P_{d,t}^* = (1-\psi)\left(v_t - x_t\right) \tag{121}$$

$$P_{f,t}^* = (1-\psi)v_t^* \tag{122}$$

Resource constraint

$$\omega d_t + (1-\omega)d_t^* = z_t + \alpha k_{t-1} + (1-\alpha)l_t \tag{123}$$

$$(1-\omega)f_t + \omega f_t^* = z_t^* + \alpha k_{t-1}^* + (1-\alpha)l_t^* \tag{124}$$

Final goods market clearing, where $\frac{i^{ss}}{y^{ss}} = \frac{\gamma-1}{\gamma}\frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$

$$\left(1-\frac{i^{ss}}{y^{ss}}\right)c_t + \frac{i^{ss}}{y^{ss}}i_t = \omega d_t + (1-\omega)f_t \tag{125}$$

$$\left(1-\frac{i^{ss}}{y^{ss}}\right)c_t^* + \frac{i^{ss}}{y^{ss}}i_t^* = \omega f_t^* + (1-\omega)d_t^* \tag{126}$$

UIP condition

$$R_t^* - R_t + x_{t|t+1} - x_t = -\Gamma\left(b_t + n_t\right) \tag{127}$$

Net foreign assets

$$b_t = \frac{1}{\beta}b_{t-1} + (1-\omega)\left(d_t^* - f_t + x_t + P_{d,t}^* - P_{f,t}\right) \tag{128}$$

2.3 Costly distribution (CD)

Marginal utility

$$u_{c,t} = \left(\nu(1-\sigma)-1\right)c_t - (1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l_t \tag{129}$$

$$u_{c,t}^* = \left(\nu(1-\sigma)-1\right)c_t^* - (1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l_t^* \tag{130}$$

Capital accumulation

$$k_t = (1-\delta)k_{t-1} + \delta i_t \tag{131}$$

$$k_t^* = (1-\delta)k_{t-1}^* + \delta i_t^* \tag{132}$$

Demand for final goods

$$\gamma P_{d,t} + d_t = \omega d_t + (1-\omega)f_t \tag{133}$$

$$\gamma P_{d,t}^* + d_t^* = \omega f_t^* + (1-\omega)d_t^* \tag{134}$$

$$\gamma P_{f,t} + f_t = \omega d_t + (1-\omega)f_t \tag{135}$$

$$\gamma P_{f,t}^* + f_t^* = \omega f_t^* + (1-\omega)d_t^* \tag{136}$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1-\mathcal{L}}l_t + c_t \tag{137}$$

$$w_t^* = \frac{\mathcal{L}}{1-\mathcal{L}} l_t^* + c_t^* \tag{138}$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \tag{139}$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \tag{140}$$

Optimal investment

$$u_{c,t} + \phi\delta\left(i_t - k_{t-1}\right) = u_{c,t|t+1} + \left[1 - \beta(1-\delta)\right] r_{t|t+1} + \beta\phi\delta\left(i_{t|t+1} - k_t\right) \tag{141}$$

$$u_{c,t}^* + \phi\delta\left(i_t^* - k_{t-1}^*\right) = u_{c,t|t+1}^* + \left[1 - \beta(1-\delta)\right] r_{t|t+1}^* + \beta\phi\delta\left(i_{t|t+1}^* - k_t^*\right) \tag{142}$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \tag{143}$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \tag{144}$$

Marginal cost

$$v_t = (1-\alpha) w_t + \alpha r_t - z_t \tag{145}$$

$$v_t^* = (1-\alpha) w_t^* + \alpha r_t^* - z_t^* \tag{146}$$

Price setting

$$p_{d,t} = v_t \tag{147}$$

$$p_{f,t} = \frac{1}{(1+\xi)(1-\xi)}\left(v_t^* + x_t + \frac{\xi}{\theta} v_t\right) \tag{148}$$

$$p_{d,t}^* = \frac{1}{(1+\xi)(1-\xi)}\left(v_t - x_t + \frac{\xi}{\theta} v_t^*\right) \tag{149}$$

$$P_{f,t}^* = v_t^* \tag{150}$$

Final goods prices, where $v^{ss} = \frac{\theta-1}{\theta(1+\xi)}$

$$P_{d,t} = (1 - \xi v^{ss}) p_{d,t} + \xi v^{ss} v_t \tag{151}$$

$$P_{f,t} = (1 - \xi v^{ss}) p_{f,t} + \xi v^{ss} v_t \tag{152}$$

$$P_{d,t}^* = (1 - \xi v^{ss}) p_{d,t}^* + \xi v^{ss} v_t^* \tag{153}$$

$$P_{f,t}^* = (1 - \xi v^{ss}) p_{f,t}^* + \xi v^{ss} v_t^* \tag{154}$$

Resource constraint

$$\omega d_t + (1-\omega) d_t^* + \xi\left[\omega d_t + (1-\omega) f_t\right] = (1+\xi)\left[z_t + \alpha k_{t-1} + (1-\alpha) l_t\right] \tag{155}$$

$$(1-\omega) f_t + \omega f_t^* + \xi\left[\omega f_t^* + (1-\omega) d_t^*\right] = (1+\xi)\left[z_t^* + \alpha k_{t-1}^* + (1-\alpha) l_t^*\right] \tag{156}$$

Final goods market clearing, where $\frac{i^{ss}}{y^{ss}} = \frac{\theta-1}{\theta} \frac{\delta\alpha}{\frac{1}{\beta} - 1 + \delta}$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t + \frac{i^{ss}}{y^{ss}} i_t = \omega d_t + (1-\omega) f_t \tag{157}$$

$$\left(1-\frac{i^{ss}}{y^{ss}}\right)c_t^*+\frac{i^{ss}}{y^{ss}}i_t^*=\omega f_t^*+(1-\omega)d_t^* \tag{158}$$

UIP condition

$$R_t^*-R_t+x_{t|t+1}-x_t=-\Gamma\left(b_t+n_t\right) \tag{159}$$

Net foreign assets

$$b_t=\frac{1}{\beta}b_{t-1}+\left(1-\xi v^{ss}\right)(1-\omega)\left(d_t^*-f_t+x_t+p_{d,t}^*-p_{f,t}\right) \tag{160}$$

2.4 Price Dispersion (PD)

Marginal utility

$$u_{c,t}=\left(\nu(1-\sigma)-1\right)c_t-(1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l_t \tag{161}$$

$$u_{c,t}^*=\left(\nu(1-\sigma)-1\right)c_t^*-(1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l_t^* \tag{162}$$

Capital accumulation

$$k_t=(1-\delta)k_{t-1}+\delta i_t \tag{163}$$

$$k_t^*=(1-\delta)k_{t-1}^*+\delta i_t^* \tag{164}$$

Demand for final goods, where $v^{ss}=\frac{1-q}{1-q+\theta}$

$$\gamma\left[(1-\theta v^{ss})P_{d,t}+\theta v^{ss}v_t\right]+d_t=\omega d_t+(1-\omega)f_t \tag{165}$$

$$\gamma\left[(1-\theta v^{ss})P_{d,t}^*+\theta v^{ss}v_t^*\right]+d_t^*=\omega f_t^*+(1-\omega)d_t^* \tag{166}$$

$$\gamma\left[(1-\theta v^{ss})P_{f,t}+\theta v^{ss}v_t\right]+f_t=\omega d_t+(1-\omega)f_t \tag{167}$$

$$\gamma\left[(1-\theta v^{ss})P_{f,t}^*+\theta v^{ss}v_t^*\right]+f_t^*=\omega f_t^*+(1-\omega)d_t^* \tag{168}$$

Consumption-leisure choice

$$w_t=\frac{\mathcal{L}}{1-\mathcal{L}}l_t+c_t \tag{169}$$

$$w_t^*=\frac{\mathcal{L}}{1-\mathcal{L}}l_t^*+c_t^* \tag{170}$$

Optimal bond holdings

$$u_{c,t}=R_t+u_{c,t|t+1} \tag{171}$$

$$u_{c,t}^*=R_t^*+u_{c,t|t+1}^* \tag{172}$$

Optimal investment

$$u_{c,t}+\phi\delta\left(i_t-k_{t-1}\right)=u_{c,t|t+1}+\left[1-\beta(1-\delta)\right]r_{t|t+1}+\beta\phi\delta\left(i_{t|t+1}-k_t\right) \tag{173}$$

$$u_{c,t}^*+\phi\delta\left(i_t^*-k_{t-1}^*\right)=u_{c,t|t+1}^*+\left[1-\beta(1-\delta)\right]r_{t|t+1}^*+\beta\phi\delta\left(i_{t|t+1}^*-k_t^*\right) \tag{174}$$

Capital-labor choice

$$k_{t-1}-l_t=w_t-r_t \tag{175}$$

$$k_{t-1}^*-l_t^*=w_t^*-r_t^* \tag{176}$$

Marginal cost

$$v_t = (1-\alpha)w_t + \alpha r_t - z_t \tag{177}$$

$$v_t^* = (1-\alpha)w_t^* + \alpha r_t^* - z_t^* \tag{178}$$

Price setting

$$P_{d,t} = v_t \tag{179}$$

$$P_{f,t} = \frac{1-q}{1-q+\theta q}(x_t + v_t^*) + \frac{\theta q}{1-q+\theta q}v_t \tag{180}$$

$$\frac{1-q}{1-q+\theta q}(v_t - x_t) + \frac{\theta q}{1-q+\theta q}v_t^* \tag{181}$$

$$P_{f,t}^* = v_t^* \tag{182}$$

Resource constraint

$$\omega d_t + (1-\omega)d_t^* + \theta\left[\omega d_t + (1-\omega)f_t\right] = (1+\theta)\left[z_t + \alpha k_{t-1} + (1-\alpha)l_t\right] \tag{183}$$

$$(1-\omega)f_t + \omega f_t^* + \theta\left[\omega f_t^* + (1-\omega)d_t^*\right] = (1+\theta)\left[z_t^* + \alpha k_{t-1}^* + (1-\alpha)l_t^*\right] \tag{184}$$

Final goods market clearing, where $\frac{i^{ss}}{y^{ss}} = \frac{(1-q)(1+\theta)}{1-q+\theta}\frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$

$$\left(1-\frac{i^{ss}}{y^{ss}}\right)c_t + \frac{i^{ss}}{y^{ss}}i_t = \omega d_t + (1-\omega)f_t \tag{185}$$

$$\left(1-\frac{i^{ss}}{y^{ss}}\right)c_t^* + \frac{i^{ss}}{y^{ss}}i_t^* = \omega f_t^* + (1-\omega)d_t^* \tag{186}$$

UIP condition

$$R_t^* - R_t + x_{t|t+1} - x_t = -\Gamma\left(b_t + n_t\right) \tag{187}$$

Net foreign assets

$$b_t = \frac{1}{\beta}b_{t-1} + (1-\theta v^{ss})(1-\omega)\left(d_t^* - f_t + x_t + P_{d,t}^* - P_{f,t}\right) \tag{188}$$

2.5 Nested CES (NCES)

Marginal utility

$$u_{c,t} = \left(\nu(1-\sigma)-1\right)c_t - (1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l_t \tag{189}$$

$$u_{c,t}^* = \left(\nu(1-\sigma)-1\right)c_t^* - (1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l_t^* \tag{190}$$

Capital accumulation

$$k_t = (1-\delta)k_{t-1} + \delta i_t \tag{191}$$

$$k_t^* = (1-\delta)k_{t-1}^* + \delta i_t^* \tag{192}$$

Demand for final goods, where $\frac{i^{ss}}{y^{ss}} = \left(\frac{\varepsilon_d^{ss}-1}{\varepsilon_d^{ss}}\tilde{\omega} + (1-\tilde{\omega})\frac{\varepsilon_f^{ss}-1}{\varepsilon_f^{ss}}\right)\frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$ and $\tilde{\omega}$ is auxiliary parameter that can be mapped onto τ via $1+\tau = \left(\frac{1-\tilde{\omega}}{\tilde{\omega}}\frac{N}{N_X}\right)^{\frac{1}{1-\theta}}\frac{\varepsilon_d^{ss}}{\varepsilon_d^{ss}-1}\frac{\varepsilon_f^{ss}-1}{\varepsilon_f^{ss}}$

$$\theta P_{d,t} + d_t = \left(1-\frac{i^{ss}}{y^{ss}}\right)c_t + \frac{i^{ss}}{y^{ss}}i_t \tag{193}$$

$$\theta P^*_{d,t} + d^*_t = \left(1-\frac{i^{ss}}{y^{ss}}\right)c^*_t + \frac{i^{ss}}{y^{ss}}i^*_t \tag{194}$$

$$\theta P_{f,t} + f_t = \left(1-\frac{i^{ss}}{y^{ss}}\right)c_t + \frac{i^{ss}}{y^{ss}}i_t \tag{195}$$

$$\theta P^*_{f,t} + f^*_t = \left(1-\frac{i^{ss}}{y^{ss}}\right)c^*_t + \frac{i^{ss}}{y^{ss}}i^*_t \tag{196}$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1-\mathcal{L}}l_t + c_t \tag{197}$$

$$w^*_t = \frac{\mathcal{L}}{1-\mathcal{L}}l^*_t + c^*_t \tag{198}$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \tag{199}$$

$$u^*_{c,t} = R^*_t + u^*_{c,t|t+1} \tag{200}$$

Optimal investment

$$u_{c,t} + \phi\delta\left(i_t - k_{t-1}\right) = u_{c,t|t+1} + \left[1-\beta(1-\delta)\right]r_{t|t+1} + \beta\phi\delta\left(i_{t|t+1} - k_t\right) \tag{201}$$

$$u^*_{c,t} + \phi\delta\left(i^*_t - k^*_{t-1}\right) = u^*_{c,t|t+1} + \left[1-\beta(1-\delta)\right]r^*_{t|t+1} + \beta\phi\delta\left(i^*_{t|t+1} - k^*_t\right) \tag{202}$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \tag{203}$$

$$k^*_{t-1} - l^*_t = w^*_t - r^*_t \tag{204}$$

Marginal cost

$$v_t = (1-\alpha)w_t + \alpha r_t - z_t \tag{205}$$

$$v^*_t = (1-\alpha)w^*_t + \alpha r^*_t - z^*_t \tag{206}$$

Price setting, where $\varepsilon_d^{ss} = \left[\frac{1}{\theta}\left(1-\frac{\tilde{\omega}}{N}\right) + \frac{1}{\gamma}\frac{\tilde{\omega}}{N}\right]^{-1}$ and $\varepsilon_f = \left[\frac{1}{\theta}\left(1-\frac{1-\tilde{\omega}}{N_X}\right) + \frac{1}{\gamma}\frac{1-\tilde{\omega}}{N_X}\right]^{-1}$

$$P_{d,t} = \frac{1}{1-\varepsilon_d^{ss}}\varepsilon_{d,t} + v_t \tag{207}$$

$$P_{f,t} = \frac{1}{1-\varepsilon_f^{ss}}\varepsilon_{f,t} + v^*_t + x_t \tag{208}$$

$$P^*_{d,t} = \frac{1}{1-\varepsilon_f^{ss}}\varepsilon^*_{d,t} + v_t - x_t \tag{209}$$

$$P^*_{f,t} = \frac{1}{1-\varepsilon_d^{ss}}\varepsilon^*_{f,t} + v^*_t \tag{210}$$

Market shares

$$S_{d,t} = P_{d,t} + d_t - \left(\tilde{\omega}\left[P_{d,t} + d_t\right] + (1-\tilde{\omega})\left[P_{f,t} + f_t\right]\right) \tag{211}$$

$$S_{f,t} = P_{f,t} + f_t - \left(\tilde{\omega}\left[P_{d,t} + d_t\right] + (1-\tilde{\omega})\left[P_{f,t} + f_t\right]\right) \tag{212}$$

$$S^*_{d,t} = P^*_{d,t} + d^*_t - \left((1-\tilde{\omega})\left[P^*_{d,t} + d^*_t\right] + \tilde{\omega}\left[P^*_{f,t} + f^*_t\right]\right) \tag{213}$$

$$S^*_{f,t} = P^*_{f,t} + f^*_t - \left((1-\tilde{\omega})\left[P^*_{d,t} + d^*_t\right] + \tilde{\omega}\left[P^*_{f,t} + f^*_t\right]\right) \tag{214}$$

Demand elasticities, where

$$\varepsilon_{d,t} = \varepsilon_d^{ss}\frac{\tilde{\omega}}{N}\left(\frac{1}{\theta} - \frac{1}{\gamma}\right)S_{d,t} \tag{215}$$

$$\varepsilon^*_{d,t} = \varepsilon_f^{ss}\frac{1-\tilde{\omega}}{N_X}\left(\frac{1}{\theta} - \frac{1}{\gamma}\right)S^*_{d,t} \tag{216}$$

$$\varepsilon_{f,t} = \varepsilon_f^{ss}\frac{1-\tilde{\omega}}{N_X}\left(\frac{1}{\theta} - \frac{1}{\gamma}\right)S_{f,t} \tag{217}$$

$$\varepsilon^*_{f,t} = \varepsilon_d^{ss}\frac{\tilde{\omega}}{N}\left(\frac{1}{\theta} - \frac{1}{\gamma}\right)S^*_{f,t} \tag{218}$$

Resource constraint, where $\bar{\omega} = \left[1 + \frac{1-\tilde{\omega}}{\tilde{\omega}}\frac{\varepsilon_d^{ss}}{\varepsilon_d^{ss}-1}\frac{\varepsilon_f^{ss}-1}{\varepsilon_f^{ss}}\right]^{-1}$

$$\bar{\omega}d_t + (1-\bar{\omega})d^*_t = z_t + \alpha k_{t-1} + (1-\alpha)l_t \tag{219}$$

$$\bar{\omega}f^*_t + (1-\bar{\omega})f_t = z^*_t + \alpha k^*_{t-1} + (1-\alpha)l^*_t \tag{220}$$

Final goods market clearing

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right)c_t + \frac{i^{ss}}{y^{ss}}i_t = \tilde{\omega}d_t + (1-\tilde{\omega})f_t \tag{221}$$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right)c^*_t + \frac{i^{ss}}{y^{ss}}i^*_t = \tilde{\omega}f^*_t + (1-\tilde{\omega})d^*_t \tag{222}$$

UIP condition

$$R^*_t - R_t + x_{t|t+1} - x_t = -\Gamma\left(b_t + n_t\right) \tag{223}$$

Net foreign assets

$$b_t = \frac{1}{\beta}b_{t-1} + (1-\tilde{\omega})\left(d^*_t - f_t + x_t + P^*_{d,t} - P_{f,t}\right) \tag{224}$$

2.6 Deep habits (DH)

Marginal utility

$$u_{c,t} = \left(\nu(1-\sigma) - 1\right)c_t - (1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l_t \tag{225}$$

$$u^*_{c,t} = \left(\nu(1-\sigma) - 1\right)c^*_t - (1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l^*_t \tag{226}$$

Capital accumulation

$$k_t = (1-\delta)k_{t-1} + \delta i_t \tag{227}$$

$$k_t^* = (1-\delta)k_{t-1}^* + \delta i_t^* \tag{228}$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1-\mathcal{L}} l_t + c_t \tag{229}$$

$$w_t^* = \frac{\mathcal{L}}{1-\mathcal{L}} l_t^* + c_t^* \tag{230}$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \tag{231}$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \tag{232}$$

Optimal investment

$$u_{c,t} + \phi\delta\left(i_t - k_{t-1}\right) = u_{c,t|t+1} + \left[1-\beta(1-\delta)\right] r_{t|t+1} + \beta\phi\delta\left(i_{t|t+1} - k_t\right) \tag{233}$$

$$u_{c,t}^* + \phi\delta\left(i_t^* - k_{t-1}^*\right) = u_{c,t|t+1}^* + \left[1-\beta(1-\delta)\right] r_{t|t+1}^* + \beta\phi\delta\left(i_{t|t+1}^* - k_t^*\right) \tag{234}$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \tag{235}$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \tag{236}$$

Marginal cost

$$v_t = (1-\alpha)w_t + \alpha r_t - z_t \tag{237}$$

$$v_t^* = (1-\alpha)w_t^* + \alpha r_t^* - z_t^* \tag{238}$$

Price setting

$$p_{d,t} = \left(1 + \frac{\zeta(1-\rho)\beta}{1-\beta\rho}\right) v_t - \frac{\zeta(1-\rho)\beta}{1-\beta\rho}\Delta_{d,t} \tag{239}$$

$$p_{f,t} - x_t = \left(1 + \frac{\zeta(1-\rho)\beta}{1-\beta\rho}\right) v_t^* - \frac{\zeta(1-\rho)\beta}{1-\beta\rho}\Delta_{f,t} \tag{240}$$

$$p_{d,t}^* + x_t = \left(1 + \frac{\zeta(1-\rho)\beta}{1-\beta\rho}\right) v_t - \frac{\zeta(1-\rho)\beta}{1-\beta\rho}\Delta_{d,t}^* \tag{241}$$

$$p_{f,t}^* = \left(1 + \frac{\zeta(1-\rho)\beta}{1-\beta\rho}\right) v_t^* - \frac{\zeta(1-\rho)\beta}{1-\beta\rho}\Delta_{f,t}^* \tag{242}$$

Composite (habit-adjusted) goods

$$d_t = \tilde{d}_t + \zeta h_{d,t-1} \tag{243}$$

$$f_t = \tilde{f}_t + \zeta h_{f,t-1} \tag{244}$$

$$d_t^* = \tilde{d}_t^* + \zeta h_{d,t-1}^* \tag{245}$$

$$f_t^* = \tilde{f}_t^* + \zeta h_{f,t-1}^* \tag{246}$$

Demand for intermediate goods

$$\tilde{d}_t = -\theta\left(p_{d,t} - P_{d,t}\right) - \zeta(1-\varphi)h_{d,t-1} + d_t \tag{247}$$

$$\tilde{f}_t = -\theta\left(p_{f,t} - P_{f,t}\right) - \zeta(1-\varphi)h_{f,t-1} + f_t \tag{248}$$

$$\tilde{d}^*_t = -\theta\left(p^*_{d,t} - P^*_{d,t}\right) - \zeta(1-\varphi)h^*_{d,t-1} + d^*_t \tag{249}$$

$$\tilde{f}^*_t = -\theta\left(p^*_{f,t} - P^*_{f,t}\right) - \zeta(1-\varphi)h^*_{f,t-1} + f^*_t \tag{250}$$

Habit formation

$$h_{d,t} = \rho h_{d,t-1} + (1-\rho)\,\tilde{d}_t \tag{251}$$

$$h_{f,t} = \rho h_{f,t-1} + (1-\rho)\,\tilde{f}_t \tag{252}$$

$$h^*_{d,t} = \rho h^*_{d,t-1} + (1-\rho)\,\tilde{d}^*_t \tag{253}$$

$$h^*_{f,t} = \rho h^*_{f,t-1} + (1-\rho)\,\tilde{f}^*_t \tag{254}$$

Shadow price of habits

$$\Delta_{d,t} = u_{c,t|t+1} - u_{c,t} + \beta\rho\Delta_{d,t|t+1} + (1-\beta\rho)\left(p_{d,t|t+1} + \tilde{d}_{t|t+1} - h_{d,t}\right) \tag{255}$$

$$\Delta_{f,t} = u^*_{c,t|t+1} - u^*_{c,t} + \beta\rho\Delta_{f,t|t+1} + (1-\beta\rho)\left(p_{f,t|t+1} - x_{t|t+1} + \tilde{f}_{t|t+1} - h_{f,t}\right) \tag{256}$$

$$\Delta^*_{d,t} = u_{c,t|t+1} - u_{c,t} + \beta\rho\Delta^*_{d,t|t+1} + (1-\beta\rho)\left(p^*_{d,t|t+1} + x_{t|t+1} + \tilde{d}^*_{t|t+1} - h^*_{d,t}\right) \tag{257}$$

$$\Delta^*_{f,t} = u^*_{c,t|t+1} - u^*_{c,t} + \beta\rho\Delta^*_{f,t|t+1} + (1-\beta\rho)\left(p^*_{f,t|t+1} + \tilde{f}^*_{t|t+1} - h^*_{f,t}\right) \tag{258}$$

Demand for composite goods, where $\frac{i^{ss}}{y^{ss}} = \frac{\theta-1}{\theta}\left(1 + \frac{\zeta(1-\rho)\beta}{(1-\beta\rho)}\right)\frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$

$$\gamma P_{d,t} + d_t = \left(1 - \frac{i^{ss}}{y^{ss}}\right)c_t + \frac{i^{ss}}{y^{ss}}i_t \tag{259}$$

$$\gamma P^*_{d,t} + d^*_t = \left(1 - \frac{i^{ss}}{y^{ss}}\right)c^*_t + \frac{i^{ss}}{y^{ss}}i^*_t \tag{260}$$

$$\gamma P_{f,t} + f_t = \left(1 - \frac{i^{ss}}{y^{ss}}\right)c_t + \frac{i^{ss}}{y^{ss}}i_t \tag{261}$$

$$\gamma P^*_{f,t} + f^*_t = \left(1 - \frac{i^{ss}}{y^{ss}}\right)c^*_t + \frac{i^{ss}}{y^{ss}}i^*_t \tag{262}$$

Resource constraint, where $\tilde{\omega} = \frac{\left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}}{\left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}+1}$

$$\tilde{\omega}\tilde{d}_t + (1-\tilde{\omega})\tilde{d}^*_t = z_t + \alpha k_{t-1} + (1-\alpha)l_t \tag{263}$$

$$\tilde{\omega}\tilde{f}^*_t + (1-\tilde{\omega})\tilde{f}_t = z^*_t + \alpha k^*_{t-1} + (1-\alpha)l^*_t \tag{264}$$

Final goods market clearing

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right)c_t + \frac{i^{ss}}{y^{ss}}i_t = \frac{1}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}}d_t + \frac{1}{1 + \left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}}f_t \tag{265}$$

$$\left(1-\frac{i^{ss}}{y^{ss}}\right)c_t^* + \frac{i^{ss}}{y^{ss}}i_t^* = \frac{1}{1+\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}}f_t^* + \frac{1}{1+\left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}}d_t^* \tag{266}$$

UIP condition

$$R_t^* - R_t + x_{t|t+1} - x_t = -\Gamma\left(b_t + n_t\right) \tag{267}$$

Net foreign assets

$$b_t = \frac{1}{\beta}b_{t-1} + (1-\tilde{\omega})\left(\tilde{d}_t^* - \tilde{f}_t + x_t + p_{d,t}^* - p_{f,t}\right) \tag{268}$$

2.7 Customer Capital (CC)

Marginal utility

$$u_{c,t} = \left(\nu(1-\sigma)-1\right)c_t - (1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l_t \tag{269}$$

$$u_{c,t}^* = \left(\nu(1-\sigma)-1\right)c_t^* - (1-\nu)(1-\sigma)\frac{\mathcal{L}}{1-\mathcal{L}}l_t^* \tag{270}$$

Capital accumulation

$$k_t = (1-\delta)k_{t-1} + \delta i_t \tag{271}$$

$$k_t^* = (1-\delta)k_{t-1}^* + \delta i_t^* \tag{272}$$

Demand for final goods

$$\gamma P_{d,t} + d_t = \omega d_t + (1-\omega)f_t \tag{273}$$

$$\gamma P_{d,t}^* + d_t^* = \omega f_t^* + (1-\omega)d_t^* \tag{274}$$

$$\gamma P_{f,t} + f_t = \omega d_t + (1-\omega)f_t \tag{275}$$

$$\gamma P_{f,t}^* + f_t^* = \omega f_t^* + (1-\omega)d_t^* \tag{276}$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1-\mathcal{L}}l_t + c_t \tag{277}$$

$$w_t^* = \frac{\mathcal{L}}{1-\mathcal{L}}l_t^* + c_t^* \tag{278}$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \tag{279}$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \tag{280}$$

Optimal investment

$$u_{c,t} + \phi\delta\left(i_t - k_{t-1}\right) = u_{c,t|t+1} + \left[1-\beta(1-\delta)\right]r_{t|t+1} + \beta\phi\delta\left(i_{t|t+1} - k_t\right) \tag{281}$$

$$u_{c,t}^* + \phi\delta\left(i_t^* - k_{t-1}^*\right) = u_{c,t|t+1}^* + \left[1-\beta(1-\delta)\right]r_{t|t+1}^* + \beta\phi\delta\left(i_{t|t+1}^* - k_t^*\right) \tag{282}$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \tag{283}$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \tag{284}$$

Marginal cost

$$v_t = (1-\alpha)w_t + \alpha r_t - z_t \tag{285}$$

$$v_t^* = (1-\alpha)w_t^* + \alpha r_t^* - z_t^* \tag{286}$$

Price setting (optimal bargaining), where $\mathcal{U} = \frac{\chi\eta}{1-\eta}[1-(1-\delta_H)\beta]+1$

$$\mathcal{U}p_{d,t} = (\mathcal{U}-1+\eta)P_{d,t} + (1-\eta)v_t \tag{287}$$

$$\mathcal{U}p_{f,t} = (\mathcal{U}-1+\eta)P_{f,t} + (1-\eta)\left(v_t^* + x_t\right) \tag{288}$$

$$\mathcal{U}p_{d,t}^* = (\mathcal{U}-1+\eta)P_{d,t}^* + (1-\eta)\left(v_t - x_t\right) \tag{289}$$

$$\mathcal{U}p_{f,t}^* = (\mathcal{U}-1+\eta)P_{f,t}^* + (1-\eta)v_t^* \tag{290}$$

Marketing capital

$$m_{d,t} = (1-\delta_m)m_{d,t-1} + \delta_m a_{d,t} \tag{291}$$

$$m_{f,t} = (1-\delta_m)m_{f,t-1} + \delta_m a_{f,t} \tag{292}$$

$$m_{d,t}^* = (1-\delta_m)m_{d,t-1}^* + \delta_m a_{d,t}^* \tag{293}$$

$$m_{f,t}^* = (1-\delta_m)m_{f,t-1}^* + \delta_m a_{f,t}^* \tag{294}$$

Optimal marketing capital conditions

$$\begin{aligned} v_t + \psi\delta_m\left(a_{d,t} - m_{d,t-1}\right) = {}& \beta(1-\delta_m)\left(v_{t|t+1} + u_{c,t|t+1} - u_{c,t}\right) + \beta\psi\delta_m\left(a_{d,t|t+1} - m_{d,t}\right) \\ & + [1-(1-\delta_m)\beta]\left(h_t + W_{d,t} - \omega m_{d,t} - (1-\omega)m_{f,t}\right) \end{aligned} \tag{295}$$

$$\begin{aligned} v_t - x_t + \psi\delta_m\left(a_{f,t} - m_{f,t-1}\right) = {}& \beta(1-\delta_m)\left(v_{t|t+1} - x_{t|t+1} + u_{c,t|t+1}^* - u_{c,t}^*\right) + \beta\psi\delta_m\left(a_{f,t|t+1} - m_{f,t}\right) \\ & + [1-(1-\delta_m)\beta]\left(h_t + W_{f,t} - \omega m_{d,t} - (1-\omega)m_{f,t}\right) \end{aligned} \tag{296}$$

$$\begin{aligned} v_t^* + x_t + \psi\delta_m\left(a_{d,t}^* - m_{d,t-1}^*\right) = {}& \beta(1-\delta_m)\left(v_{t|t+1}^* + x_{t|t+1} + u_{c,t|t+1} - u_{c,t}\right) + \beta\psi\delta_m\left(a_{d,t|t+1}^* - m_{d,t}^*\right) \\ & + [1-(1-\delta_m)\beta]\left(h_t^* + W_{d,t}^* - (1-\omega)m_{d,t}^* - \omega m_{f,t}^*\right) \end{aligned} \tag{297}$$

$$\begin{aligned} v_t^* + \psi\delta_m\left(a_{d,t}^* - m_{d,t-1}^*\right) = {}& \beta(1-\delta_m)\left(v_{t|t+1}^* + u_{c,t|t+1}^* - u_{c,t}^*\right) + \beta\psi\delta_m\left(a_{f,t|t+1}^* - m_{f,t}^*\right) \\ & + [1-(1-\delta_m)\beta]\left(h_t^* + W_{d,t}^* - (1-\omega)m_{d,t}^* - \omega m_{f,t}^*\right) \end{aligned} \tag{298}$$

Customer list

$$d_t = (1-\delta_H)d_{t-1} - \delta_H\left[\omega m_{d,t} + (1-\omega)m_{f,t}\right] + \delta_H\left(m_{d,t} + h_t\right) \tag{299}$$

$$f_t = (1-\delta_H)f_{t-1} - \delta_H\left[\omega m_{d,t} + (1-\omega)m_{f,t}\right] + \delta_H\left(m_{f,t} + h_t\right) \tag{300}$$

$$d_t^* = (1-\delta_H)d_{t-1}^* - \delta_H\left[(1-\omega)m_{d,t}^* + \omega m_{f,t}^*\right] + \delta_H\left(m_{d,t}^* + h_t^*\right) \tag{301}$$

$$f_t^* = (1-\delta_H)f_{t-1}^* - \delta_H\left[(1-\omega)m_{d,t}^* + \omega m_{f,t}^*\right] + \delta_H\left(m_{f,t}^* + h_t^*\right) \tag{302}$$

Optimal customer list conditions

$$W_{d,t} = \frac{1-(1-\delta_H)\beta}{\mathcal{U}-1}\left(\mathcal{U}p_{d,t} - v_t\right) + (1-\delta_H)\beta\left(u_{c,t|t+1} - u_{c,t} + W_{d,t|t+1}\right) \tag{303}$$

$$W_{f,t} = \frac{1-(1-\delta_H)\beta}{\mathcal{U}-1}\left(\mathcal{U}\left[p_{f,t} - x_t\right] - v_t^*\right) + (1-\delta_H)\beta\left(u^*_{c,t|t+1} - u^*_{c,t} + W_{f,t|t+1}\right) \tag{304}$$

$$W^*_{d,t} = \frac{1-(1-\delta_H)\beta}{\mathcal{U}-1}\left(\mathcal{U}\left[p^*_{d,t} + x_t\right] - v_t\right) + (1-\delta_H)\beta\left(u_{c,t|t+1} - u_{c,t} + W^*_{d,t|t+1}\right) \tag{305}$$

$$W^*_{f,t} = \frac{1-(1-\delta_H)\beta}{\mathcal{U}-1}\left(\mathcal{U}p^*_{f,t} - v^*_t\right) + (1-\delta_H)\beta\left(u^*_{c,t|t+1} - u^*_{c,t} + W^*_{f,t|t+1}\right) \tag{306}$$

Retailer value

$$J_{d,t} = \frac{1}{\chi}\left(\frac{\mathcal{U}-1+\eta}{\eta}P_{d,t} - \mathcal{U}p_{d,t}\right) + (1-\delta_H)\beta\left(u_{c,t|t+1} - u_{c,t} + J_{d,t|t+1}\right) \tag{307}$$

$$J_{f,t} = \frac{1}{\chi}\left(\frac{\mathcal{U}-1+\eta}{\eta}P_{f,t} - \mathcal{U}p_{f,t}\right) + (1-\delta_H)\beta\left(u_{c,t|t+1} - u_{c,t} + J_{f,t|t+1}\right) \tag{308}$$

$$J^*_{d,t} = \frac{1}{\chi}\left(\frac{\mathcal{U}-1+\eta}{\eta}P^*_{d,t} - \mathcal{U}p^*_{d,t}\right) + (1-\delta_H)\beta\left(u^*_{c,t|t+1} - u^*_{c,t} + J^*_{d,t|t+1}\right) \tag{309}$$

$$J^*_{f,t} = \frac{1}{\chi}\left(\frac{\mathcal{U}-1+\eta}{\eta}P^*_{f,t} - \mathcal{U}p^*_{f,t}\right) + (1-\delta_H)\beta\left(u^*_{c,t|t+1} - u^*_{c,t} + J^*_{f,t|t+1}\right) \tag{310}$$

Free entry by retailers

$$\omega J_{d,t} + (1-\omega)J_{f,t} = v_t \tag{311}$$

$$(1-\omega)J^*_{d,t} + \omega J^*_{f,t} = v^*_t \tag{312}$$

Resource constraint, where $\tilde{\mathcal{W}} = \frac{\delta_m \delta_H W^{ss}}{1-(1-\delta_m)\beta}\frac{\mathcal{U}-1+\eta}{\eta}$

$$\omega d_t + (1-\omega)d^*_t + \tilde{\mathcal{W}}\left(\omega a_{d,t} + (1-\omega)a_{f,t}\right) + \chi\delta_H h_t = \left(1 + \tilde{\mathcal{W}} + \chi\delta_H\right)\left(z_t + \alpha k_{t-1} + (1-\alpha)l_t\right) \tag{313}$$

$$(1-\omega)f_t + \omega f^*_t + \tilde{\mathcal{W}}\left(\omega a^*_{f,t} + (1-\omega)a^*_{d,t}\right) + \chi\delta_H h^*_t = \left(1 + \tilde{\mathcal{W}} + \chi\delta_H\right)\left(z^*_t + \alpha k^*_{t-1} + (1-\alpha)l^*_t\right) \tag{314}$$

Final goods market clearing, where $\frac{i^{ss}}{y^{ss}} = \left(1 + \tilde{\mathcal{W}} + \chi\delta_H\right)\frac{\eta}{\mathcal{U}-1+\eta}\frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right)c_t + \frac{i^{ss}}{y^{ss}}i_t = \omega d_t + (1-\omega)f_t \tag{315}$$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right)c^*_t + \frac{i^{ss}}{y^{ss}}i^*_t = \omega f^*_t + (1-\omega)d^*_t \tag{316}$$

UIP condition

$$R^*_t - R_t + x_{t|t+1} - x_t = -\Gamma\left(b_t + n_t\right) \tag{317}$$

Net foreign assets, where $\mathcal{M} = \eta \frac{\mathcal{U}-1}{\mathcal{U}-1+\theta} \frac{\delta_H}{1-(1-\delta_H)\beta} \frac{\delta_m}{1-(1-\delta_m)\beta}$

$$\begin{aligned} b_t = {} & \frac{1}{\beta} b_{t-1} + (1-\omega) \frac{\eta \mathcal{U}}{\mathcal{U}-1+\eta} \left(d_t^* - f_t + x_t + p_{d,t}^* - p_{f,t} \right) \\ & + (1-\omega) \mathcal{M} \left(v_t + a_{f,t} - v_t^* - x_t - a_{d,t}^* \right) \end{aligned} \tag{318}$$

3 Data Sources and Replication

Data sources. Data for the model section come from the replication package of Drozd and Nosal (2012), available online from the *American Economic Review*. Most statistics are taken from that paper, and we use their estimate of the TFP process. Original data sources are: OECD Main Economic Indicators (`https://www.oecd.org/en/publications/main-economic-indicators_16097319.html`), Bureau of Labor Statistics (import and export price indices, `https://www.bls.gov/mxp/`), International Monetary Fund Direction of Trade Statistics (`https://data.imf.org/en/datasets/IMF.STA:IMTS`, currently IMTS, previously DOTS), and Bureau of Economic Analysis (BEA) National Income and Product Accounts and input–output tables (`https://www.bea.gov/industry/input-output-accounts-data`).

In addition, as cited in the paper, we use the S&P Compustat North America (Fundamentals Annual) dataset provided by Wharton Research Data Services (WRDS), and Bureau of Economic Analysis input–output tables at the detail level (BEA 402 Industry I–O Use Tables, benchmark years 2007, 2012, and 2017). The NAICS crosswalk files are in the same folder and were downloaded from the BEA website in January 2025 (`2017_to_2022_NAICS.xlsx`, `2022_to_2017_NAICS.xlsx`).

Replication of Table 1 (markups). Compustat: Run the Jupyter notebook `compustat_markups.ipynb` in the replication package (Data folder). Due to proprietary data restrictions, the folder does not include the S&P 500 Compustat file. The code contains an automated download from WRDS. Prior to the download, WRDS credentials need to be entered when prompted. I–O tables: See Excel file `2017_402_sectoral_markups.xlsx` (sheet `Table_1`, Data folder). The file is linked, and the source data come from the 2017 SUT tables downloaded from the BEA website in January 2025 (`Use_SUT_Framework_2017.xlsx`).

Replication of Tables 4– (model results). Run the Dynare codes from the replication package. Instructions are in `README.txt`.