Long-run Trade Elasticity and the Trade-Comovement Puzzle

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ONLINE APPENDIX: NOT FOR PUBLICATION

A Relation of $\mathcal{S}$ to correlation coefficient

To a first order approximation, we have shown in Section A.1 that our model implies the following dynamic system

$$\begin{aligned} \hat{y} &= (1-\mathcal{S})\hat{A} + \mathcal{S}\hat{A}^* \\ \hat{y}^* &= (1-\mathcal{S})\hat{A}^* + \mathcal{S}\hat{A} \end{aligned}$$

where

$$0 < \mathcal{S} < 1/2.$$

That is, note that the coefficients in (48) add up to one, and that $\mathcal{S}$ corresponds to the one on $\hat{A}^*$ (to a first order approximation). For simplicity, normalize the variance of symmetric shock,

$$var(\hat{A}^*) = var(\hat{A}) = 1,$$

and note that

$$0 \leq cov(\hat{A}, \hat{A}^*) = corr(\hat{A}, \hat{A}^*) \leq 1.$$

Define

$$corr(y, \hat{y}) = \frac{cov\,(y, \hat{y}^*)}{var\,(\hat{y})},$$

and derive

$$\begin{aligned} cov\,(\hat{y}, \hat{y}^*) &= cov((1-\mathcal{S})\hat{A} + \mathcal{S}\hat{A}^*, (1-\mathcal{S})\hat{A}^* + \mathcal{S}\hat{A}) = \\ &= \left((1-\mathcal{S})^2 \ + \mathcal{S}^2\right) cov(\hat{A}, \hat{A}^*) \ + 2(1-\mathcal{S})\mathcal{S}, \end{aligned}$$

and

$$var\,(\hat{y}^*) = var\,(\hat{y}) = var((1-\mathcal{S})\hat{A} + \mathcal{S}\hat{A}^*) = ((1-\mathcal{S})^2 + \mathcal{S}^2 + 2(1-\mathcal{S})\mathcal{S}corr(\hat{A}, \hat{A}^*))$$

to obtain

$$corr\,(\hat{y}, \hat{y}^*) = \frac{\left((1-\mathcal{S})^2 \ + \mathcal{S}^2\right) corr(\hat{A}, \hat{A}^*) \ + 2(1-\mathcal{S})\mathcal{S}}{(1-\mathcal{S})^2 + \mathcal{S}^2 + 2(1-\mathcal{S})\mathcal{S}corr(\hat{A}, \hat{A}^*)}.$$

Observe that the above expression is strictly decreasing in $\mathcal{S}$ given Assumptions 1 and 2, since

$$\frac{\partial corr\left(\hat{y},\hat{y}^*\right)}{\partial \mathcal{S}} = -\frac{2(1-corr(\hat{A},\hat{A}^*)^2)(2\mathcal{S}-1)}{(1-2(1-corr(\hat{A},\hat{A}^*))(1-\mathcal{S})\mathcal{S})^2} < 0,$$

and hence that the correlation coefficient is strictly increasing in $\mathcal{S}$. For detailed derivations of the above expressions refer to our Mathematica notebook available online.

B Alternative definition of trade in Section 2

We omit all algebraic deriviation, which can be found in the Mathematical notebook posted online. The polar case of all trade costs being explicit; that is,

$$x = \frac{f}{d+f(1+\tau)}.$$

In this case gravity applies globally and there is no restriction on ρ, although we will establish our main result by restricting $\rho > 1$.

It is not possible to explicitly solve for $\tau(\bar{x})$ and hence we use implicit differentiation to calculate

$$\frac{d\tau(\bar{x})}{d\bar{x}} = \frac{\tau+1}{\bar{x}\left((\rho-1)(\tau+1)\bar{x}-\rho\right)}.$$

Note that the inverse of this expression is

$$\frac{d\bar{x}}{d\tau} = \frac{\bar{x}\left((\rho-1)(\tau+1)\bar{x}-\rho\right)}{\tau+1},$$

which implies that for all values of ρ we have a negative relation between trade cost and trade, and hence the restriction introduce in the benchmark model no longer applies. The comovement coefficient S in this case depends on both τ and $\bar{x}$, and so to obtain total effect of trade changes in trade cost must be taken explicitely into account:

$$S(\bar{x},\tau) = (\tau+1)\bar{x}\left(\frac{2\rho\phi\left(2(\tau+1)\bar{x}-1\right)+(\tau+1)\bar{x}\left(\tau\bar{x}+\bar{x}-1\right)\left(2(\rho-1)(\tau+1)\bar{x}-2\rho+1\right)}{(\tau+1)\bar{x}\left(\tau\bar{x}+\bar{x}-1\right)-2\rho\phi}+1\right).$$

Accordingly, we define trade-comovement link $\mathcal{L}$ as

$$\mathcal{L} = \frac{\partial S(\tau,\bar{x})}{\partial \bar{x}} + \frac{\partial S(\tau,\bar{x})}{\partial \tau}\frac{d\tau}{d\bar{x}}.$$

As is clear from the formula for S, analyzing this model is more challenging. To make progress, show that $\mathcal{L}$ is always negative for $\phi = 0$, establishing trade-comovement puzzle, and then show that in the limit $\phi \to \infty$ it is strictly positive. Given continuity of this expression, for sufficiently high ϕ, the relationship between trade and comovement must be positive. It is easy to verify by plotting the function that it occurs for actually low values of ϕ. The figure plots $\mathcal{L}$ for all values of $\bar{x}$ and all values of ϕ (for $\tau = .38$ and $\rho = 10$). We now prove this result formally.

Proposition 3 *For $\phi = 0$ we have $\mathcal{L} < 0$, implying trade-comovement puzzle.*

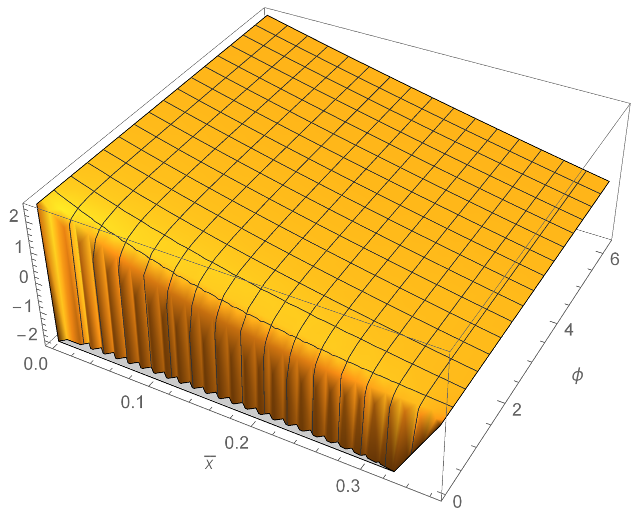

Figure 5: $\mathcal{L}$ as a function of $\bar{x}$ and ϕ, for $\tau = .38$ and $\rho = 10$.

Proof.

Note that after plugging in $\phi = 0$ we have:

$$\mathcal{L} = 2(\rho-1)(\tau+1)\left(2(\tau+1)\bar{x}-1\right)\left(\frac{1}{(\rho-1)(\tau+1)\bar{x}-\rho}+1\right).$$

The sign of this expression is determined by

$$\begin{aligned} sign(\mathcal{L}) &= sign(-2(\rho-1)\left(\frac{1}{(\rho-1)(\tau+1)\bar{x}-\rho}+1\right)) \\ &= sign(\frac{2(\rho-1)^2\left(1-(1+\tau)\bar{x}\right)}{(\rho-1)(\tau+1)\bar{x}-\rho}) \\ &= sign((\rho-1)(\tau+1)\bar{x}-\rho) \\ &= sign(\rho((\tau+1)\bar{x}-1)-(\tau+1)\bar{x}). \end{aligned}$$

which is always negative. ■

Proposition 4 *There exists sufficiently large $\bar{\phi}$ so that for all $\phi > \bar{\phi}$ we have $\mathcal{L} > 0$ for any $\rho > 1$ (as a sufficient condition).*

Proof. Having shown that $\mathcal{L} < 0$ for $\phi = 0$, given continuity of $\mathcal{L}$ with respect to ϕ, it suffices to show that the limit as $\phi \to \infty$ is strictly positive. We calculate the sign of the limit to obtain the following evaluation

$$\begin{aligned} sign\left(\lim_{\phi\to\infty}\mathcal{L}\right) &= sign\left((\tau+1)\left(2-4(\tau+1)\bar{x}\right)\left(\frac{1}{(\rho-1)(\tau+1)\bar{x}-\rho}+1\right)\right) \\ &= sign\left(1+\frac{1}{(\rho-1)(\tau+1)\bar{x}-\rho}\right) \end{aligned}$$

which is always positive for $\rho > 1$ (sufficient condition, not necessary). This is clearer by rewriting

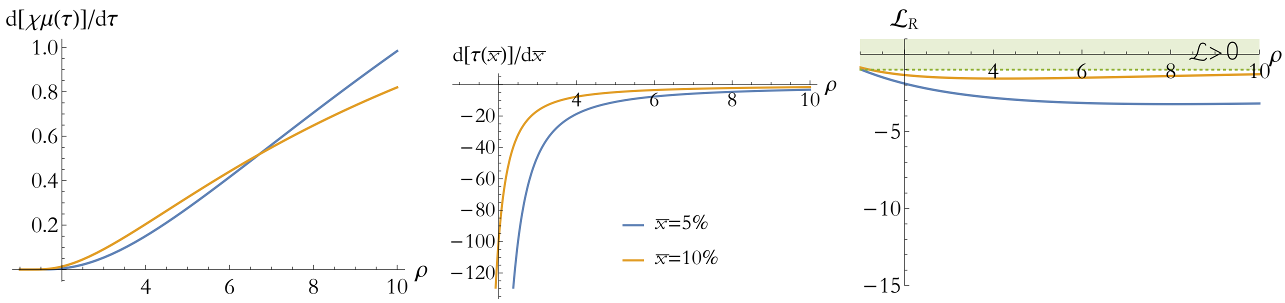

Figure 6: $\mathcal{L}_R = \frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}\frac{d\tau}{dx}|_{\tau(\bar{x})}$ as function of elasticity $\rho \geq 1$ for CRRA utility function in (36) ($\sigma = 2$, $\zeta = 1/3$.)

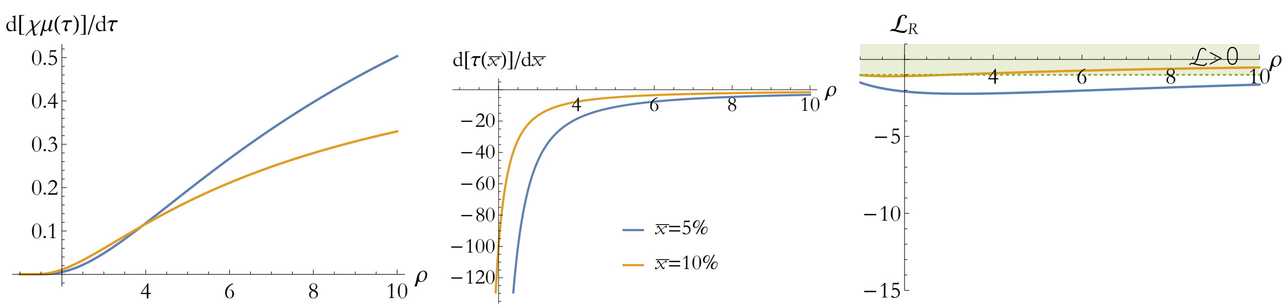

Figure 7: $\mathcal{L}_R = \frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}\frac{d\tau}{dx}|_{\tau(\bar{x})}$ as function of elasticity $\rho \geq 1$ for CRRA utility function in (36) ($\sigma = 5$, $\zeta = 1/3$.)

the last term as

$$sign\left(1 - \frac{1}{\rho(1-(\tau+1)\bar{x}) + (\tau+1)\bar{x}}\right) > 0.$$

(It is possible to show that $\mathcal{L} > 0$ is increasing in ϕ for $\rho > 1$ (sufficient condition), and hence that $\mathcal{L}$ crosses zero only once, but we omit it.)

■

C Non-separable utility function in Section 2

Here we characterize the forces behind the trade-comovement puzzle under the assumption that the utility function is as in (36) (with $\sigma > 1$) and $\Phi = 0$. We show that, qualitatively, the results of Section 2 stands, although we note that this utility function alleviates the puzzle for unreasonably high setting of bilateral trade intensity. We derive the formulas and compare the results to those in the paper. We also prove Proposition 3 in the paper.

We begin by redoing Figure 1 from the paper for the utility function in (36) for $\sigma = 2$ and $\sigma = 5$ (and $\psi = 1/3$). $\sigma = 2$ case implies negative trade comovement link due to the effect of risk-sharing and $\sigma = 5$ implies negative trade-comovement link for 5% trade intensity and essentially zero link for 10%. Overall, non-separable utility alleviates the puzzle. Higher value of risk aversion also alleviates the puzzle, but the improvement is small for a plausible range of values.

The decomposition considered in the paper for nonseperable utility function implies the following values of the coefficients:

$$\alpha = 1 \qquad \eta = -(1-\zeta)\bar{x} \tag{52}$$

$$\pi = \frac{1}{2(\bar{x}(\rho+\zeta-1)-\rho)} \qquad \theta = \frac{1-2\zeta\bar{x}}{2\zeta\bar{x}(\bar{x}(\rho+\zeta-1)-\rho)} \tag{53}$$

$$\mu = \frac{\psi\bar{x}(-2\rho((\sigma-1)\zeta+1)+2\bar{x}(\rho(\sigma-1)\zeta+\rho+(\sigma-1)(\psi-1)\zeta-1)+1)}{4\zeta\bar{x}(-\rho\sigma+(\rho-1)\sigma\bar{x}+(\sigma-1)\zeta\bar{x}+1)-1} \tag{54}$$

It can be verified that:

$$\begin{aligned}\mathcal{L}_R \;=\; & ((\psi-1)(-4\zeta\bar{x}^2(2(\rho-1)(\sigma-1)\zeta^2-(4\rho-3)(\sigma-1)\zeta+(\rho-1)(\sigma-2)) - \\ & 4\bar{x}(\rho(\sigma-1)\zeta+\rho+(\sigma-1)(\zeta-1)\zeta-1)+2\rho((\sigma-1)\zeta+1)-1))/ \\ & (4\zeta\bar{x}\,(\bar{x}(\zeta-\sigma(\rho+\zeta-1))+\rho\sigma-1)+1)^2\end{aligned}$$

and

$$\begin{aligned}\mathcal{L}_C \;=\; & (\zeta(4(\zeta-1)\bar{x}(\bar{x}((\sigma-1)\zeta^2((2\rho-1)\sigma-1) + \\ & \sigma\zeta(\sigma-3\rho(\sigma-1))-\rho\sigma+\sigma-\zeta)+(\sigma-1)(\zeta-1))+\sigma(-\zeta)+\sigma+\zeta-2)+1)/ \\ & (4\zeta\bar{x}\,(\bar{x}(\zeta-\sigma(\rho+\zeta-1))+\rho\sigma-1)+1)^2\end{aligned}$$

Taking the limit $\bar{x} \to 0$, we obtain (27). This implies that the trade-comovement puzzle arises for low levels of trade for all parameter values.

Figure **??** plots all the coefficient of the decomposition and compares them side-by-side between the two specifications of the utility function. It is clear that, qualitatively, the relations are identical. The only difference is their relative magnitude, in particular much weaker risk-sharing channel's connection to trade (slope of $\chi\mu$ in the lower- left-panel). But, this turns out to be a double-edged sword as far as trade-comovement puzzle goes. On the one hand, it reduces the direct effect of risk sharing, but it also reduces its adverse effect on the complementarity channel through its indirect effect, making it weaker through $\eta\theta\mu$ (slope of lower- right- panel). Overall, nonseperability alleviates the puzzle but does not resolve it, especially for empirically relevant ranges of bilateral trade intensity (.e.g., 10% or lower). Figure 8 shows the negative region of $\mathcal{L}$ for $\sigma = 2$ and $\sigma = 5$.

(Derivations of the above expressions can be find in the Mathematica notebook available online.)

D Deriviation of (38)-(46)

The Lagrangian of the domestic country household is

$$\begin{aligned} L \;=\; & \sum_{s^t} \Pr(s^t)\beta^t[\log G(d(s^t), f(s^t)) - \Phi(d(s^t), f(s^t)) - l(s^t) \\ & - \lambda(s^t)((d(s^t)+p(s^t)f(s^t)(1+\tau)+Q(s^{t+1})B(s^{t+1})-w(s^t)l(s^t)-B(s^t))],\end{aligned} \tag{55}$$

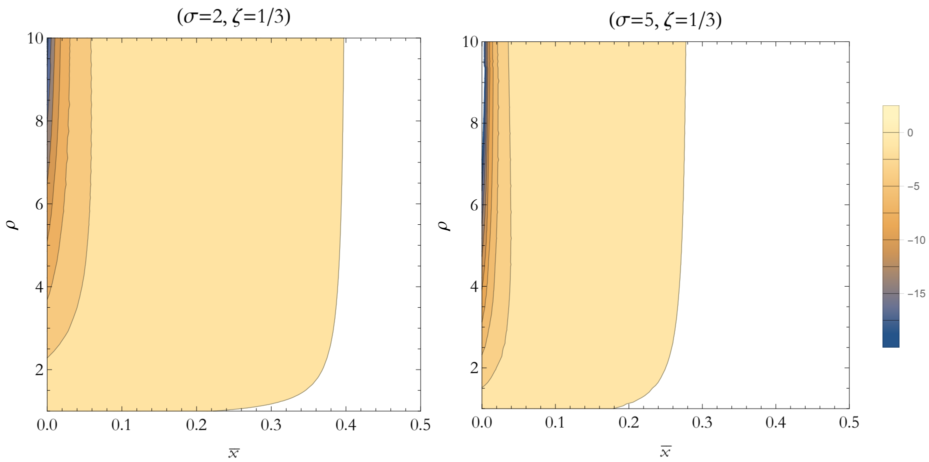

Figure 8: The negative region of trade-comovement link $\mathcal{L}$ for nonseperable CRRA utility function, $\sigma = 2, \zeta = 1/3$ (left panel) and $\sigma = 5, \zeta = 1/3$ (right panel).

and the Lagrangian of the foreign country household is

$$\begin{aligned} L^* &= \sum_{s^t} \Pr(s^t)\beta^t[\log G(f^*(s^t), d^*(s^t)) - \Phi(f^*(s^t), d^*(s^t)) - l^*(s^t) \\ &\quad - \lambda^*(s^t)((p(s^t)f^*(s^t) + d^*(s^t)(1+\tau) + Q(s^{t+1})B^*(s^{t+1}) - p(s^t)w^*(s^t)l^*(s^t) - B^*(s^t))]. \end{aligned} \tag{56}$$

The equilibrium must satisfy the first order conditions of the domestic country household Lagrangian with respect to $d(s^t), f(s^t), l(s^t), B(s^{t+1}), \lambda(s^t)$, an analogous set of conditions for the foreign country, zero profit conditions,

$$w(s^t) = A(s^t) \tag{57}$$

$$w^*(s^t) = A^*(s^t), \tag{58}$$

and feasibility conditions

$$d(s^t) + d^*(s^t)(1+\tau) = y(s^t) = A(s^t)l(s^t) \tag{59}$$

$$f^*(s^t) + f(s^t)(1+\tau) = y^*(s^t) = A^*(s^t)l^*(s^t). \tag{60}$$

We use the first order for l and l^* to drop Lagrange multipliers, which, using (57) and (58), are

$$\begin{aligned} \lambda(s^t) &= A(s^t)^{-1} \\ \lambda^*(s^t) &= (A^*(s^t)p(s^t))^{-1}, \end{aligned}$$

and note that the remaining first order conditions with respect to d,f,d^*,f^* imply

$$\frac{\partial[\log G(d(s^t),f(s^t)) - \Phi(d(s^t),f(s^t)) - l(s^t)]}{\partial d(s^t)} = A(s^t)^{-1} \tag{61}$$

$$\frac{\partial[\log G(d(s^t),f(s^t)) - \Phi(d(s^t),f(s^t)) - l(s^t)]}{\partial f(s^t)} = p(s^t)(1+\tau)A(s^t)^{-1} \tag{62}$$

$$\frac{\partial[\log G(f^*(s^t),d^*(s^t)) - \Phi(f^*(s^t),d^*(s^t)) - l^*(s^t)]}{\partial f^*(s^t)} = A^*(s^t)^{-1} \tag{63}$$

$$\frac{\partial[\log G(f^*(s^t),d^*(s^t)) - \Phi(f^*(s^t),d^*(s^t)) - l^*(s^t)]}{\partial d^*(s^t)} = (p\left(s^t\right)A^*(s^t))^{-1}(1+\tau), \tag{64}$$

and the first order conditions with respect to B_{+1}, B^*_{+1} imply

$$\lambda(s^t) = \lambda^*(s^t). \tag{65}$$

Combining with the formulas for shadow values, we obtain

$$\frac{\partial[\log G(d(s^t),f(s^t)) - \Phi(.,.)] - l^*(s^t}{\partial d(s^t)}(1+\tau) = \frac{\partial[\log G(f^*(s^t),d^*(s^t)) - \Phi(.,.) - l^*(s^t)]}{\partial d^*(s^t)}. \tag{66}$$

E Business cycle implications of the quantitative model

To verify that our model accounts for the trade-comovement relationship without sacrificing the performance in other respects, we report a set of business cycle statistics generated from our model. The results, presented in Table 9, report median business cycle statistics from our simulated model, as well as medians in our dataset. As the inspection of the table shows, the model matches the statistics fairly well, at least as well as the frictionless model and often better. One notable improvement is the prediction that output is more correlated internationally than consumption, addressing the so called 'quantity anomaly'.[36]

F Volatility ratio across countries in our sample

Table 10 presents estimates of the volatility ratio in our sample.

G Mapping of national accounts onto quantitative model

GDP in constant prices (steady state prices) in corresponds in our quantitative model to

$$L_D(P_d d + P_f f + P_g g) + \sum_{i=F,W} x_i p_i^d L_i d_i - p_D^f L_D f - p_D^g L_D g + v_D(a_D^f + a_D^g) - x_F v_F a_F^d - x_W v_W a_W^d,$$

[36]Identified in Backus, Kehoe and Kydland (1992).

Table 9: business cycle Statistics: Data and Models[a]

Statistic	Data Median[b]	Benchmark Median	Frictionless Median
A. Correlation			
domestic with foreign			
TFP (measured)	0.44	0.54	0.52
GDP	0.52	0.53	0.52
Consumption	0.41	0.45	0.57
Employment	0.42	0.46	0.54
Investment	0.50	0.38	0.45
GDP with			
Consumption	0.71	0.92	0.93
Employment	0.60	0.81	0.99
Investment	0.71	0.98	0.98
Net exports	-0.20	-0.63	-0.69
Terms of trade with			
Net exports	-0.31	-0.89	-0.54
B. Volatility relative to GDP			
Consumption	0.79	0.28	0.26
Investment	3.04	3.90	3.66
Employment	0.71	0.83	0.52
Net exports	0.59	0.20	0.14

[a]Statistics based on logged and Hodrick-Prescott filtered time series with a smoothing parameter $\lambda = 1600$.
[b]Unless otherwise noted, data column refers to the median in our sample of countries for the period 1980Q1-2011Q4.

consumption and investment in constant prices corresponds to[37]

$$L_D(P_{d,t}d_t + P_{f,t}f + P_{g,t}g_t)\frac{c_t}{G(d_t, f_t, g_t)}, \tag{67}$$

$$L_D(P_{d,t}d_t + P_{f,t}f + P_{g,t}g_t)\frac{i_t}{G(d_t, f_t, g_t)}, \tag{68}$$

and employment index corresponds to $l_{i,t}$. Notice that investment in marketing does not enter the expenditure side measurement of GDP. This assumption is consistent with the methodology of national income accounting, in which expenses on R&D, marketing, advertising are all treated as intermediate inputs − see SNA (1993) Par. 1.49, 6.149, 6.163, 6.165. While *R&D* expenses have been capitalized in the U.S., this is the prevalent convention across countries in our sample.

[37]Consumption and investment in period zero prices are not equal to c and i. The reason is that the Euler's Law does not apply for period zero (steady state) prices. However, quantitatively the difference is essentially zero.

Table 10: Volatility ratio in a cross-section of major industrialized countries

	Detrending method	
Country	Hodrick-Prescott filter (1600)	Linearly detrended
Australia	0.88	0.78
Austria	2.76	2.31
Belgium	1.21	1.27
Canada	1.27	1.24
Denmark	1.17	1.52
Finland	1.67	1.31
France	0.77	0.86
Germany	1.38	1.36
Italy	1.07	1.12
Japan	0.68	0.63
Korea	0.59	0.65
the Netherlands	0.99	0.77
Norway	1.18	1.21
Portugal	1.07	1.04
Spain	1.89	1.21
Sweden	1.59	2.14
Switzerland	1.05	0.87
United Kingdom	0.90	0.67
United States	1.20	0.88
Median	1.17	1.12

H Data sources

Bilateral trade statistics were taken from International Monetary Fund, Direction Of Trade Statistics, 2005. From Source OECD, Quarterly National Accounts: Gross Fixed Capital Formation ("P51: Gross fixed capital formation", "VOBARSA: Millions of national currency, volume estimates, OECD reference year, annual levels, seasonally adjusted"), GDP in constant prices ("B1_GE: Gross domestic product - expenditure approach", "VOBARSA: Millions of national currency, volume estimates, OECD reference year, annual levels, seasonally adjusted"). Our measure of labor is civilian employment or employment from Quarterly National Accounts or the International Labor Organization (based on data availability). GDP is available from 1980Q1 to 2011Q4 for all countries in our sample. Employment data is missing for some countries for some years (see Online Appendix for more details what data we used). Since labor data is often not seasonally adjusted, we apply the X-12-ARIMA Seasonal Adjustment Program from census.gov.

Nominal GDP series come from World Development Indicators, World Bank. Gross Fixed Capital Formation, GDP in constant prices and Civil Employment series come Source OECD.org, Quarterly National Accounts. Series for physical capital have been constructed using the perpetual inventory method with a constant depreciation of 2.5%. Aggregate GDP for blocks of countries has been computed from growth rates of GDP in constant prices (recent years, varies by country) weighted by the nominal GDP of each country in 2004 (we applied the growth rates backwards).