"The stability of the proportion of the national dividend accruing to labour is... a bit of a miracle."

— J.M. Keynes (1939)

Understanding Growth Through Automation: The Neoclassical Perspective

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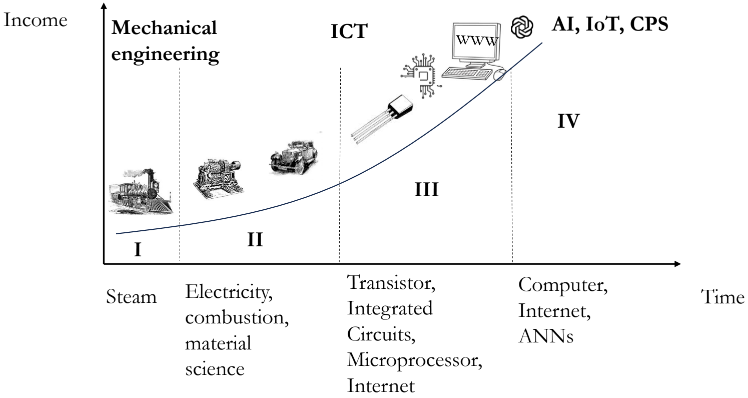
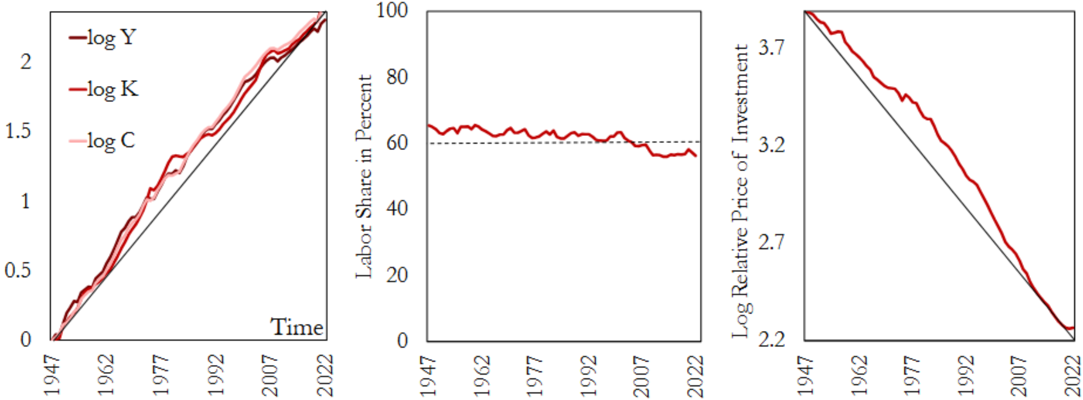
Capital as Engine of Growth



Main Innovation Pathway: **Mechanical/Electric Power** $\Longrightarrow$ **Interface** $\Longrightarrow$ **Capital Deployment in Tasks**

Economic Manifestation: **Capital Productivity** $\uparrow$ $\Longrightarrow$ **Automation** & **Existing Task Augmentation/Deepening**

Stylized Facts on (Frontier) Industrial Growth



(a) Balanced Growth (b) Stable Labor Share (c) Capital-Augmenting Change

Neoclassical Benchmark: Success and Deficiencies

$$\max \int_0^\infty e^{-\rho t} u(c_t) dt \quad \text{s.t.} \quad \dot{K}_t = (Z_t K_t)^{1-\alpha} L_t^\alpha - C_t, \quad Z_t = e^{gt}$$

Knife-Edge Foundation

- Unit elasticity $(\sigma = 1)$ *assumed*

(Stability remains a ”miracle”)

Agnostic on Mechanism

- Reduced-form production
- Capital as generic aggregate
- Exogenous ”manna” growth

This Paper

Task- and Power Law-based microfoundation of neoclassical growth theory.

Highlights:

- Stability as an Emergent Property: Derives aggregate Cobb–Douglas dynamics and stable labor shares from task-level competition.
- Full Industrial Arc: Endogenizes the transition from pre-industrial stagnation to takeoff and modern growth with automation.
- Innovation as Random Growth Process: Explicit probability model of capital-focused R&D and its interaction with economic incentives.
- Micro-level Predictions: Endogenous task displacement and a theoretical link to granular, occupation-level automation exposure.

Building Blocks and Mechanism

Endogenous extensive and intensive margins of capital-oriented innovations:

- Automation: Technology improves to automate new tasks, displacing labor
  $\Rightarrow$ labor share down
- Augmentation/Deepening: Technology improves within already-automated tasks, making the remaining labor more essential
  $\Rightarrow$ labor share down

Random-based innovation process on task space & vast task space:

- Power Law (PL)-induced and balance through coexistence of forces.

Related Literature

Task-Based Growth & Automation

- **Framework:** Zeira (1998), Acemoglu-Restrepo (2018)
- **AI & Growth:** Aghion et al. (2019), Jones-Liu (2024)
- **Baumol Forces:** Baumol (1967), Ngai-Pissarides (2007)

CD-Pareto and PL in Economics

- **CD-Pareto:** Houthakker (1955), Jones (2005), Lagos (2007)
- **PL:** Luttmer (2007), Gabaix (2009)

Differentiation:

1. Emergent Share Stability from PL

No knife edge required on economic forces.

2. BGP with Capital-Augmenting Progress

Task-level rebalancing (not sectoral).

3. Robust to directed R&D

Directed R&D = random bundled task = mixing on task space.

Model

Task-based production, innovation, and aggregation

Task-Based Production

Output requires completing a continuum of tasks $i \in [0,1]$.

$$\left(\int_0^1 a(i)^{\frac{\gamma-1}{\gamma}}\, di\right)^{\frac{\gamma}{\gamma-1}} \geq A$$

- Key assumptions:
  - → Tasks are complementary: production is constrained by bottlenecks ($0 \leq \gamma < 1$).
  - → Labor and capital are perfect substitutes within tasks: Defining property of a "task."
  - → Technology summarized by capital requirement $q(i)$. Labor requirement is 1.

Completing task i at intensity $a(i)$ requires a(i) units of labor or q(i)a(i) units of capital.

Innovation at the Task Level

Each task $i \in [0, 1]$ is defined by its unit capital requirement $q_t(i)$.

Assumption 2: Task-Level Innovation

Capital requirements follow a Geometric Brownian Motion with a reflecting barrier:

$$d \log q_t = -g\, dt + \sigma\, dW_t, \qquad q_t \geq q_0(t) = e^{-\phi g t}$$

Key parameters

- **Drift ($g > 0$):** Average pace of automation.
- **Diffusion (σ):** Idiosyncratic uncertainty.
- **Frontier (q_0):** Technology frontier growth.

Task Turnover and Rebirth

Tasks become obsolete at Poisson rate χ and are reborn at the frontier:

$$q^{\text{new}}(t) = q_0(t).$$

Intuition:

- most tasks drift toward the frontier over time;
- some tasks receive unfavorable shocks and remain far from it;
- turnover prevents the distribution from collapsing.

Model of extreme innovations such as AI/Software

Pareto Distribution

Lemma 1

Normalized task complexity ($x_t \equiv q_t e^{\phi g t}$) has a stationary Pareto distribution:

$$p(x) = \zeta x^{-\zeta-1}, \qquad x \geq 1.$$

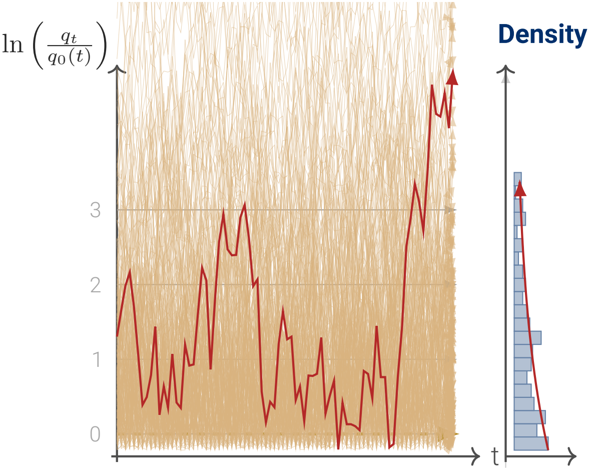
where

$$\zeta = \frac{g(1-\phi) + \sqrt{g^2(1-\phi)^2 + 2\sigma^2\chi}}{\sigma^2} \qquad \left(\chi \to 0 : \; \zeta = \frac{2g(1-\phi)}{\sigma^2}\right)$$

Most tasks lie near the frontier; thick tail of hard-to-automate tasks remains indefinitely.

Example

- Simulation of 1000 tasks
- Density after T=1000
- Detrended $\ln(q/q_0)$



Optimal Task Allocation

Let w, r be given (shadow) prices of each factor

- firm cost minimization or, equivalently
- planning problem of allocating fixed inputs K, L

Then, given any increasing $q(i)$-optimal task allocation implies

- perform with capital (automate) iff

$$r \cdot q(i)a(i) < w \cdot a(i).$$

Optimal Task Allocation

Lemma 2 (Optimal task allocation)

Cost minimization implies a cutoff rule

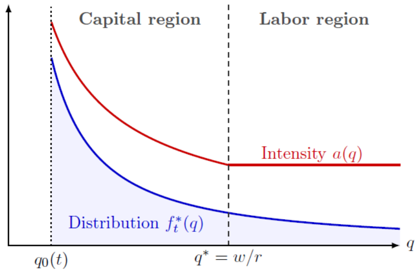
$$q^* = \frac{w}{r}.$$

The optimal task intensity satisfies

$$a(q) = \begin{cases} a(q^*)\left(\dfrac{q}{q^*}\right)^{-\gamma}, & q < q^*, \\ a(q^*), & q \geq q^*. \end{cases}$$

where

$$a(q^*) = A\left(\frac{1-\gamma}{\beta}\frac{q_0}{q^*}^{\zeta} - \frac{\zeta}{\beta}\frac{q_0}{q^*}^{1-\gamma}\right)^{\frac{\gamma}{1-\gamma}}$$



Blue: task distribution Red: optimal intensity

Aggregate Production Function

Production function $Y = F(K, Y; q_0)$ is implicitly defined by input requirements

$$\frac{L}{Y} = \int_{q_0}^{q^*} a(q) dPr(q; q_0) = a\left(q^*\right)\left(\frac{q_0}{q^*}\right)^{\zeta}$$

$$\frac{K}{Y} = \int_{q_0}^{q^*} a(q) q dPr(q; q_0) = Y \zeta \frac{a\left(q^*\right) q^*}{\beta}\left(\frac{q_0}{q^*}\right)^{\zeta}\left(1 - \left(\frac{q_0}{q^*}\right)^{\beta}\right)$$

Aggregation Properties

As automation expands, the mass of labor tasks shrinks, but

1. tasks already automated also become cheaper to perform with capital;
2. with a Pareto tail of hard-to-automate tasks, these two forces balance.

   (has to do with fractal property of Pareto distribution, invariance under stretching)

Limit of the labor share as economy grows is a constant:

$$s_L = \left[1 + \frac{\zeta}{1-\gamma-\zeta} D\left(\frac{q_0}{q^*}\right)\right]^{-1} \to \frac{1-\gamma-\zeta}{1-\gamma}.$$

Cobb-Douglas as the Long-Run Limit

Proposition 1 (Cobb–Douglas as the long-run limit)

Let $D_b = \{(K, L) : K/L > b\}$.

As $q_0 \to 0$*, the task-based production function* $F(K, L; q_0)$ *converges on* D_b *to the Cobb–Douglas limit*

$$F^{CD}(K, L; q_0) = A\, q_0^{-(1-\alpha)} L^{\alpha} K^{1-\alpha},$$

uniformly in relative error. In the limit, the labor share converges to

$$\alpha = \frac{1 - \gamma - \zeta}{1 - \gamma}.$$

Moreover, derivatives of F *converge analogously to those of* F^{CD}.

- Implied balanced growth is consistent with ongoing automation.

Mechanism

Identifying the Key Forces and Effects

Illustrative Example: A Four-task "Leontief" Economy

Setup: $Y = 10$. Each task $i \in \{1, 2, 3, 4\}$ performed by **capital** or **labor**.

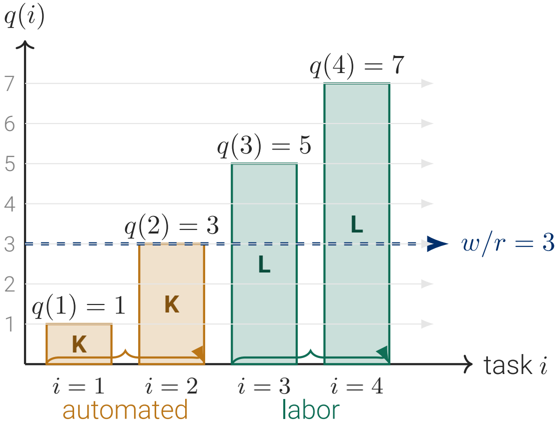
Must perform all tasks for $Y > 0$.

- $q(i)$ = capital requirement to automate task i
- w = wage; r = user cost of capital on BGP
- $r = \rho + g + \delta \equiv 1$ (normalization)
- Price of output = numeraire $\equiv 1$

Baseline schedule: $q(1) = 1,\ q(2) = 3,\ q(3) = 5,\ q(4) = 7.$

Factor prices: $w = 3$, $r = 1$, so $w/r = 3$.

Baseline



Initial equilibrium (baseline):

Tasks 1,2 automated ($q \leq 3$).

Tasks 3,4 use labor ($q > 3$).

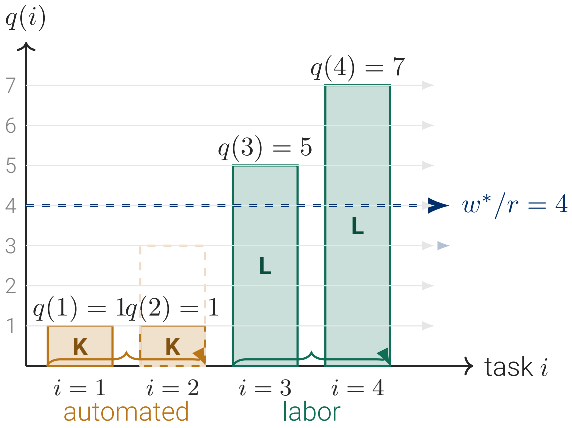
$K = q(1) + q(2) = 4; \quad L = 2.$

Cost $= 4 + 6 = 10 = Y$.

$\Pi = 0$ (Zero profit).

$$LS = \frac{Lw}{Y} = \frac{6}{10} = \frac{3}{5} = 60\%$$

Augmentation/Deepening: $q(2): 3 \to 1$



Long run ($\Pi = 0$, $w^* = 4$):

Tasks 1,2 automated ($q \leq 4$).

Tasks 3,4 use labor ($q > 4$).

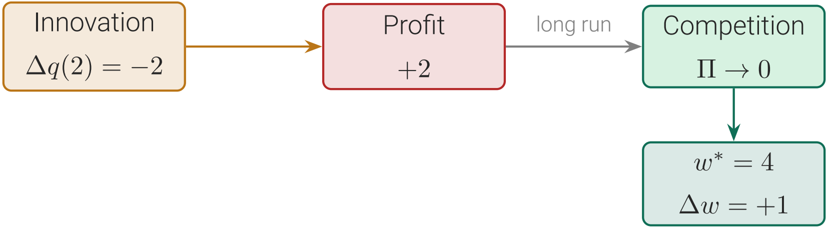
$K' = q(1) + q(2) = 2; \quad L' = 2.$

Cost $= 2 + 8 = 10 = Y$.

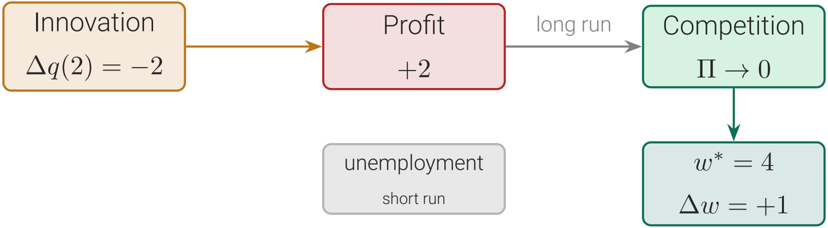
$\Pi = 0$ (Zero profit).

$$LS = \frac{w^* \cdot L'}{Y} = \frac{8}{10} = \frac{4}{5} = 80\%$$

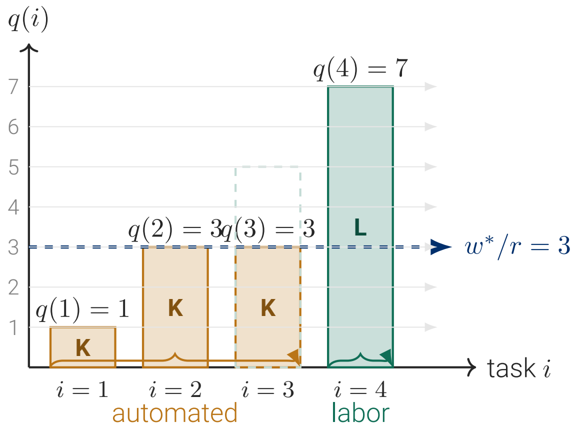
Augmentation/Deepening: Chain of Adjustments



Augmentation/Deepening: Chain of Adjustments



Automation: $q(3) : 5 \to 3$



Long run ($\Pi = 0$, $w^* = 3$):

Tasks 1,2,3 automated ($q \leq 3$).

Task 4 uses labor ($q > 3$).

$K'' = 1 + 3 + 3 = 7; \quad L'' = 1.$

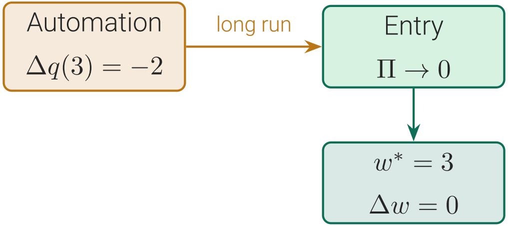
Cost $= 7 + 3 = 10 = Y$.

$\Pi = 0.$

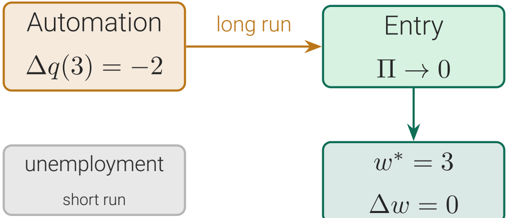
SR=LR: wage unchanged.

$$LS = \frac{w^* \cdot L''}{Y} = \frac{3}{10} = 30\%$$

Automation: Chain of Adjustments



Automation: Chain of Adjustments



Combined effect

	Baseline	**Augmentation** $q(2): 3 \to 1$	**Automation** $q(3): 5 \to 3$	**Combined** *both*
Shock	–	intensive margin	extensive margin	both
K	4	2	7	**5**
L	2	2	1	1
w^*	3	**4**	3	**5**
Labor Lw	6	**8**	**3**	**5**
Capital Kr	4	**2**	**7**	**5**
LS	$3/5 = 60\%$	$4/5 = 80\%$	$3/10 = 30\%$	$\mathbf{1/2 = 50\%}$
ΔLS	–	$+1/5$	$-3/10$	$\mathbf{-1/10}$

Growth Implications

Pre-industrial trap, takeoff, and transition

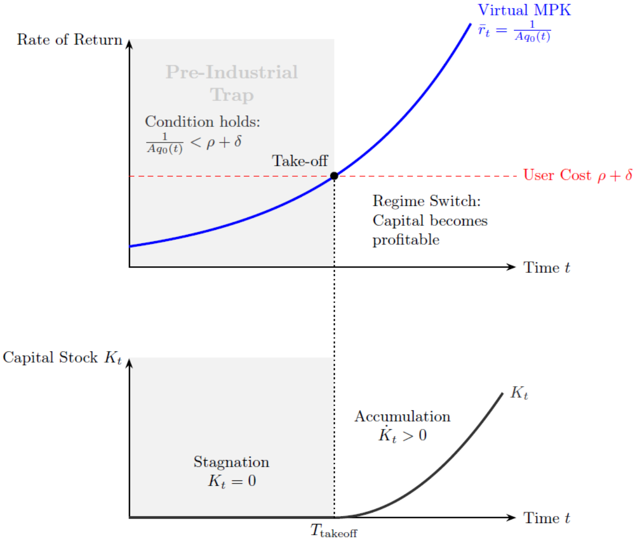
What's Different: Neoclassical vs. Task-Based Growth

- **Neoclassical Model:** Inada conditions imply $\rightarrow$ capital always accumulated.
- **Task-Based Model:** The marginal product of capital is **bounded**
  - $\rightarrow$ even the "easiest" task requires a finite amount of capital $q_0(t)$;
  - $\rightarrow$ if the maximum possible return is below the user cost, capital is not adopted:

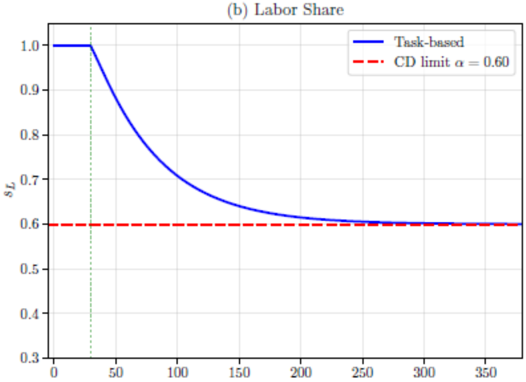
$$\bar{r}_t = \frac{1}{q_0(t)} < \rho + \delta.$$

- Distinct regime and pre-industrial sink:
  - $\rightarrow$ technological frontier $q_0(t)$ is too high for profitable use of capital
  - $\rightarrow$ stagnation because no use of capital, slow growth, and possibly no capital-augmenting progress (no learning-by-doing)

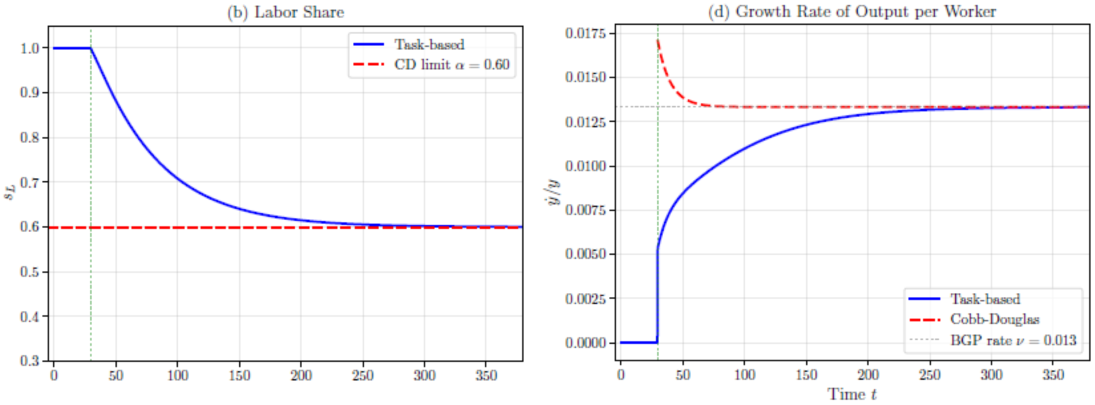
Pre-Industrial Sink and Endogenous Takeoff



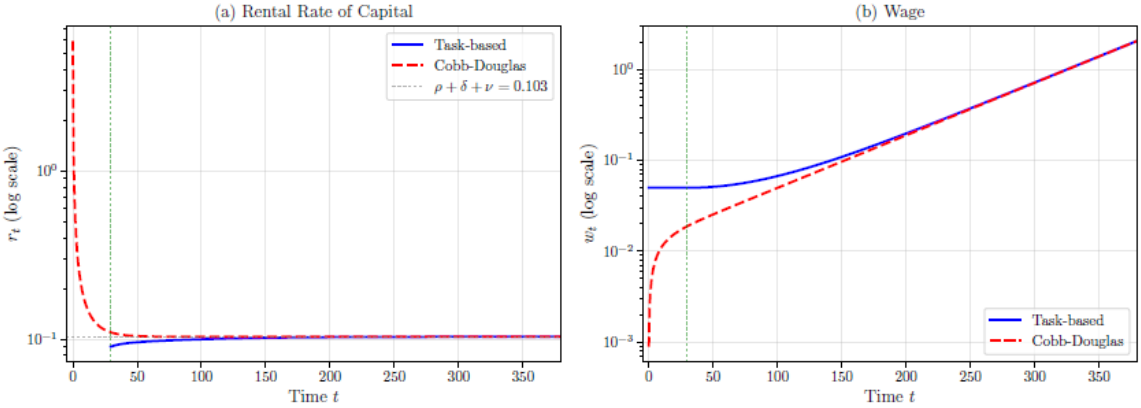
Transitional Dynamics: Task-Based Economy vs. Cobb–Douglas



Transitional Dynamics: Task-Based Economy vs. Cobb–Douglas



Factor Prices Along the Transition



Evidence from Occupation-Level Data

Testing Model’s Tail Implication

From the Model to Occupation-Level Data

Treat each occupation as a representative labor task with $q > q^*$.

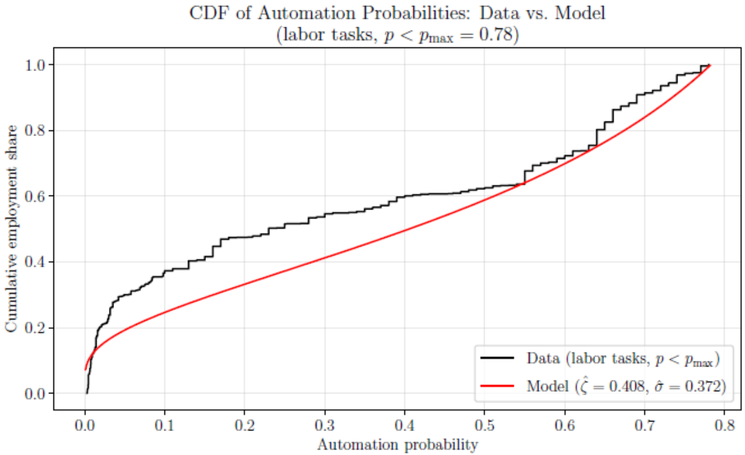
- Define its distance from the automation cutoff: $y \equiv \log\left(\frac{q}{q^*}\right) > 0$.
- Because labor tasks come from a Pareto tail, log-distance is exponential.
- Over a horizon of T years, the automation probability is $p(y) \approx \Phi\left(\frac{(g+\nu)T-y}{\sigma\sqrt{T}}\right)$.
- Intuition:
  - → many occupations lie relatively close to the cutoff;
  - → fewer occupations are very far from it;
  - → the number declines at a constant proportional rate.

Data and Calibration

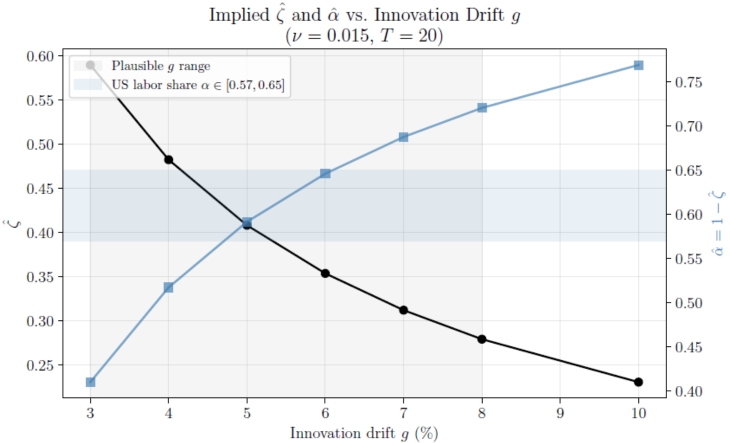
Object	Value	Source / rationale
Balanced-growth rate, ν	1.5% per year	BLS real hourly compensation
Automation horizon, T	20 years	Horizon in Frey–Osborne
Innovation drift, g	5%	Quality-adjusted decline in equipment prices

- Data: Frey–Osborne automation probabilities for 702 US occupations.
- Estimated from the data: tail parameter ζ.

Model Fit: Automation Probabilities Across Occupations



Implied Tail Index and Labor Share



Baseline estimate: $\hat{\zeta} \approx 0.41 \;\Rightarrow\; \hat{\alpha} \approx 0.59.$

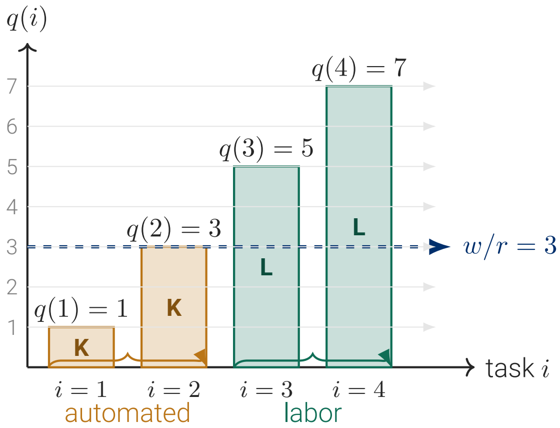
Remarks: AI as Automation-biased Technological Change

Frey and Osbornein 2013 identified these “future” automation roadblocks:

- Engineering Bottlenecks: The model’s inputs were based on three categories of tasks that computers still struggle with, which they called ”engineering bottlenecks”
- Perception and Manipulation: Handling objects in unstructured environments.
- Creative Intelligence: Originality and fine arts.
- Social Intelligence: Negotiation, persuasion, and care-giving.

Gen-AI/gen-AI robotics hits precisely these areas and no other

Remarks: AI as Automation-biased Technological Change



AI $\Rightarrow$ q(3), q(4)

AI productivity high $\Rightarrow$ $LS \downarrow$ AI productivity so so $\Rightarrow$ $LS \downarrow\downarrow\downarrow$

Conclusion

- **Emergent Stability:** A stable labor share is consistent with sustained automation along the BGP, provided capital productivity follows a random growth process.

- **Endogenous Automation:** Automation does not inherently depress the labor share when it is part of a broader, "balanced" innovation process across the task distribution.

- **AI Pivot:** Unlike historical automation, AI likely represents an **automation-biased** technological change that may break the historical "miracle" of stability.

- **Exposure Validation:** The model's predicted labor share and fat tail aligned with occupation-level exposure data (e.g., Frey & Osborne 2017).