Why Tease? The Role and Ramifications of Credit Card Teaser Rates*

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Abstract

Households who borrow on credit cards at promotional “teaser” rates often do so in anticipation of later receiving balance transfer offers extending attractive promotional pricing. Lenders make profits when such borrowers are caught the expiration of promotional rates, resulting in a “cat-and-mouse dynamic” in a large segment of the U.S. credit card market. Here we document the prevalence of teaser rates and promotional repricing through balance transfers prior to the Great Recession and develop an equilibrium model that can account for this feature of the data. We then employ the model to study the collapse of balance transfers during the Great Recession and assess its contribution to deleveraging on credit cards, thus far exclusively attributed to demand-side forces.

Keywords: credit cards, deleveraging, financial crisis, Great Recession, consumer default, balance transfers, unsecured credit

JEL codes: E21, D91, G20

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1 Introduction

Households who borrow on credit cards at promotional “teaser” rates often do so in anticipation of later receiving balance transfer offers extending attractive promotional pricing. Lenders make profits when such borrowers are ultimately caught paying step-up rates that follow the expiration of teaser rates.1 Thus far there has been little empirical and theoretical research on the potential ramifications of this salient feature of the U.S. credit card market. The current vintage of economic models of unsecured lending abstracts from such dynamic by exogenously restricting contracts to constant-interest and constant-maturity contracts.2 Here we document the prevalence of teaser rates and promotional repricing through balance transfers and develop an equilibrium model that can account for this feature. We employ the model to study the collapse of balance transfers during the Great Recession and assess its contribution to deleveraging on credit cards.

Our model rationalizes teaser rates and balance transfers as an equilibrium phenomenon. The key mechanism is that borrowers discount the future quasi-geometrically and hence systematically overestimate their ability to pay down debt in the future. Knowing it, lenders offer low promotional “tease” rates early so as to lure borrowers into the contract and make money after the step-up rate kicks in and the borrowers still find themselves owing money. Importantly, our mechanism is supported by the micro-level evidence suggesting that borrowers prefer sub-

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1 As documented by Evans and Schmalensee (2005), balance transfers and the popularity of introductory “teaser” rates have been rising rapidly since the 1990s, to subsequently become the hallmark of the U.S. credit card market. By early 2000s, Evans and Schmalensee (2005) report that a whopping 17 percent of outstanding balances were being transferred per annum. Since only a fraction of credit card balances are debt, and given credit cards are open-end credit contracts.

2 Some equilibrium model do consider endogenous maturity. For example, Drozd and Nosal (2007) and Chatterjee, Corbae, Nakajima and Ríos-Rull (2007) both consider endogenous switching and repricing. However, they do not model teaser rates and predatory role of balance transfers in response to teaser rate pricing.
optimal offers under geometric discounting. Specifically, in an important paper Ausubel and Shui (2005) show that consumers who accept new credit card offers with introductory offers end up paying more than they would have paid if they picked a different offer, which they argue is only consistent with quasi-geometric discounting. Here we turn their insight into an equilibrium model of credit card lending. As we show, this feature is necessary in our framework. In particular, when preferences are geometric, lenders neither front-load nor back-load due default risk premia.

The calibrated model accounts for the prevalence of promotional pricing of credit card debt that we estimate to be about xx percent (blank due to confidentiality clearance pending) relative to debt prior to the Great Recession. It also accounts for the fact that much of promotional balances are balance transfer initiated, with the flow of balance transfers over the average duration of promotional periods in the data closely matching the stock of promotional balances. The model generates a gap in APR on promotional balances and regular balances of xx percent (blank due to confidentiality clearance pending), which also accords well with evidence. In addition, the model matches the gross credit card indebtedness and chargeoffs on credit cards.

Substantively, we use the model to show that the presence of promotional “teaser” pricing leads to a vulnerability of the credit card market to credit supply disruptions. Specifically, we explore this implication of the model in light of the financial crisis of 2007-2008 and the Great Recession, which, as we show, resulted in a major collapse of balance transfer activity and deleveraging on credit cards. We attribute this collapse to supply forces, as we find little evidence that it is related to observed borrower risk characteristics, such as their risk
scores (Vantage score), credit utilization, credit card balances, other debts and balance, delinquency/bankruptcy history, etc.\textsuperscript{3}

In the model we assume that credit card lenders in 2008 faced a credit crunch that precluded them from originating credit card portfolios that break even intertemporarily, and instead impose a temporal break-even condition. Other than that they can extend new loans. At the same time, we raise the probability of income shocks hitting consumers to account for the rising chargeoff rate on credit card debt during this time period. This additional risk must be priced into new contracts which makes new credit that priced temporal default risk in full more expensive. We explore the role of expectations regarding the duration of the shock.

Our exercise shows that the repricing shock, as we label it, can explain a sizable portion of nonchargeoff driven deleveraging during the Great Recession. In part this is also attributed to the fact that quasi-geometric borrowers are more sensitive to such an interest rate hike.\textsuperscript{4}

The insight of our paper is relevant from a policy perspective. Thus far deleveraging on credit cards has been largely attributed to demand forces.\textsuperscript{5} If that’s, the inner working of the the credit card market are not relevant for deleveraging. But, if that’s not the case, as our analysis suggests, the crisis has revealed a vulnerability of the existing regulatory framework that regulators should take a notice of.

\textsuperscript{3}The decline is also consistent with related industry evidence showing a similar decline in pre-screened mail solicitations with balance transfer options despite rising take-up rates, which would have not happened had borrowers simply lost interest in offers.

\textsuperscript{4}It is important to stress that the cause of the decline might have been also related to CARD Act of 2009 rather than the recession. Our analysis does not distinguish between the two and it is not essential for what we do to identify the exact source of tightening in the credit card market.

\textsuperscript{5}See, for example, the discussion in Brown, Haughwout, Lee and Van der Klaauw (2013) as well as that in Demyanyk and Koepke (2012). The authors of both studies contemplate the potential role of cuts to unutilized lines and some rate hikes that took place prior to CARD Act of 2009 but overall conclude that amid major decline in credit inquiries demand must have played the most prominent role.
1.1 Related literature

The literature on the role of teaser rates and repricing in credit card market is scant. A notable exception is Ausubel and Shui (2005), which we rely on in terms of providing evidence. Drozd and Nosal (2007) also analyze a model with repricing, but they do not study teaser rates. Relative to Ausubel and Shui (2005), our paper provides an equilibrium model of teaser rates and applies it to substantively study deleveraging during the crisis. Deleveraging in the credit card market attracted some attention in economics literature. For example, Athreya, Sánchez, Tam and Young (2015) analyze the effect of employment risk and the CARD Act on delinquency and study how this affected unsecured lending. Among empirical studies, Santucci (2016) looks at a pool of credit card balances drawn in 2009 and tracks them over time to attribute deleveraging to debt paydown and debt discharges. He finds that about a third of debt has been discharged, which is consistent with what we report for the aggregate data. Brown et al. (2013) provide an overview of household deleveraging and discuss the key hypothesis explaining it. To the best our knowledge, our paper is unique in exploring the role of balance transfers and teaser pricing in the context of deleveraging. The issue of time-varying price schedules and repricing does arise in standard models of unsecured lending, such as Livshits, MacGee and Tertilt (2007), Chatterjee et al. (2007), or Athreya (2002), as these models assume exogenous one-period maturity of contracts.
2 Theory

Here we lay out our baseline theory. In the next sections we use it to illustrate the mechanism that leads to promotional “teaser” pricing and calibrate it to the data to explore its quantitative predictions.

2.1 Environment

Time is discrete and the horizon is infinite. The economy is populated by a large number of consumers and a large number of lenders. Consumers live for a finite number of $3 \leq T < \infty$ periods and each period receive some exogenous income stream determined by a random variable $s = 1, 2, \ldots, S$ that follows a finite-state Markov process. Consumer need credit to smooth income shocks and lenders can extend credit to consumers. Lenders have unlimited access to funds at a fixed interest rate $r \geq 0$ and maximize profits. Lending market is competitive. The law of large numbers is assumed. There is no aggregate uncertainty.

2.1.1 Lending protocol

Credit comes exclusively in the form of state defaultable but otherwise state-noncontingent unsecured credit lines. Credit lines are characterized by credit limit $L \geq 0$ up to which funds can be drawn, a promotional rate $F$ that only applies in the first period and a step-up interest rate $R$ that is at least as high as the promotional rate and applies thereafter. Lines are open-ended but lenders can reprice them under the legal restriction that rates cannot be

\footnote{As is standard in the literature, we abstract from the presence of other credit instruments. A less stark interpretation of this assumption is that secured borrowing is frictionless, absent of capital gains or losses, and unsecured credit is exclusively used to finance purchases of consumption goods that cannot be used as a collateral.}
hiked above $R$ and credit limits must at least the accommodate outstanding debt, unless a
different arrangement has been authorized by the borrower.

The above commitment restriction is motivated by and consistent with the Credit Card
Accountability Responsibility and Disclosure (CARD) Act of 2009, which came into effect early
in 2010. It is arguably also consistent with the prevalent industry practices prior to the CARD
Act.\footnote{According to CARD Act lenders cannot raise interest rate on debt and cannot cut utilized credit lines (up to five years or until expiration of the account, whichever comes first). Lender can set promotional rates that expire after a specified period of time not shorter than 6 months and they can cut unutilized lines at any time (or close them). Before the CARD Act, term changes were allowed but lenders often recognized the value of reputation and often maintained terms. Anecdotal evidence suggests that term changes largely occurred in response to consumer noncompliance of some sort (e.g., delinquency, late payment) or a failure to respond to opt out notices. For example, Capital One and Citi, to large credit card lenders, were voluntarily committing themselves to offer opt out options from rate changes other than the ones triggered by noncompliance (e.g. late payments, overdraft etc...) prior to the CARD Act. In early 2008, Chase, another major credit card lender, adopted an internal rule of not responding to any credit history changes when reviewing terms. OCC in its advisory letters openly discouraged all national banks from the practice of changing terms on credit cards (see OCC Advisory Letter, AL 2004-10). For a more detailed evidence regarding term changes before the CARD Act, see case-studies described in Appendix to H.R. 5244 “The Credit Cardholders’ Bill of Rights: Providing New Protections for Consumers,” Hearing before the Subcommittee on Financial Institution and Consumer Credit of the Committee on Finance Services, U.S. House of Representatives, One Hundred Tenth Congress Session, April 17, 2008, Serial no. 110-109. Pages 280, 327, 371, 373-379, and 410 are of particular interest. See also the excellent monograph by Evans and Schmalensee (2004).}

2.1.2 Timing of events

The timing of events within each period is as follows (Figure 1): 1) Upon entering each
period, consumers publicly observe their state, which is comprised of their age $t$, their credit
line on hand $C$, outstanding debt $B$, and the current realization of a random variable $s$ that
determines within period uncertainty (income). 2) Lending market opens and consumers re-
ceive the market offer $M$. 3) Consumer decide whether to accept the market offer ($\lambda = 1$)
or reject it ($\lambda = 1$) with no recall. 4) Incumbent lenders then reprice the original contract
extending repriced offer $I$. 5) With the current contract $C'$ on hand, which is either $M$ or $I$
Figure 1: Timing of events within the period.

Notes: The figure illustrates timing of events within each period. The consumer’s initial state is described by open credit line \( C \), debt \( B \) and current random state \( s \). The future period state \( C', B' = b, s' \) is the result of random shocks, as well as debt refinancing, default and consumption decisions made by the consumer within the period.

depending on \( \lambda \), consumers choose whether to default (\( \delta = 1 \)) or repay (\( \delta = 0 \)) and determine their consumption \( c \) and the current level of borrowing \( b \). In the following period, \( b \) becomes \( B \), \( C' \) becomes \( C \) and the above timing is repeated.

We now lay out formal details of the consumer problem and the lender problem and conclude with the definition of equilibrium.

### 2.2 Consumer problem

There are of two types of consumers in the model: geometric \( \eta = 1 \) and quasi-geometric \( 0 < \eta < 1 \). Geometric consumer type discounts temporal utility flows \( u(c_t) \) using the vector \((\beta, \beta^2, \beta^3, \ldots)\), where \( 0 < \beta \leq 1 \). In contrast, quasi-geometric consumer type discounts tempo-
ral utility flows using the vector \((\eta \beta, \eta \beta^2, \eta \beta^3, \ldots)\). Accordingly, quasi-geometric consumer type evaluates consumption streams according to preferences given by

\[ u(c_t) + \eta \beta [u(c_{t+1}) + \beta u(c_{t+2}) + \beta^2 u(c_{t+3}) + \ldots] \]

and hence incorrectly presumes that her future preferences will be

\[ u(c_{t+1}) + \beta u(c_{t+2}) + \beta^2 u(c_{t+3}) + \ldots, \]

while in actuality they are

\[ u(c_{t+1}) + \eta \beta [u(c_{t+2}) + \beta u(c_{t+3}) + \ldots]. \]

As a result, quasi-geometric consumers make time-inconsistent choices based on a false presumption that their “future selves” are more patient than they actually are.

In the baseline setup, we assume that consumer type is permanent characteristic, with the share of each type being \(\phi\) and \(1 - \phi\), respectively. However, type can also be stochastic and depend on the realization of the random variable \(s\). We consider such an extension in the quantitative analysis of our model.

We now formally lay out the \(\eta\)-type consumer problem. We begin from the interim stage of the model’s timing.
2.2.1 *Ex-ante* consumer problem

Consumers first choose whether to retain or refinance existing credit line $C$ depending on the continuation value. The market moves first and the incumbent second, and there is no recall. The equilibrium decision function to refinance is denoted by $\lambda^{\eta}_t(C, B, s)$ and it depends on consumer’s state as of the beginning of the period.

Formally, let $M_t^{\eta}(C, B, s)$ be the equilibrium contract that the market extends to the consumer of type $\eta$ in state $(C, B, s; t)$ and let $I_t^{\eta}(C, B, s)$ be the equilibrium repriced (incumbent’s) offer in case the consumer does not refinance. We define these object formally when we discuss lender problem. The consumer chooses $\lambda$ to solve

$$V_t^{\eta}(C, B, s) = \max_{\lambda=0,1} \left\{ \lambda U_t^{\eta}(M_t^{\eta}(C, B, s), C, B, s) + (1 - \lambda)U_t^{\eta}(I_t^{\eta}(C, B, s), C, B, s) \right\},$$

for any $0 < \rho < 1$, and for $\rho = 1$ we assume $\lambda = 1$ since the market does not open.

2.2.2 *Interim* consumer problem

Consider next the consumer problem after the lending market closed and/or after the incumbent lender has already repriced the line. Let $C = (F, R, L)$ be the original contractual terms that the consumer had with the incumbent and let $C' = (F', R', L')$ be the current terms that apply in the given period, which, may come from the incumbent or a new lender.

The consumer of type $\eta$ in the interim state $(C', C, B, s; t)$ then chooses $\delta_t^{\eta}(C', C, B, s)$, $c_t^{\eta}(C', C, B, s)$ and $b_t^{\eta}(C', C, B, s)$ to maximize:

$$U_t^{\eta}(C', C, B, s) = \max_{\delta=0,1} \delta U_t^{\eta}(C', C, B, s)$$

(2)
where the continuation value contingent on $\delta$ is given by

$$\delta U^\eta_t(C', C, B, s) = \max_{(c, b) \in \Gamma} \{u_t(c) - \delta \chi(s) + \eta \beta [\delta \mathbb{E}_s V^1_{t+1}(C_0, 0, s') + (1 - \delta) \mathbb{E}_s V^1_{t+1}(C', b, s')] \}. \quad (3)$$

$V^1_{t+1}(\cdot)$ is the continuation value of a consumer of type $\eta = 1$ to imply quasi-geometric discounting when $\eta < 1$ and $\Gamma$ is the budget constraint given by the following inequalities:

$$c \leq Y_t(s, \delta) - B + b - [\rho R + (1 - \rho) F] b^+(1 - \delta), \quad (4)$$

$$b \leq L'.$$

where $Y_t(s, \delta)$ is income, $B$ is current debt, $b$ is future debt and $b^+$ is shorthand for $\max\{0, b^+\}$. As is clear from the expressions, defaulting ($\delta = 1$) wipes out all debt at the expense of an exogenous utility punishment $\chi(s)$ and beginning the next period with exogenous penalty contract $C_0 = (\bar{R}, \bar{R}, 0)$, where $\bar{R} \geq r$ is exogenously assumed penalty rate.

The first inequality of the budget constraint implies that consumption depends on income $Y$, which is a function of random state $s$, consumer age $t$ and default decision $\delta$, and current borrowing $b$ net of debt $B$ and interest payments that are paid when the consumer repays ($\delta = 0$). Importantly, interest payments on $b^+$ are split between the issuer of $C$ credit line and the new credit line depending on the value of parameter $\rho$. That is, $\rho R b^+$ is the interest paid to the issuer of the original $C$ line and $(1 - \rho) F b^+$ is the interest paid to the issuer of the new line $C'$. This key assumption makes the step up rate $R$ in our model have a bite even when the consumer refinances. The physical interpretation is that there is a delay $\rho$ that implies that even when the consumer refinances the incumbent still collects interest rate on a fraction
\(\rho\) of the period. We refer to this feature as \textit{refinance-timing friction}. In the baseline setup we assume \(\rho\) is deterministic but in the quantitative section we do consider an extension with stochastic \(\rho\) that depends on \(s\).

The second inequality is the borrowing constraint and it is determined by \(L'\) and not \(L\). This is a simplification and it is without loss in our setup. Note that when the consumer refinances with a new lender the incumbent lender does not bear any default risk, as refinancing occurs prior to defaulting. Hence, relaxing consumer’s borrowing constraint is always going to be profitable for the incumbent lender and the consumer would have requested a limit hike in the interim.

Finally, note that we have restricted attention to \(F\) being a promotional rate, and hence we require that \(R \geq (1 - \rho)F\). We summarize it in the assumption below.

\textbf{Assumption 1} \textit{F is restricted to be a promotional rate, hence }\(R \geq (1 - \rho)F\).

\subsection*{2.3 Lender problem}

There are two types of lenders in the model: \textit{incumbent lenders}, who already have an open credit line with the customer, and \textit{new lenders}, who compete in the market to extend refinance offers that poach customers from incumbents.\(^8\) Both lenders may supply the current offer \(C'\) depending on whether the consumer decides to refinance or stay with the incumbent.

\(^8\)This is not the case for borrower who transition from borrowing to lending. In such case the incumbent trivially has \(L = 0\), in which case there is no poaching.
2.3.1 New lenders

The market is competitive and hence the market offer maximizes consumer utility subject to a zero profit condition. Formally, let $\Pi_\eta t(C', C, B, s)$ be the profit function of a new lender whose offer $C'$ has been accepted. Then,

$$M_\eta t(C, B, s) = \arg \max_{C'} U_\eta t(C', C, B, s) \quad (5)$$

subject to

$$\Pi_\eta t(C', C, B, s) = 0. \quad (6)$$

We define $\Pi_\eta t$ at the end of the section.

2.3.2 Incumbents

Incumbents reprice after the market closes and they have a monopoly power over the consumer subject to the legal restriction that the new interest rate cannot exceed $R$ and the new credit limit must be at least $B$, unless the lender obtains an authorization from the borrower by making an appropriate take-it-or-leave-it offer. Accordingly, the repriced offer from the incumbent maximizes incumbent’s profits subject to the requirement that the utility associated with the repriced contract is at least as high as the one implied by the worst legal contract $(R, R, B)$. Formally, if the consumer does not accept the market offer,

$$I_\eta t(C, B, s) = \arg \max_{C'} \Pi_\eta t(C', C, B, s) \quad (7)$$
subject to
\[ U_i^n(C', C, B, s) \geq U_i^n((R, R, B), C, B, s). \] (8)

The incumbent lender reprices also when the consumer refinances. However, in such a case the problem is trivial since the incumbent can only change the credit limit and not the interest rate, since the consumer leaves on the portion \((1 - \rho)\) of the period to which the rate change applies. Accordingly, if \(L\) constrains borrowing, it is always profitable for the incumbent to raise credit limit so as to match the one extended by the new lender and possibly increase borrowing (note that borrowing cannot decrease).

Moreover, as a side comment, note that even if the incumbent lender could change rate \(R\), which would make sense to allow for, charging \(R\) would still be optimal. Note that, since the new lender generally enters on the increasing portion of interest rate revenue, and so \((1 - \rho)F'b^+\) is increasing in \(F\). If this was not the case lowering \(F\) would be optimal, since it must unambiguously raise profits of the new lender and borrower’s utility as well. But this implies that \(\rho Rb_i^{n+}\) is increasing in \(R\) and hence that charging \(R\) is optimal to maximize profits.

We summarize it in the lemma below.

**Lemma 1** If consumer refinances \((\lambda = 1)\) to a non-degenerate contract \((i.e., b^+ > 0)\), rate \(R\) and credit limit \(L \geq L'\) maximize incumbent’s profits.

We next define \(\Pi_i^n\) to close the lender problem.

**2.3.3 Lender profits**

To define the profit function \(\Pi_i^n\), we exploit the property that incumbent lenders who face imminent default optimally reprice the loan prior to defaulting by cutting any unused
portion of the the credit line, implying that the borrower defaults only on \( B \). We summarize this straightforward result in the lemma below and build it into our notation. Note that the immediate corollary of this lemma is that slack credit limits have no bite in our setup. Hence, without loss, we can assume that the market contract always involves a tight credit limit, i.e., \( b^+ = L' \). We use this property in the analysis of the model and exploit it in the computation of its equilibrium.

**Lemma 2** If the consumer defaults on incumbent’s contract, the incumbent’s repriced contract \( I^\eta_t(C, B, s) = (F', R', L') \) optimally cuts credit limit to \( L' = B \).

**Corollary 1** It is without loss to assume that the equilibrium market contract \( M^\eta_t(C, B, s) = (F', R', L') \) always involves a tight borrowing constraint, i.e., \( L' = b^+ (M^\eta_t(C, B, s), C, B, s) \).

The profit function \( \Pi^\eta_t(C', C, B, s) \) of a new lender whose contract \( C' = (F', R', L') \) has been accepted by the consumer is defined as follows: 1) if the consumer defaults in the current period, i.e., \( \delta^\eta_t(C', C, B, s) = 1 \),

\[
\Pi^\eta_t(C', C, B, s) = -b^+ (C', C, B, s)(1 + r);
\]

and 2) if the consumer repays in the current period,

\[
\Pi^\eta_t(C', C, B, s) = (1 - \rho)(F' - r)b^+ (C', C, B, s) + \mathbb{E}_u[\tilde{\Pi}^\eta_{t+1}(C', b^+ (C', C, B, s), s')]/(1 + r),
\]

where \( \tilde{\Pi}^\eta_t(\cdot) \) is the profit function of an incumbent lender as of the beginning of the next period, i.e., when the new lender extending current line \( C' \) in the current period \( t \) becomes incumbent
lender in the future period $t + 1$.

The profit function $\hat{\Pi}_t^n(C, B, s)$ of an *incumbent lender* who extended contract $C = (F, R, L)$ in the previous period to the consumer and will reprice to $I_t^n(C, B, s) = (F', R', L')$ is defined as follows: 1) if the consumer decides *not* to refinance in the current period and default, i.e. $\delta_t^n(I_t^n(C, B, s), C, B, s) = 1$ and $\lambda_t^n(C, B, s) = 0$,

$$\hat{\Pi}_t^n(C, B, s) = -B(1 + r);$$

2) if the consumer decides neither to refinance in the current period nor default, i.e., $\lambda_t^n(C, B, s) = 0$ and $\delta_t^n(I_t^n(C, B, s), C, B, s) = 0$, profit is

$$\hat{\Pi}_t^n(C, B, s) = (\rho R + (1 - \rho)F' - r)b_t^n(I_t^n(C, B, s), C, B, s) + \mathbb{E}_s[\hat{\Pi}_{t+1}(I_{t+1}^n(C, s), b_t^n(I_t^n(C, B, s), B, s), s')]/(1 + r),$$

since the incumbent will reprice before the period start; and 3) if the consumer decides to refinance in the current period, i.e. $\lambda_t^n(C, B, s) = 1$,

$$\Pi_t^n(C, B, s) = \rho(R - r)b_t^{n+}(I_t^n(C, B, s), C, B, s),$$

since the incumbent bears no default risk in such a case.
2.4 Equilibrium

Equilibrium is the collection of policy functions \( c^\eta_t(C', C, B, s) \), \( b^\eta_t(C', C, B, s) \), \( \delta^\eta_t(C', C, B, s) \), \( \lambda^\eta_t(C, B, s) \), \( C^\eta_t(C, B, s) \), as well as the associated value and profit functions, such that \( \delta^\eta_t(C', C, B, s), c^\eta_t(C', C, b, s), b^\eta_t(C', C, b, s) \) solve (2), \( \lambda^\eta_t(C, B, s) \) solves (1) given (2), and \( M^\eta_t(C, B, s) \) and \( I^\eta_t(C, B, s) \) derived given by (5) and (7), respectively.

2.5 Model’s mechanism in a three-period example

Here we consider a three period version of our model that assumes time-invariant log utility function \( u(c) = \log(c) \) and a two-state Markov income process: \( Y(s = 1) = Y \), and \( Y(s = 2) = \overline{Y} \), where the low income \( Y \) is an absorbing state, occurs with probability \( p_L = p > 0 \), and high income occurs with probability \( p_H = 1 - p \). The initial income is \( Y \) and the consumer starts from some nontrivial amount of debt \( B \) that she wishes to rollover through the remaining periods, i.e., to make the problem nontrivial, we assume throughout that the consumer borrows in equilibrium to make the problem nontrivial. Fig:figure-setup diagrammatically illustrates the consumer problem in this setup.

Furthermore, to make the contracting problem tractable, we assume an extremely convex cost of defaulting \( \chi(Y) \) as a function of income state. That is, we set \( \chi = 0 \) when income is low to ensure consumer always defaults on debt when income is low and assume \( \chi \) is arbitrarily high when income is high to prevent default in the high state all along. For convenience, we set \( \beta = 1 \) and assume lenders discount future profits at the same rate as the consumer, which is \( \beta = 1 \) in the case of geometric consumer type and \( \eta \beta = \eta < 1 \) in the case of naivete quasi-geometric consumer type.
2.5.1 Optimal contracting problem

We are interested in the pricing of credit card offers by the lender who extends credit in the first period. This lender faces a nontrivial default risk in both the first period and in the second period and also the risk of refinancing in the second period. We now define the problem of this lender.

Let the contract extended by this lender be denoted by $C = (F, R, L)$. Let the value function of the consumer associated with this contract be $U(F, R, L)$ and let the profit function of the first period lender be described by $\Pi(F, R, L)$. These functions are given by equations (10) and (11), as shown in the lemma below. The lender chooses $F, R, L$ to maximize

$$\max_{F,R,L} U(F, R, L)$$

subject to

$$\Pi(F, R, L) = 0.$$ 

**Lemma 3** First period lender’s profit function is

$$\Pi(F, R, L) = \begin{cases} 
((1 - \rho)F - p)b_1^+ + (\rho R' - p)b_2^+ & \text{if } \lambda = 1 \\
((1 - \rho)F - p)b_1^+ + (R' - p)b_2^+ & \text{otherwise}.
\end{cases}$$
and consumer’s value function is

\[
U(F, R, L) = \max_{\lambda=0,1} \max_{b_1 \leq L, b_2} \ u(\overline{Y} - B + b_1 - (1 - \rho)Fb_1^+) + \\
\eta[p\{u(\overline{Y} - b_1 + b_2 - (\rho R + (1 - \rho)F'(\lambda, R))b_2^+) + pHu(\overline{Y} - b_2) + pu(\overline{Y})\}]
\]

\[
\eta[p[(u(\overline{Y}) + \beta u(\overline{Y}))]].
\]

where

\[F'(\lambda, R) = \begin{cases} 
\frac{p}{1 - \rho} & \text{if } \lambda = 1 \\
\min\{R, \frac{p}{1 - \rho}\} & \text{otherwise.}
\end{cases}\]

Moreover, for all \(R > \frac{p}{1 - \rho}\) the consumer refinances, implying \(\lambda = 1\), and otherwise stays with the incumbent.

The proof of the lemma is straightforward. The profit from the repriced or refinanced contract extended in the second period is \(\pi = ((1 - \rho)F' - p)b_2^+\), and hence the market offer implies \((1 - \rho)F' = p\) and slack credit limit that makes \(b_2\) unconstrained. Furthermore, if the consumer does not refinance, for the incumbent lender to break-even, we must have \(R \geq p\), as otherwise the condition in Assumption 1 that \((1 - \rho)F \geq R\) would be violated, since incumbent’s profit is negative when \(R < p\) and \((1 - \rho)F < p\). This is clear from formula (10).

Finally, refinancing decision depends on \(R\) as the incumbent can extract all surplus from the consumer, and since her profit from second period on is \(\pi = ((1 - \rho)F' - p)b_2^+\), she will set \(F'\) as high as possible, i.e. either \(F' = R\) or the incumbent will put the consumer at indifference by matching market offer with \((1 - \rho)F' = p\).
2.5.2 Equilibrium contract

We now turn to the characterization of the first period lender problem in (9). We consider each consumer type separately and characterize how lender price default risk across periods.

2.5.3 Geometric consumers ($\eta = 1$)

We begin from the benchmark case of geometric consumers, i.e., we assume $\eta = 1$. To make the problem nontrivial, we make an assumption that $B$ is sufficiently high (and $p$ not too high) to imply interior borrowing in high state for the equilibrium contract, whatever it is.

**Assumption 2** $B$ is sufficiently high and $p$ is not too high to ensure interior borrowing $0 < b_1 < \bar{Y}$, and ex post second period borrowing $b_2^+ > 0$ in high income state for the equilibrium contract.

We prove a stark result that lenders should price in default risk contemporaneously into rates and that there are no balance transfers in equilibrium. What this result implies is that first period’s lender contemporaneous profit is zero period-by-period, that is,

\[
((1 - \rho)F - p)b_2^+ = 0, \\
(R - p)b_2^+ = 0,
\]

Moreover, since $R = p$ implies that the incumbent’s repriced offer is $F' = p$, the borrower does not reprice since the market offer is $F' = p/(1 - \rho) > p$. Hence, an even stronger result applies: Borrowers do not refinance even if new lenders face significantly lower cost of funds than incumbents. We summarize it in Proposition 1 below.
Proposition 1  In case of geometric discounting ($\eta = 1$), lenders fully price in contemporaneous default risk into current rates, i.e., set $(1-\rho)F = p$, and there are is no refinancing (balance transfers) in equilibrium even if future lender’s cost of funds is lower by up to $p(1/(1-\rho)-1)$.

The intuition behind 1 is simple, albeit the proof is tedious. Note that the borrower’s Euler equation is

$$(1 - (1 - \rho)F)u'(c_1) = (1 - p)u'(c_2),$$

and hence involves a wedge $(1 - p)/(1 - (1 - \rho)F) > 1$ for any $(1 - \rho)F < p$ that leads to higher $b_1$. This is because when the borrower defaults she does not internalize repayment. From the lender’s perspective who maximize borrower utility subject to the cost of funds this is suboptimal, as the cost of funds that are defaulted on should be internalized by the borrower. In other words, lenders would rather have the borrower choose $b_1$ according to

$$u'(c_1) = u'(c_2),$$

but requires that $(1 - \rho)F = p$, and hence default risk must be priced in contemporaneously to rates.

The alternative solution to setting $(1 - \rho)F = p$ would be to impose a binding credit limit to ensure the above condition. However, this alone raises no revenue for the lender and rates must be set accordingly to recoup default losses. It then turns out that setting $(1 - \rho)F = p$ is still optimal, since this is the maximum level of the first period rate that is nondistortionary because of the binding borrowing constraint. But since this interest rate by definition relaxes the borrowing constraint, $L$ is in effect nonbinding.
2.5.4 Quasi-geometric consumers ($\eta = 1$)

We now turn to the characterization of the naivete quasi-geometric consumers ($\eta < 1$), and show that in such a case $(1 - \rho)F < p$ and balance transfers may arise in equilibrium.

**Proposition 2** In case of quasi-geometric discounting ($\eta < 1$), lenders incompletely price in contemporaneous default risk into current rates, i.e., set $(1 - \rho)F < p$. Moreover, for $p$ not too high, the optimal contract features $F = 0$ for sufficiently low $\eta > 0$, and there is refinancing (balance transfers) in equilibrium for sufficiently low $\rho$.

The intuition behind this result can be easily grasp by considering what happens when the borrower incorrectly believes that she will be saving between the second and the third period while in actuality she borrows. In such a case, it is clear that setting $F = 0$ is optimal for lenders as long as there exists sufficiently positive $R > p/\rho$ that lets first period lender break even. For $p$ sufficiently low, and $B$ that warrants borrowing, as Assumption 2 implies, this is the case and hence $F = 0$.

More generally, the result is more nuanced because the consumer my assume she will borrow in the future period. However, there is a positive wedge between the borrowing level that enters the profit function of the first period lender and the one expected borrowing by the consumer. In particular, the lender knows that consumer borrowing solves

$$
\hat{b}^2_2 = \arg \max_{b_2} \{ u(\bar{Y} - b_1 + \hat{b}_2 - (\rho R + (1 - \rho)F'(\lambda, R)\hat{b}_2^+)) + (1 - p)\eta u(\bar{Y} - \hat{b}_2) \},
$$

(12)
while the borrower ex ante incorrectly assumes

\[ b_2 = \arg \max_{b_2} \{ u(\bar{Y} - b_1 + \hat{b}_2 - (\rho R + (1 - \rho)F'(\lambda, R))\hat{b}_2^+ + (1 - p)u(\bar{Y} - \hat{b}_2)\}. \]  (13)

Importantly, note that Proposition 2 implies that if the flow of credit supply in the economy is disrupted so that balance transfer offers do not arrive as expected, the borrower the faces higher interest rate on debt than she assumed ex ante, causing deleveraging. We exploit this particular dynamic in light of the deleveraging on credit cards during the Great Recession.

### 3 Data [Incomplete]

We will focus on two key facts in the data.

**Fact 1. Promotional activity and balance transfers collapsed in the credit cared market during the Great Recession**

A balance transfer occurs when debt carried on one account is moved to another account. Since balance transfers are costly, they are usually initiated by lenders who extend introductory offers on balance transfers. About 10% of offers charge no fees, and most charge about 3% percent fee in exchange for an introductory interest on transferred balances for an average of about 15 months. However, introductory and/or balance transfer promotions were nearly always offered with 0 percent interest before the crisis.\(^9\) Hence, the incentive that borrowers have to execute balance transfer is the combination of lower payments and/or a liquidity relief. 2001.

\(^9\)See evidence reported by Grodzicki (2015), as found in national survey of mail-out credit offers administered, Mintel Comperemedia (Mintel). The recent report on Credit Card by market by the CFPB provides a wealth of information about balance transfers. See also ?, pp. 176-190. CFPB reports that the majority of balance transfer offers offered 0 APR.
Figure 2: Balance transfer activity a measured by the average number of balance transfers executed by credit card users in a given year. Source: Experian.

To document the decline in the balance transfer activity during the Great Recession, we use credit bureau micro-data that reports the number of balance transfers executed by a borrower in a given year. The database is a representative panel of 100,000 individual credit histories collected by Experian from 2001 until 2013, tracked over time in 2 year intervals, with many variables looking two years before to connect consecutive observations. The sample is representative as of 2013 and loses its representatives going backwards. The data report for every consumer the number of balance transfers executed during preceding 12 months and balances on all her revolving accounts. While we know the number of balance transfers, we do not know their volume nor how much credit card debt the individual carried. We also cannot

---

10Credit report data implies that the target population for our analysis consists of all US residents with a credit history in 2001. The target population excludes individuals who have never applied for or qualified for a loan back in 2001 or after 2001. The credit report data included in our panel consists of information on bank card accounts that have been updated by the creditor within the last 12 months. Since minimum payments are updated on all current accounts with balances, this limitation is not a binding one for our analysis.
cleanly distinguish debt from balances, albeit we can attempt to approximate it.\textsuperscript{11} \textsuperscript{11} reports that substantial amounts are being transferred, with the average exceeding $4,300 in 2017 and preceding years.

Figure 2 presents the average number of balance transfers per credit card user.\textsuperscript{12} \textsuperscript{12} As we can see, there is a clear break from the rising trend in 2007. The decline continues until about 2010, when balance transfer activity bottoms out but does not recover until 2013 (when our data ends).

To assess whether the decline is demand- or supply-driven, we give the hypothesis of demand-driven channel a change by estimating a regression model of balance transfers with several crucial variables that lenders see and which they likely deem important when extending credit. Unfortunately we do not see income, which lenders may factor into their decision. However, we do not expect that borrowers reveal their current income shocks as credit applications ask for income in the preceding year. Hence it would have to be that early hike in unemployment accounts for most of the shock, which is not plausible, and in part we could also thing of it as difficult to distinguish combination of income and supply shock that is a function of income. The estimated model for the period preceding the crisis accounts for about 7 percent of variation in the data. The Appendix provides more details.

We assess whether the decline can be explain by changing risk attributes of borrowers. Unfortunately we do not see income, which lenders may factor into their decision. However, we do not see other variables such as credit scores, delinquency, debt etc. The estimated linear

\textsuperscript{11}Debt are balances carried for at least two payment cycles.
\textsuperscript{12}We shifted data by one year so that the value pertains to the year in which it is report year. Since data is biannual we linearly interpolated the value between two consecutive years. Data values are for year 2000, 2002, etc.
probability is in the Appendix provides more details.

As Figure 3 shows, the model is far off from the decline in balance transfer activity seen in the data. This suggests to us that the hypothesis that the decline has been driven by a major change in borrower composition arguably implied by their behavior, or even shocks, is not consistent with our findings.

![Figure 3: Balance transfer activity as measured by the average number of balance transfers executed by credit card users in a given year. Source: Experian.](image)

Our finding is corroborated by the fact that mail-in solicitation are the main tool of customer acquisition and new credit card solicitations offering promotional balance transfers dropped by about 70%, while the response rate to solicited offers generally rose during this time period (the response rates to direct mail solicitations went from 0.467% in 2006 to 0.575% in 2009).\(^\text{13}\) If lenders use some kind of automated model that based on borrower changing at-

\(^{13}\)Sources: Synovate Mail Monitor, Argus, Information & Advisory Services, LLC (files.consumerfinance.gov/f/2011/03/Argus-Presentation.pdf), and Mintel Media (www.comperemedia.com).
tributes shut down solicited offers, our flexible procedure should pick it up by replicating this model. If, on the other hand, borrowers received offers but reject the response rate to solicited offers for reasons that our model cannot pick up, response rate to offers should have plummeted, and it did not.

Fact 2. U.S. households deleveraged on credit cards during the Great Recession

![Figure 4: Revolving debt growth before and after chargeoffs. Source: Federal Reserve Board of Governors.](image)

Figure 4 compares the growth rate of credit card debt before and after taking out credit card charge-offs. To construct this series, we use the basic law of motion for debt which states that debt $B_t$ at $t$ is equal to debt at $t - 1$ less debt discharged between $t$ and $t - 1$ (net of recoveries) and debt $b_t$ drawn at $t$ (or paid back if negative):

$$B_t = B_{t-1} - \delta_t B_{t-1} + b_t,$$
which implies the following decomposition of the growth of debt:

\[
\frac{b_t}{B_t} = \frac{B_t - B_{t-1}}{B_t} + \delta_t,
\]

Accordingly, we define

\[
\frac{B_t - B_{t-1}}{B_t} + \delta_t,
\]

as the annual growth rate of credit card debt after charge-offs, and

\[
\frac{B_t - B_{t-1}}{B_t}
\]

as the annual growth rate of debt before charge-offs. The figure illustrates these two series.14

As is clear from the figure, credit card debt contracted sharply during the Great Recession period both when measured before and after charge-offs. Debt growth before charge-offs fell from about 12 percent on average between 2000 and 2007 to about negative 8 percent in 2009, bottoming out in 2011. The decline in debt growth before charge-offs was only by about 30 percent smaller than the decline in debt growth after charge-offs, which suggests that consumers borrowed less than usually or paid down debt. While 2011 brought recovery to the credit card market, the figure shows that the growth of credit card debt remained depressed for much longer.

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14To construct these growth rates, we measure credit card debt by seasonably adjusted stock of revolving debt reported by Federal Reserve Board of Governors (FRB), which mainly includes credit card debt according to FRB. To measure the charge-off rate, we use seasonally adjusted charge-off rate on credit cards (all banks) also reported by FRB.
4 Calibration

[to be completed]

5 Quantitative findings

[to be completed]

6 Application: Credit card deleveraging of 2007-2011

[to be completed]

References


Clausen, Andrew and Carlo Strub, “Envelope theorems for non-smooth and non-concave optimization,” *ECON - Working Papers 062, Department of Economics - University of Zurich April* 2012.
Appendix

A. Model of Balance Transfer Activity

We used the following linear probability model to predict the number of balance transfers in Section 2. To begin with, the model shows poor overall explanatory power, suggesting that balance transfers do not cleanly depend on bureau-data variables. It is nonetheless informative as lenders use some kind of models based on bureau-data to solicit customers. These are the attributes that they see and likely use.

B. Omitted Proofs

B.1 Appendix B: Omitted Proofs

Proof of Proposition 1: 

We proceed in two steps.

First, we show that no backloaded contract with \((1 - \rho)F < p\) can be optimal.

Second, we show that \((1 - \rho)F = p\) satisfies first order conditions of the lender’s contracting problem.

Third, we note the following to complete the argument. The lender problem in (9) is continuous and hence by Weierstrass theorem for appropriately compactified contract space so that on the boundary points yield low value of the program a unique interior point that satisfies first order conditions must be the maximum. We compactify the contract space by assuming lower bound \(L = 0\) and note that upper bound \(L\) is irrelevant since \(L\) can be dropped when it is nonbinding by Lemma 2 and Corollary
Table 1: Linear model of the annual number of balance transfers.

<table>
<thead>
<tr>
<th>Explanatory variable</th>
<th>Regression coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest revolving balance</td>
<td>3.88e-6 (2.3e-7)</td>
</tr>
<tr>
<td>Bankcard balances (total)</td>
<td>-9.6e-7 (9.0e-8)</td>
</tr>
<tr>
<td>Bankcard utilization</td>
<td>-0.0245 (0.0005)</td>
</tr>
<tr>
<td>Bankcard total monthly min payments to balances</td>
<td>-0.0057 (0.0021)</td>
</tr>
<tr>
<td>Severe bankcard delinquency on record</td>
<td>-0.0647 (0.0028)</td>
</tr>
<tr>
<td>Bankruptcy on record (indicator)</td>
<td>-0.0055 (0.0044)</td>
</tr>
<tr>
<td>Severe bankcard delinquency in next 24 months (indicator)</td>
<td>-0.0216 (0.0031)</td>
</tr>
<tr>
<td>Bankruptcy in next 24 months (indicator)</td>
<td>-0.0222 (0.0063)</td>
</tr>
<tr>
<td>Recent credit card delinquency on record (last 24 months)</td>
<td>-0.0368 (0.0036)</td>
</tr>
<tr>
<td>Recent major credit card delinquency on record (last 24 months)</td>
<td>-0.0156 (0.0055)</td>
</tr>
<tr>
<td>Bankcard accounts ever discharged or settled (number)</td>
<td>0.0003 (0.0008)</td>
</tr>
<tr>
<td>Bankcard debt recovery score</td>
<td>-0.0002 (0.0000)</td>
</tr>
<tr>
<td>Current Vantage v3 credit score</td>
<td>-0.0006 (0.0000)</td>
</tr>
<tr>
<td>Home equity lines (number, open)</td>
<td>0.0512 (0.0020)</td>
</tr>
<tr>
<td>Revolving accounts (number, open)</td>
<td>0.0307 (0.0003)</td>
</tr>
<tr>
<td>Autoloans (number, open)</td>
<td>0.0017 (0.0012)</td>
</tr>
<tr>
<td>Mortgage (indicator, open)</td>
<td>0.0113 (0.0019)</td>
</tr>
<tr>
<td>Mortgage balance (open, total)</td>
<td>1.23e-8 (6.4e-9)</td>
</tr>
<tr>
<td>Court judgments all (number on record)</td>
<td>-0.0957 (0.0800)</td>
</tr>
<tr>
<td>Foreclosures all (number on record)</td>
<td>-0.0734 (0.0504)</td>
</tr>
<tr>
<td>Collection inquiries all (number on record)</td>
<td>-0.0094 (0.0017)</td>
</tr>
</tbody>
</table>

Year FE x State FE

<table>
<thead>
<tr>
<th>Regression coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
</tr>
<tr>
<td>N</td>
</tr>
<tr>
<td>R2</td>
</tr>
</tbody>
</table>

1, and finally assume bounds on \((1 - \rho)F = 1\) and \(R = 1\) high enough to imply effectively hard borrowing constraint at zero. Such extreme rates clearly yield a lower value of the program under Assumption 2 relative to proposed contract in the proposition, since the consumer actually borrows for that contract and low rates are strictly better than higher rates.

Finally, fourth, we note that neither discrete refinancing decision nor default decision invalidates differentiability nor the use of envelope theorem since these decisions involve a maximum of concave value functions, implying a finite number of downward kinks in the sense of Clausen and Strub (2012). By the results they prove, all value functions are differentiable and the envelope theorem applies globally despite the fact that discrete choices are being made. The basic argument here is that the consumer will never choose to be at the downward kink, which is the only point where the value function is not differentiable and the envelope theorem fails. Clausen and Strub (2012) establish this more generally for recursive setups that stack countably many kinks. (Note that Assumption 2 plays an important role to ensure global differentiability and the applicability of the envelope theorem when we invoke it since this reasoning does no apply to kink at zero debt since it is an upward kink.)
Step 1: No backloaded contract with \((1 - \rho)F < p\) is optimal.

To show this, assume that \(F, R, L\) is the optimal backloaded contract. We will construct an infinitesimal deviation from this contract that is zero profit and raises the utility of the borrower, contradicting its optimality.

To that end, consider lowering \(R\) by infinitesimal \(dR < 0\) and raising \(F\) by some \(dF > 0\), so that profits remain zero, while keeping \(L\) unchanged. (A similar argument but somewhat less intuitive may be obtained by considering the Lagrangian to lender problem (9).

First, note the following properties of this variation. By Lemma 2 and Corollary 1, the borrowing constraint can be assumed tight without loss. Since \(L\) is tight and remains unchanged under this deviation, and in general when partial derivative with respect to \(R\) or \(F\) is taken, \(b_1\) can only fall but cannot go up, and hence if \(b_1(F, R)\) denotes consumer’s optimal borrowing policy, we can conclude the following: i)

\[
b_{1R} = \frac{\partial b_1(F, R)}{\partial R} \leq 0,
\]

\[
b_{1F} = \frac{\partial b_1(F, R)}{\partial F} \leq 0.
\]

ii) Furthermore, by consumption smoothing, we know that

\[
b_{2b_1} = \frac{\partial b_2(b_1, R)}{\partial b_1} > 0.
\]

Finally, iii) we also have

\[
b_{2R} = \frac{\partial b_2(b_1, R)}{\partial R} < 0,
\]

which follows from Assumption 2. We can explicitly calculate second period policy function:

\[
b_2(b_1, R) = \frac{(1 - p)b_1 + (p - \rho R)\bar{Y}}{(2 - p)(1 - \rho R)},
\]

which implies

\[
b_{2R} = -\frac{(1 - p)(\bar{Y} - b_1)}{(2 - p)(1 - \rho R)^2} < 0,
\]

given Assumption 2 requires \(b_1 < \bar{Y}\). (This general property applies to entire CRRA family of utility functions.)

Case 1a. Consider first nonbinding \(L\) and suppose \((1 - \rho)F < p\) is such that it implies \(R > p/(1 - \rho)\) so that the consumer refinances, i.e., \(\lambda = 1\). By implicit function theorem, which applies as long as Assumption 2 is ensured, observe that the implied change in \(F\) due to our variation that keeps profits constant is

\[
dF_{d\pi = 0} = \left(\frac{dF}{dR}\right)_{d\pi = 0} = -\frac{\Pi_R}{\Pi_F}dR.
\]

Now, the impact on the utility function is

\[
dU = U_RdR + U_FdF_{d\pi = 0},
\]

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which we must show is (strictly) positive \( dU > 0 \). Note that, since \( U_R dR > 0, U_F dF < 0 \), this is the case iff

\[
U_F dF \left( \frac{U_R}{U_F} \right) (\frac{dF_d\pi=0}{dR}) + 1 > 0
\]

\[
U_R \left( \frac{dF_d\pi=0}{dR} \right) + 1 < 0
\]

which requires

\[
\frac{U_R}{U_F} > \frac{\Pi_R}{\Pi_F} > 1.
\]

We will now show this is the case. To that end, we introduce general notation

\[
\Pi(F, R, L) := ((1 - \rho)F - p)b_1^+(F, R) + (1 - p)(R - p)b_2^+ (b_1(F, R), R)),
\]

and note that we also use derivative wrt \( b_1 \), which is

\[
\Pi_{b_1}(F, R, L) := (1 - \rho)F - p + (1 - p)(\rho R - p)b_2^+_{b_1}.
\]

In above expressions we drop \( L \) from policy function, as it is irrelevant given we only consider changes to \( F \) and \( R \). (This would apply to binding \( L \).)

By envelope theorem that applies since \( L \) is tight but nonbinding and borrowing constraint is slack in second period by Lemma 3, we calculate

\[
\frac{U_R}{U_F} < \frac{(1 - p)b_2^+}{(1 - p)b_1^+}.
\]

Similarly, we calculate by differentiating the profit function that

\[
\frac{\Pi_R}{\Pi_F} = \frac{(1 - p)b_2^+ + \beta \Pi_{b_1}b_1^+ R + (1 - p)(\rho R - p)b_2^+_{b_1} (b_1(F, R), R)}{(1 - p)b_1^+ + \Pi_{b_1}b_1^+ F},
\]

and note the following additional properties: iv) Since the borrowing constraint is nonbinding we must have \( \Pi_{b_1} \ge 0 \). This follows from the fact that, had \( \Pi_{b_1} < 0 \), since the constraint is nonbinding, it would pay to lower \( L \), which would contradict optimality of this contract. Note that the impact on the objective function is infinitesimal as it is at its peak with respect to \( b_1, b_2 \), while profit would unambiguously increase since lower \( L \) would implies lower \( b_1 \) (as it is tight for the current contract). Hence, \( \Pi_{b_1} b_{1R}^+ < 0 \) and \( \Pi_{b_1} b_{1F}^+ < 0 \) by property i) above. v) Since for our contract \( \rho R - p > 0 \), we have \( \beta(1 - p)(\rho R - p)b_2^+_{b_2} < 0 \). Finally, vi) \( \Pi_F > 0, \Pi_R > 0 \), as otherwise the lenders would operate on the decreasing portion of the profit function and it would be beneficial to simply lower \( R \) (\( F \)) for the same \( F \) (\( R \)) and \( L \), contradicting optimality of the assumed contract. Accordingly, by property iv), we obtain

\[
\frac{\Pi_R}{\Pi_F} > \frac{(1 - p)b_2^+ + \Pi_{b_1}b_{1R}^+ + (1 - p)(\rho R - p)b_2^+(b_1, R)}{(1 - p)b_1^+}.
\]

and hence we have

\[
\frac{U_R}{U_F} > \frac{(1 - p)b_2^+}{(1 - p)b_1^+ + \Pi_{b_1}b_{1R}^+ + (1 - p)(\rho R - p)b_2^+(b_1, R)} > 1,
\]

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where the latter inequality follows from properties v-vi. We have thus obtained that the assumed zero profit deviation raises utility of the consumer, contradicting optimality of the original contract.

**Case 1b.** Consider now nonbinding $L$ and suppose $(1 - \rho)F < p$ is such that it implies $R \leq p/(1 - \rho)$ so that the consumer does not refinance, i.e., $\lambda = 0$.

This case boils down to exactly analogous reasoning. In step of the argument where $\rho$ appears replace it by $\rho = 1$. Note by Lemma 3 that the incumbent reprices to $R$, and hence second period interest rate is simply $R$.

If the variation we consider is at the indifference point between refinancing or not refinancing additional care needs to be taken. Note that that since the contract $R = p/(1 - \rho)$ is a zero profit from repriced contract second period on, lenders profit function is continuous and only changes slope. Same argument applies to the utility function, as is clear from (11),(10). We have shown that for both slopes the variation raises utility of the borrower. Since lowering $R$ transitions consumer from refinancing to no refinancing by Lemma 3, what matters is the direction derivative and we do not require that the left- and right-hand side derivatives equal at the transition point.

**Case 2.** Consider now binding $L$ and assume any $(1 - \rho)F < p$ implying that the consumer either refinances or does not.

Since we have shown that refinancing does not affect the reasoning consider the last step of the reasoning fleshed out in case 1a, i.e.

$$\frac{\Pi_R}{\Pi_F} = \frac{(1 - p)b^+_2 + \Pi b^+_{1R} + (1 - p)(\rho R - p)b^+_2R}{(1 - \rho)b^+_1 + \Pi b^+_1 F}.$$ 

Note that in the binding case we additionally have $b^+_{1R} = 0, b^+_1 F = 0$, and hence

$$\frac{\Pi_R}{\Pi_F} = \frac{(1 - p)b^+_2 + (1 - p)(\rho R - p)b^+_2R}{(1 - \rho)b^+_1}$$

which leads to an analogous but tighter evaluation

$$\frac{(U_R)(\Pi_R)}{(U_F)(\Pi_F)} = \frac{(1 - p)b^+_2}{(1 - p)b^+_2 + (1 - p)(\rho R - p)b^+_2R(b_1(R), R)} > 1.$$ 

We similarly contradict the optimality of the original contract.

**Step 2:** The contract $(1 - \rho)F = p, R = p, L$ nonbinding, satisfies first period lender’s first order conditions for optimality. (Binding $L$ is trivially suboptimal.)

To that end, consider the Lagrangian

$$L = U(F, R, L) - \lambda \Pi(F, R, L),$$

where

$$\Pi(F, R, L) = ((1 - \rho)F - p)b^+_1(F, R, L) + (1 - p)(R - p)b^+_2(b_1(F, R, L), R))$$

by equation (10). Note that for the given contract the consumer does not refinance, which follows from Lemma 3. Evaluating the Lagrangian at $(1 - \rho)F = p, R = p$, we derive the following first
order conditions:

\[
U_F = \lambda (1 - \rho) b_1^+, \\
U_R = \lambda (1 - p) b_2^+, \\
U_L = 0,
\]

The last condition is clear, since it follows from the fact that \( L \) is nonbinding and hence has no value for the consumer. We verify the first two conditions, which require

\[
\frac{U_R}{U_F} = \frac{(1 - p) b_2^+}{(1 - \rho) b_1^+},
\]

As before, by Envelope Theorem applied to the consumer problem (since \( L \) does not bind), we derive

\[
\frac{U_R}{U_F} = \frac{(1 - p) b_2^+}{(1 - \rho) b_1^+},
\]

which finishes the proof.

As noted at the beginning, the candidate contract found in Step 2 must be the maximum, as any other interior contract does not satisfy first order conditions for optimality and boundary points are strictly worse for sufficiently low \( p \). □

This case boils down to exactly analogous reasoning. In step of the argument where \( \rho \) appears replace it by \( \rho = 1 \). Note by Lemma 3 that the incumbent reprices to \( R \), and hence second period interest rate is simply \( R \).

**Proof of Proposition 2.**

We will now show that the contract identified in Step 2 in the proof of Proposition 1 as optimal is no longer optimal. Hence, some \((1 - \rho)F < p\) must be optimal.

To that end, consider again the Lagrangian but in this case consider quasi-geometric consumer. In such case the consumer believes he is choosing \( b_2 \) given by equation (13) but in actuality lenders know her choice is \( b_2^\eta \) given by equation (12), where we note that \( b_2^\eta > b_2 \) (for a given \( b_1 \) which is common across the two cases). We also add discounting to lender problem so that the lender discounts the future just like the consumer, as assumed in text. The modified profit function is

\[
\Pi^\eta(F, R, L) = ((1 - \rho)F - p) b_1^+(F, R, L) + \eta(1 - p)(R - p) b_2^{\eta^+}(b_1(F, R, L), R)).
\]

The Lagrangian is

\[
L = U(F, R, L) - \lambda \Pi^\eta(F, R, L),
\]

which at \((1 - \rho)F = p, R = p, L\) tight but nonbinding gives

\[
\frac{U_R}{U_F} = \frac{\eta(1 - p) b_2^{\eta^+}}{(1 - \rho) b_1^+}, \\
U_L = 0.
\]
Analogously, using Envelope Theorem applied to the consumer problem given nonbinding \( L \), we obtain
\[
\frac{U_R}{U_F} = \frac{\eta (1 - p) b_2^+}{(1 - \rho) b_1^+},
\]
which is a contradiction since \( b_2^+ > b_1^+ \). Given these derivatives the lender would like to deviate from the given contract by lowering \( F \) while raising \( R \), which is a feasible direction, hence the proposed contract is no longer optimal.

Consider now \( \eta \) arbitrarily low. In such a case the consumer would like to borrow an essentially unlimited amount, and by defaulting on it, lenders would need to recoup default premium on repayment path. Since there is a maximum revenue that can be collected from the consumer, it is infeasible for \( L \) to be nonbinding for \( \eta \) arbitrarily low. Consider now the case of binding \( L \) and consider an analogous variation to the one considered in proof of Proposition 1. The reasoning works regardless whether consumer refinances or not.

We can repeat all steps with the new profit function and obtain
\[
\frac{\Pi_R}{\Pi_F} = \beta \frac{(1 - p) b_2^+ + (1 - p)(R - p) b_2^{2R}}{(1 - \rho) b_1^+}.
\]
Since \( b_1 \) is the same here, as it is set ex ante, this leads to the following evaluation for the deviation considered in proof of Proposition 1
\[
\left( \frac{U_R}{U_F} \right) / \left( \frac{\Pi_R}{\Pi_F} \right) = \frac{(1 - p) b_2^+}{(1 - p) b_2^+ + (1 - p)(R - p) b_2^{2R} (b_1, R)}.
\]
Now, we can show that generally extreme impatience leads the consumer to almost borrow all her future income, i.e.,
\[
b_2^+ \rightarrow_{\eta \to 0} Y
\]
and leads to complete insensitivity to interest rate changes
\[
b_2^{2R} \rightarrow_{\eta \to 0} 0.
\]
This can be explicitly derived for the log utility. Hence, in the limit we have
\[
\left( \frac{U_R}{U_F} \right) / \left( \frac{\Pi_R}{\Pi_F} \right) \rightarrow_{\eta \to 0} \frac{(1 - p) b_2^+}{(1 - p) Y} < 1,
\]
since \( b_2^+ < Y \), and thus the opposite deviation is optimal, i.e., lenders would like to raise \( R \) and lower \( F \) as long as it is possible. This leads to \( F = 0 \).

Any \((1 - \rho)F < p\) implies \( R > p \) when the consumer does not refinance. This follows from lender’s zero profit condition. Assume by contradiction that the consumer does not refinance for all \( \rho \). Since \( b_1, b_2^+, b_2 \) is independent of \( \rho \) assuming the consumer does not refinance, we can write \( R = p + \varepsilon \), for some \( \varepsilon > 0 \) is independent of \( \rho \). Now, the market offer is \( F' = p/(1 - \rho) \), and hence for sufficiently low \( \rho \), we will have \( \rho R + (1 - \rho)F' > R \), since \( \rho (p + \varepsilon) + (1 - \rho) p/(1 - \rho) > p + \varepsilon \) for sufficiently low \( \rho \). This contradicts the the consumer does not refinance for all \( \rho \). In particular, if \( F = 0 \), we have \( \Pi^\eta \rightarrow_{\eta \to 0} (1 - p)(R - p)Y - pb_1^+ \), and thus zero profit condition requires \( R = p + \frac{p}{1 - p} b_1^+ \).