Understanding International Prices: Customers as Capital *

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ABSTRACT

The paper develops a new theory of pricing-to-market driven by dynamic frictions of building market shares. Our key innovation is a capital theoretic model of marketing in which relations with customers are valuable. We discipline the introduced friction using data on differences between short-run and long-run price elasticity of international trade flows. We show that the model accounts for several pricing puzzles of international macroeconomics.

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I. Introduction

Standard international macroeconomic models, while being successful in accounting for business cycle dynamics of quantities, have so far failed to account for the movements of international relative prices. In the data, three patterns are evident. First, real export prices[1] and real import prices are highly positively correlated, and both are positively correlated with the real exchange rates. Second, the terms of trade is much less volatile than the real exchange rates.[2] Third, there are large and persistent movements in the real exchange rates. These movements, often interpreted as deviations from the law of one price at the aggregate level, are mimicked by persistent deviations from the law of one price at more disaggregated levels.

Neither real business cycle models nor sticky price models have thus far been able to account for these patterns. In the standard real business cycle model, the real export price is negatively correlated with the real import price and the real exchange rate, the terms of trade is more volatile than the real exchange rate, and while real exchange rates are persistent, the law of one price holds at the disaggregated level. While sticky price models can, under certain assumptions, generate some of these features, they fail to generate anywhere near the persistence observed in the data[3].

Our reading of the above evidence is that it suggests the presence of frictions of bringing goods to a market, which break the law of one price (LOP hereafter) by endogenously segmenting the markets. This view is strongly supported by the micro-level evidence suggesting that international markets remain highly segmented, and exporters price to market in which they sell. Marston (1990), Knetter (1993), Goldberg & Knetter (1997) and, more recently, Burstein & Jaimovich (2009), provide evidence that when the real exchange rate depreciates, the prices of exported goods in most product markets systematically rise relative to the prices of the similar (or identical) goods sold at home. Consequently, the literature has interpreted

[1]By *real* prices we mean nominal prices evaluated relative to standard measures of the overall domestic price level.

[2]Consider the Jan 2006-Jan 2008 real depreciation. During this time, the U.S. real effective exchange depreciated by 11%, while the terms of trade for manufactured goods increased by only 0.5%. Export and import price indices for manufactured articles increased by 8.7% and 9.2%, respectively. (Sources: BLS for prices, IMF IFS for exchange rate).

[3]Chari, Kehoe & McGrattan (2002) show this in a canonical sticky price model. The model generates too little persistence of real exchange rate. In this model real exchange rate is isomorphic with deviations from LOP due to local currency pricing. Introducing real rigidities can improve the model's fit, but as shown by Johri & Lahiri (2008), does not fully resolve the problem.

this result as evidence that markups on exported goods tend to systematically rise when the real exchange rate depreciates, and fall when it appreciates.

Motivated by the above evidence, our paper proposes an equilibrium theory in which micro founded search and matching frictions result in endogenous market segmentation and deviations from LOP of the kind suggested by the empirical literature. The key mechanism is that firms need to explicitly build market shares by matching with their customers, and this process is costly and time consuming. As a result, price arbitrage through quantities traded is inhibited, and the real exchange rate fluctuations endogenously vary markups on the exported goods. Quantitatively, due to persistent pricing-to-market implied by our theory, the model successfully accounts for the volatility of the terms of trade relative to the real exchange rate, and implies a strong positive correlation between the real export price, the real import price, and the real exchange rate. Business cycle behavior of quantities is shown to be on par with the standard theory.

We develop our model in a fully-fledged international business cycle framework, to quantitatively relate the impact of our frictions on the standard business cycle facts. To make the model quantitative, and assess the relevance of this channel, we propose a novel approach to put discipline on the new features of the model by bringing in the data on the discrepancy between the low short-run and high long-run estimates of the price elasticity of trade flows. This discrepancy, well documented in the international trade literature, is often referred to as the *elasticity puzzle* (see Ruhl (2008)). Since in our model the elasticity puzzle is intimately related to the idea of market share sluggishness, we exploit it to independently discipline the key friction. Specifically, high long-run measured elasticity maps in our model onto a high intrinsic elasticity of substitution between home and foreign tradables (Armington elasticity), while our novel friction of building market shares is disciplined by the evidence that is typically used in the standard theory to justify the choice of low Armington elasticity[4].

The idea of sluggish market shares that we pursue here is not entirely new to economics.

[4] Other papers capable of addressing this puzzle include Ruhl (2008) and Ghironi & Melitz (2005). In these papers, adjustment is instantaneous and differences between measured elasticities are driven by the distinction between responses to temporary versus permanent shocks (due to fixed cost of entry). By contrast, in our setup, the disconnect comes from a sluggish response of market shares to shocks, and is gradual also after permanent shocks.

In fact, such frictions have long been considered a promising avenue since at least the 1980s. For instance, Krugman (1986, p. 32), in his seminal contribution to the subject, states: *The best hope of understanding pricing to market seems to come from dynamic models of imperfect competition. At this point, my preferred explanation would stress the roles of [...] the costs of rapidly adjusting the marketing and distribution infrastructure needed to sell some imports, and demand dynamics, resulting from the need of firms to invest in reputation.* In addition, such frictions have long found support in the anecdotal evidence about international trade relations between firms, and more recently, also in the evidence on firms' market share growth after entry into a new market. The anecdotal evidence, (e.g. H. Hakansson (1982), Turnbull & M. T. Cunningham (1981), and Egan & Mody (1992)), primarily based on surveys with the CEOs, has pervasively stressed the importance of long-lasting producer-supplier B2B relationships, informational frictions involved in finding new suppliers, and the presence of highly individualized relationships that make switching particularly costly. Recent evidence on firms' market share growth following entry into a foreign market, discussed in Eaton, Eslava, Kugler & Tybout (2007) and Ruhl & Willis (2008), shows that, as expected in the presence of such frictions, the buildup of market shares is a slow process. Moreover, for the U.S. domestic market, using data from the Census of Manufactures, Foster, Haltiwanger & Syverson (2008) recently argue that the demand-side frictions are actually the key factor responsible for the slow growth of demand after plant entry into a new geographical area, with 'new' plants lagging behind 'old' plants even 9 years after entry. Despite this evidence, thus far, theoretical treatments of such frictions remain scant in macro literature. In this paper, we aim at filling this gap by proposing a tractable theory of such demand-side frictions, and relating them to the failure of arbitrage.

The detailed structure of our model is as follows. First, international trade takes place only in matches between retailers and intermediate good producers. Second, intermediate good producers explicitly build their customer base by choosing spending on broadly interpreted marketing (market research, design and customization of the product, distribution infrastructure, advertising, technical support). Marketing is valuable to the producers because search of retailers is undirected and the meeting probabilities are determined by the relative marketing capital held by each producer. Matches with the retailers are long-lasting and each producer, as a state variable, carries an endogenous list of customers to whom he can sell a finite quantity of the good. Accumulation of marketing capital, and thus the buildup of market shares, is not frictionless and takes time, as producers face what we term as *market expansion friction*. Due to the bilateral monopoly problem which arises within each match, dock and wholesale prices are determined in the model by *bargaining* between producers and retailers.

Market expansion friction and bargaining are the two key features that give rise to pricing to market in our model. Bargaining is critical, as it creates an explicit link between the producer prices and the valuation of the goods by the local buyers (retailers). In particular, for intermediate values of bargaining power θ, bargaining implies that the difference between the export price p_x and home wholesale price of the same good p_d depends on the difference between the foreign retailer's valuation of the domestic good measured in the domestic numéraire, xP_d^*, and the domestic retailer's valuation P_d measured in the same unit, according to

$$p_x - p_d = \theta(xP_d^* - P_d).$$

As a result of this link, any deviation in retailer's valuation of the domestic good across international markets, leads in our model to a similar deviation of the producer prices of these goods. Our second key feature, the market expansion friction, which affects the speed at which market shares adjust in equilibrium, is what makes these deviations sustainable, and consistent with the producer profit maximization. That is because producers in our model, in order to sell their goods, need to match with additional customers. This process is time consuming and costly, and hence gives rise to a time-varying and target-market specific shadow 'marketing' cost. This shadow cost makes arbitrage unprofitable, even though transacted prices may suggest otherwise.

The way these effects play out in the general equilibrium of our model is as follows. Since the low short-run price elasticity of trade flows is determined by the market expansion friction (sluggish market shares) rather than low Armington elasticity, market shares move in our model almost exactly as much as in the standard models with low Armington elasticity (consistent with short-run elasticity of trade flows). However, unlike in these models, due to high Armington elasticity set to account for high long-run price elasticity of trade flows, local retailer valuations P_d^*, P_d remain highly insensitive to the underlying market share movements. Since our search and matching environment gives rise to movements of the real exchange rate x which are very similar to standard models with low elasticity (and for analogous reasons), by the bargaining relation stated above, producer prices behave consistently with the empirical patterns we document. Quantitatively, as highlighted in Section V, the distinct feature of this mechanism is that it can deliver a large degree of pricing to market even for low level of average markups (as PTM is largely independent from the exact level of markups).

Related literature Our model is related to a number of contributions in the pricing-to-market and incomplete pass-through literature. Dynamic pricing-to-market models with frictions similar to ours include partial equilibrium models by Krugman (1986), Froot & Klemperer (1989), Kasa (1992), and Alessandria (2004), as well as general equilibrium quantitative models by Lapham (1995) and Ravn, Schmitt-Grohe & Uribe (2007).

Relative to this first group papers, we propose a quantitative general equilibrium model in which sluggish market shares endogenously arise from the underlying search and matching frictions. In addition, we show that such frictions have the potential to reconcile the international macroeconomics approach with static trade theory by accounting for the discrepancy between the measured price elasticities of trade. Among these papers, Krugman (1986), Kasa (1992), and Lapham (1995) consider a simple convex adjustment cost directly imposed on aggregate exports. Since our work takes off from this approach, in Section VII.we provide a quantitative evaluation of this friction when embedded in a general equilibrium framework. We show that while such formulation can deliver deviations from LOP, it cannot be disciplined the same way as our formulation (by elasticity puzzle), and generally falls short of the price data relative to our formulation.

Another approach related to ours is taken in Ravn, Schmitt-Grohe & Uribe (2007). That paper explores the role of 'deep habit' formation in generating pricing to market, a friction generally regarded as a reduced form way of modeling the process of customer acquisition, on which we focus. In this context, the key difference is that in the 'deep habit' model, producers use low prices to lure consumers to their variety of good, while in our paper, producers have a separate 'marketing' technology of enlarging customer base, and thus lower prices are not the only instrument of expanding it[5].

A separate part of the literature develops quantitative models of pricing to market driven by static market structures and static frictions, built on the Dixit-Stiglitz environment (Alessandria (2009), Atkeson & Burstein (2008), and Corsetti, Dedola & Leduc (2008)). In contrast to this literature, we propose a conceptually different dynamic friction, and explore its unique prediction on the dynamics of prices and quantities to parameterize the strength of the endogenous impediments to price arbitrage that it creates[6].

II. Puzzles

This section sets the quantitative goal for our theory by defining the discrepancy between the predictions of the standard international macroeconomic model[7] and international price data. We use data for both disaggregated prices and aggregate prices. Our aggregate data is based on HP-filtered quarterly price data for the time period 1980 to 2005, and our sample includes the time series for the following countries: Belgium, Australia, Canada, France, Germany, Italy, Japan, the Netherlands, United Kingdom, United States, Sweden, and Switzerland. Our disaggregated data are based on the disaggregated producer and wholesale price data for Japan [8].

A. Export-Import Price Correlation Puzzle

One of the basic predictions of the standard theory for international relative price movements is that the price of the exported goods, evaluated relative to the overall home price level, *comoves negatively* with the similarly constructed import price. Intuitively, this implication follows from the fact that, by law of one price, export prices are tied in the model

[5]This distinction has important consequences. In the 'deep habit' model, with regular productivity shocks, times of persistent real exchange rate depreciation are typically times when future habit is valuable, and hence producers typically cut markups when the real exchange rate depreciates, and raise them in time of real exchange rate appreciation. This gives rise to more than a complete pass-through of exchange rate to export prices. Habit model work well only with a particular formulation of demand shocks developed by Ravn, Schmitt-Grohe & Uribe (2007).

[6]In Drozd & Nosal (2010), we discuss the quantitative performance of these and other leading pricing to market frictions in a unified dynamic business cycle environment. Here, we briefly evaluate the performance of our model vis-à-vis the criteria defined in that paper in Section VI.

[7]Backus, Kehoe & Kydland (1995). See Stockman & Tesar (1995) for a version with non-tradable goods.

[8]Our data sources are described in the online appendix.

to the prices of domestically-produced and domestically-sold goods, and import prices are tied to corresponding prices abroad, expressed in home units. As a result, whenever the real exchange rate depreciates[9], import prices rise relative to home prices due to their direct link to the overall foreign price level. At the same time, export prices fall relative to home prices, as home prices also involve more expensive imported goods. This implication can be easily derived from a simple model with only tradable goods[10], and follows independently from the supply side of the model (and shocks). As shown in Drozd & Nosal (2008), this anomaly can not be explained by non-tradable goods due to insufficient relative price movements between tradable and non-tradable goods in the data.

To contrast this prediction with the data, we measure export and import price by export and import price deflators[11] (P_X, P_M, respectively), and deflate them by all-items CPI index to obtain measures of real prices: $p_x \equiv P_X/CPI$ and $p_m = P_M/CPI$, (different measures of overall price level yield very similar results). Using these constructs, in the data we find exactly the opposite pattern to the prediction of the model. Specifically, across all 12 OECD countries in our sample the correlation of p_x and p_m is positive, covering the range from 0.57 (Australia) to 0.94 (Belgium and Netherlands), with a median of 0.87. Since these prices are also quite volatile, the pattern is robust to any alternative approach to measurement[12].

B. Terms of Trade Relative Volatility Puzzle

The second related prediction of the standard theory is the excess volatility of the terms of trade $p = \frac{P_M}{P_X}$ (price of imports in terms of exports) relative to the real exchange x. In this respect, the standard theory predicts that the terms of trade should be exactly equal to the PPI-based real exchange rate[13], and thus exactly as volatile. The reason is that, by LOP, the price index of exported goods is equal to the home producer price index and the price

[9] An increase in the foreign overall price level relative to the overall home price level.

[10] To see this, note that in a standard model with two tradable goods, approximation of CPI as weighted average of domestic and foreign goods implies ($0 < \omega < 1$) real export and real import price are given by: $p_x = \frac{P_d}{CPI} = \frac{P_d}{P_d^{\omega} P_f^{1-\omega}} = (\frac{P_d}{P_f})^{(1-\omega)}$ and $p_m = \frac{P_f}{CPI} = \frac{P_f}{P_d^{\omega} P_f^{1-\omega}} = (\frac{P_f}{P_d})^{\omega}$, clearly negatively correlated.

[11] Constructed from the time series for constant- and current-price import and export prices at the national level. Formal definitions are stated in the Appendix.

[12] Complete table and a comprehensive set of robustness checks are available in the online appendix.

[13] The PPI-based real exchange rate is the foreign producer price index relative to the home producer price index, when both measured in common numéraire.

index of the imported goods is equal to the foreign country producer price index measured in the home numéraire units. In contrast, in the data export and import prices are highly positively correlated and the terms of trade—defined as their ratio—carries a significantly smaller volatility than the volatility of the CPI based real exchange rates[14]. In our sample of countries, the median volatility of the terms of trade relative to the CPI-based real exchange rate is 0.54 with a range from 0.21 (Sweden) to 0.83 (Germany)[15].

C. Pricing-to-Market Puzzle

In addition to the aggregate anomalies shown above, there is ample direct evidence that the law of one price is systematically violated between countries regardless of the level of disaggregation. This is the basis for our interpretation of the aggregate facts. Here, we document these properties using a sample of the disaggregated price data from the Japanese manufacturing industry. In general, we are not the first ones to analyze these facts. Related contributions which study deviations from law of one price on disaggregated level include Knetter (1993), Goldberg & Knetter (1997), Marston (1990), or more recently, Burstein & Jaimovich (2009) who look at the scanner level wholesale prices[16].

Our dataset here includes quarterly time series for producer/wholesale level price indices for 31 highly disaggregated and highly traded manufactured commodity classifications. For each commodity classification, we have information on the price of the good when exported (export price EPI) and when sold on the domestic market (domestic wholesale price DPI)[17].

To emphasize the analogy to our aggregate analysis, we next construct objects similar to the aggregate real export price indices considered earlier, computed for each commodity classification separately. More specifically, for each commodity i, we divide its export price index by the overall Japanese CPI and use the identity relation $p_x^i \equiv \frac{EPI_i}{DPI_i}\frac{DPI_i}{CPI}$ to decompose

[14]The same conclusions also hold true if we use the PPI-based real exchange rates or the nominal exchange rates. The table with all reported correlations for this exercise is available in the online Appendix.

[15]When we clean the US import price data from the influence of the highly volatile crude oil prices the volatility of the terms of trade relative to the real exchange rate falls below 1/3.

[16]More precisely, replacement cost self-reported by the retailers.

[17]Standard PPI or WPI [wholesale price index] measures would include export prices or import prices, respectively. Price indices used here come from the producer survey data and together account for 59% of the value of Japanese exports and 18% of the value of domestic shipments (as of year 2000). Examples of commodities are: ball bearings, copying machines, silicon wafers, agricultural tractors, etc. For a complete list, see the online appendix.

the fluctuations of the real export price of each commodity into two distinct components: (i) the pricing-to-market term $\frac{EPI_i}{DPI_i}$ – capturing the deviations of the export price of commodity i from its corresponding home price – and (ii) the residual term $\frac{DPI_i}{CPI}$ – capturing the deviations of the home price of commodity i from the overall CPI.

Before we discuss any results pertaining to the above decomposition, we should first note that the commodity-level prices p_x^i exhibit similar patterns as the aggregate data: the median relative volatility of p_x^i to the real exchange rate is as high as 88%, and the median correlation of p_x^i with the real exchange rate is as high as 0.82. With our decomposition at hand, we can now look what happens behind the scene.

Variance driven by pricing-to-market To measure the contribution of the volatility of terms (i) and (ii) above to the volatility of the export price index, we use variance decomposition. Specifically, we evaluate the variance of logged and HP-filtered quarterly time series of each term relative to the sum of their variances (with smoothing parameter $\lambda = 1600$), omitting the covariance terms, as the two terms actually co-vary negatively in the data.

Under LOP, we expect that the first term $\frac{EPI_i}{DPI_i}$ to be almost constant, and all the variation in the real export prices p_x^i to come from the fluctuations of the residual term $\frac{DPI_i}{CPI}$. The data shows the opposite pattern. The pricing-to-market term $\frac{EPI_i}{DPI_i}$ carries about 93% of the total volatility (sum of the variances of the two terms), and the residual term $\frac{DPI_i}{CPI}$ carries only 7%.

Pricing-to-market related to the real exchange rate The data also leaves little ambiguity as to which term drives the high positive correlation of real export prices p_x^i with the real exchange rate (median=0.82). The median correlation of $\frac{EPI_i}{DPI_i}$ with the Japanese real exchange rate is as high as 0.84, and the median correlation of the residual term $\frac{DPI_i}{CPI}$ is actually slightly negative (-0.15).

III. Model

The overall structure of the model is similar to Backus, Kehoe & Kydland (1995) model (BKK hereafter). Time is discrete and horizon T is arbitrarily large. There are two ex-ante symmetric countries labeled *domestic* and *foreign*. Each country is populated by identical households. The only source of uncertainty in the economy are country-specific productivity shocks. Tradable goods are country-specific: d is produced in the domestic country, and f in the foreign country. d and f are traded internationally at the *wholesale* level between *producer* and *retailers*[18]. Retailers resell these goods in a perfectly competitive *retail* market to households.

In terms of notation, we distinguish foreign country-related variables from the domestic ones using an asterisk. The history of shocks $s_i \in S$ up to and including period t is denoted by $s^t = (s_0, s_1, ..., s_t)$, where the initial symmetric realization s_0, the time invariant probability $\mu(s^t)$, as well as the discrete and finite set $S \subset N^2$, are given. In the presentation of the model, whenever possible, we exploit symmetry and present the model from the domestic country's perspective.

A. Uncertainty and Production

Each country is assumed to have access to a constant returns to scale production function $zF(k,l)$ that uses country-specific capital k and labor l, and is subject to a country-specific stochastic technology z following an exogenous AR(1) process, i.e. $\log z(s^t) = \psi \log z(s^{t-1}) + \varepsilon_t$ and $\log z^*(s^t) = \psi \log z^*(s^{t-1}) + \varepsilon_t^*$, where $0 < \psi < 1$ is a common persistence parameter, and $s_t \equiv (\varepsilon_t, \varepsilon_t^*) \in S$ is an i.i.d. discrete random variable with zero mean and compact support.

Since the production function is assumed to be constant returns to scale, we summarize the production process by an economy-wide marginal cost v. Given domestic factor prices w, r and domestic shock z, the marginal cost, equal to per unit cost, is given by:

$$v\left(s^t\right) \equiv \min_{k,l} \left\{ w\left(s^t\right) l + r\left(s^t\right) k \ \text{ subject to } \ z\left(s^t\right) F(k,l) = 1 \right\}. \tag{1}$$

B. Households

Each country is populated by a unit measure of identical, finitely-lived households. In each period, they choose the level of consumption c, investment in physical capital i, labor supply l, purchases of tradable goods d, f, and purchases of a set of one-period s_{t+1}-contingent

[18]Interpretation of retailers in our model should not be confined to the retail sector only. The label is introduced to clearly distinguish the two sides of matching. By retailers we mean all other producers who participate in the overall production process of bringing goods to the consumer. Distribution of the value added across different types of producers is not critical for any of the results.

bonds $b\left(s_{t+1}|s^t\right)$, to maximize the expected discounted lifetime utility

$$U=\sum_{t=0}^{T}\beta^t\sum_{s^t\in S^t}u(c(s^t),l(s^t))\mu(s^t),$$

where u satisfies the usual assumptions and $0<\beta<1$. The preferences over domestic and foreign goods are modeled by the Armington aggregator $G\left(d,f\right)$ with an assumed exogenous elasticity of substitution (Armington elasticity) γ, and an assumed home-bias parameter ω,

$$G\left(d,f\right)=\left(\omega d^{\frac{\gamma-1}{\gamma}}+(1-\omega)f^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}},\ \gamma>0,\ \frac{1}{2}<\omega<1. \tag{2}$$

Households combine goods d and f through the above aggregator into a composite good which they use for consumption and investment, according to the aggregation constraint $c(s^t)+i(s^t)=G(d(s^t),f(s^t))$. Physical capital follows the standard law of motion, $k(s^t)=(1-\delta)k(s^{t-1})+i(s^t)$, with $0<\delta<1$. Asset markets are complete, and the budget constraint of the domestic household is given by

$$\begin{aligned}&P_d\left(s^t\right)d\left(s^t\right)+P_f\left(s^t\right)f\left(s^t\right)+\sum_{s_{t+1}\in S}Q(s_{t+1}|s^t)b(s_{t+1}|s^t)\mu(s^{t+1})\\&=b(s^t)+w\left(s^t\right)l\left(s^t\right)+r\left(s^t\right)k\left(s^{t-1}\right)+\Pi\left(s^t\right),\ \text{all } s^t.\end{aligned} \tag{3}$$

The expenditure side of the budget constraint consists of purchases of domestic and foreign goods and purchases of one-period-forward s_{t+1}-state contingent bonds. The income side consists of income from maturing bonds purchased at history s^{t-1}, labor income, rental income from physical capital, and the dividends paid out by local firms.

The foreign budget constraint is analogous, with the exception of an additional adjustment of the price of bonds given instead by $Q^*(s_{t+1},s^t)\equiv\frac{x(s^{t+1})}{x(s^t)}Q(s_{t+1}|s^t)$. In this equation, $x(s^t)$ is the ideal real exchange rate, which translates foreign units to domestic ones, as implied by the assumptions of integrated world asset market and the composite consumption in each country being the numéraire[19].

[19]Specifically, we assume that he ideal-CPI in each country is normalized to one. The ideal CPI is defined by the lowest cost of acquiring a unit of composite consumption (c in the domestic country, c^* in the foreign

Summarizing, given the initial values of the state variables, households choose their allocations to maximize lifetime utility U subject to the aggregation constraint, the law of motion for physical capital, the budget constraint (3), the standard no–Ponzi scheme condition, and the numéraire normalization. In further analysis, we will use two optimality conditions derived from the household problem: the demand equations linking retail prices to household valuations

$$P_d\left(s^t\right) = G_d\left(s^t\right), \quad P_f\left(s^t\right) = G_f\left(s^t\right), \tag{4}$$

and the standard efficient risk sharing condition,

$$x\left(s^t\right) = x(s_0)\frac{u_c^*\left(s^t\right)}{u_c\left(s^t\right)}, \tag{5}$$

where $u_c\left(s^t\right), G_d\left(s^t\right), G_f\left(s^t\right)$ denote partial derivatives and by ex-ante symmetry $x(s_0) = 1$. Since (5) is known to have counterfactual implications for the properties of the real exchange rate in the model, we later consider alternative formulations of the model and show that none of our results depend on the exact channel generating real exchange rate movements.

C. Producers

Tradable (intermediate) goods d and f are country specific and are produced by a unit measure of atomless competitive producers residing in each country. They employ local capital and labor, and use local technology, which gives rise to production cost given by (1).

The novel feature introduced in this paper is that producers need to first match with the retailers in order to sell their goods. Matching is costly and time consuming, and trade involves bargaining. In the sections that follow, we describe the details of matching and state the profit maximization problem of the producers. We provide a formal treatment of the bargaining problem in a later section, as it is not essential to define the producer problem.

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country). Since the foreign budget constraint is expressed in foreign consumption, and so is foreign b^*, integrated asset markets imply that $Q(s_{t+1}, s^t)^* = x(s^{t+1})Q(s_{t+1}, s^t)/x(s^t)$. In the data, the real exchange rate is measured using fixed-weight CPI rather than ideal CPI indices. Quantitatively, this distinction turns out not to matter in this particular class of models.

List of customers and market shares To match with retailers, the producers have access to an explicitly formulated *marketing technology* and accumulate a form of capital labeled *marketing capital*, m. Marketing capital is accumulated separately in each country a producer sells in, and the marketing capital a producer holds in each country relative to other producers, determines the contact probabilities with the searching retailers. For example, an exporter from the domestic country with marketing capital $m_d^*(s^t)$ in the foreign country attracts a fraction $m_d^*(s^t)/(\bar{m}_d^*(s^t) + \bar{m}_f^*(s^t))$ of the searching retailers from the foreign country, where $\bar{m}_f^*(s^t)$ and $\bar{m}_d^*(s^t)$ denote the average levels of marketing capital an f and d good producer holds in that country.

Formally, given the measure $h(s^t)$ of searching retailers in a given country, who are *potential customers*, the arrival of *new customers* to the customer list of a given producer is given by $h(s^t)m_d^*(s^t)/(\bar{m}_d(s^t) + \bar{m}_f^*(s^t))$. We assume that each match with a retailer is long-lasting and is subject to an exogenous destruction rate δ_H, and thus the evolution of the endogenous list of customers $H_d(s^t)$ is described by the following law of motion:

$$H_d(s^t) = (1 - \delta_H)H_d(s^{t-1}) + \frac{m_d(s^t)}{\bar{m}_d(s^t) + \bar{m}_f(s^t)}h(s^t),\ 0 < \delta_H \leq 1 \tag{6}$$

The size of this list is critical for the producer, as it determines the amount of goods this producer can sell in a given market (country). Specifically, we assume here that in each match, one unit of the good can be traded per period—to reflect the fact that each match is somewhat specific to a particular task at hand[20]. Thus, sales of a given producer cannot exceed the size of the customer list H. For example, the sales constraint[21] of a producer of good d in the foreign country with a customer list $H_d^*(s^t)$ is given by $d^*(s^t) \leq H_d^*(s^t)$.

Marketing capital Producers accumulate marketing capital m to attract searching retailers. Given last period's level of marketing capital $m_d(s^{t-1})$ and the current level of instanta-

[20]One interpretation could be that each match trades a different good, and there is a Dixit-Stiglitz aggregator on the retail level. In such case, the implied capacity constraint would be continuous rather than a discrete zero/one. We can conjecture that the results of the paper would not differ much as long as this capacity constraint would be tight enough—looser/tighter capacity constraints would work similarly to a lower/high value of ϕ. We therefore omit such considerations from the paper.

[21]Due to always positive markups, this condition *always* binds on the simulation path.

neous marketing input $a_d(s^t)$, current period marketing capital $m_d(s^t)$ is given by

$$m_d(s^t) = (1-\delta_m)\, m_d(s^{t-1}) + a_d(s^t) - \frac{\phi}{2} m_d\left(s^{t-1}\right)\left(\frac{a_d\left(s^t\right)}{m_d\left(s^{t-1}\right)} - \delta_m\right)^2. \qquad (7)$$

The above specification nests two key features: (i) decreasing returns from the instantaneous marketing input $a_d(s^t)$ and (ii) capital-theoretic specification of marketing. These features, parameterized by the market expansion friction parameter ϕ and depreciation rate δ_m, are intended to capture the idea that marketing-related assets like brand awareness, reputation or distribution network are capital for a firm and their buildup takes time. While here we consider a simple implementation, one can think more generally about the friction of expanding market shares as arising from a process in which firms are bidding for a share in a finite capacity communication channel.

Profit maximization Producers sell goods in the domestic country for the wholesale price p_d and in the foreign country for the wholesale export price $p_x \equiv xp_d^*$, measured in domestic numéraire. These prices are determined by bargaining with the domestic and foreign retailers. However, to set up the profit maximization of the producers, we can abstract from bargaining at this stage and assume that the prices are taken as given[22].

The instantaneous profit function Π of the producer is determined by the difference between the profit from sales in each market and the total cost of marketing, i.e. $\Pi = (p_d - v)d + (xp_d^* - v)d^* - vA_d(s^t)a_d - xv^*A_d^*(s^t)a_d^*$. The state-dependent input requirements that show up in the formula, A_d and A_d^*, are introduced only for the sake of later analytical characterization of the model. Unless explicitly noted, we assume $A_d(s^t) = A_d^*(s^t) = 1$ (all s^t).

Given the instantaneous profit function Π, the representative producer from the domestic country, who enters period t in state s^t with the customer lists $H_d(s^{t-1})$, $H_d^*(s^{t-1})$ and

[22] This is because the producer can perfectly anticipate the outcome of bargaining at every contingency, and cannot strategically influence it beforehand by making a different choice—as we will see later, neither the state variables nor decision variables chosen in the problem below affect the outcome of bargaining. This property follows from the 3 key assumptions of the model: (i) production, marketing and search are all constant returns to scale activities, (ii) atomistic agents, (iii) expensed search cost and marketing cost cannot be retrieved by breaking a match.

marketing capitals $m_d(s^{t-1})$, $m_d^*(s^{t-1})$, chooses the allocation $a_d(s^t)$, $a_d^*(s^t)$, $m_d(s^t)$, $m_d^*(s^t)$, $d(s^t)$, $d^*(s^t)$, $H_d(s^t)$, $H_d^*(s^t)$, to maximize the present discounted stream of future profits given by $\sum_{\tau=t}^{T} \int Q(s^\tau)\Pi(s^\tau)\,\mu(ds^\tau|s^t)$, subject to the law of motion for marketing capital, sales constraints ($d(s^t) \leq H_d(s^t), d^*(s^t) \leq H_d^*(s^t)$), and the laws of motion for customer lists (6). The discount factor $Q(s^t)$ is defined by the recursion on the conditional pricing kernel derived from the household's problem, $Q(s^t) = Q(s_t|s^{t-1})Q(s^{t-1})$.

D. Retailers

In each country there is a sector of atomless retailers who purchase goods from producers and resell them in a local competitive market to households. It is assumed that new retailers who enter into the market must incur an initial search cost χv in order to find a producer with whom they can match and trade (or specialize in a task useful for this particular producer). Each match lasts until it exogenously dissolves with a per-period probability δ_H. As long as the match lasts, the producer and the retailer have an option to trade one unit of the good per period[23]. In equilibrium, the industry dynamics is governed by a free entry and exit condition, which endogenously determines the measure h of new entrants (searching retailers). Trade between households and retailers takes place in a local competitive market at prices P_d for good d and P_f for good f. In equilibrium, these prices are given by (4), and throughout the paper we refer to them as *retail prices*.

In each period, there is a mass of retailers already matched with the producers H and a mass of new entrants h (searching retailers). A new entrant, upon paying the up-front search cost χv, meets with probability π a producer from the domestic country and with probability $1 - \pi$ the producer from the foreign country (selling in the domestic country). The entrant takes this probability as given, but in equilibrium it is determined by the marketing capital levels accumulated by the producers, according to

$$\pi(s^t) = \frac{\bar{m}_d(s^t)}{\bar{m}_d(s^t) + \bar{m}_f(s^t)}. \tag{8}$$

[23]One can more generally think of each match as effectively providing a different type of intermediate good with a low elasticity of substitution. d is then an integral over all matches. The link between exchange rate and prices will be qualitatively robust to this modification—albeit not as tractable as our formulation.

The measures of matched retailers H evolve in consistency with (6).

As is clear the above formulation of the matching process, search by the retailers is guided directly only by the marketing capital accumulated by the producers. Thus, in our model, prices and the anticipated surplus from trade with each type of producer, only indirectly influence the basket of goods which is consumed on the aggregate level[24]. This a central feature of our model environment, and a key departure from the standard models.

Bargaining and wholesale prices We assume that each retailer bargains with the producer over the total future surplus from a given match. This surplus is split in consistency with Nash bargaining solution with continual renegotiation. Nash bargaining as a surplus splitting rule is an important assumption for our results, but not the only modeling option delivering our results. Any departures from our setup can be mapped onto a time-varying Nash bargaining power, and as long as its variation is independent from exchange rate movements or limited in size, our results would largely remain unchanged.

To set the stage for the bargaining problem, we first need to define the value functions from the match for the producer and for the retailer. We assume that they trade at history s^t at some arbitrary wholesale price p, and in the future they will trade according to an equilibrium price schedule $p(s^t)$. The value functions are

$$W_d\left(p; s^t\right) = \max\left\{0,\, p - v\left(s^t\right)\right\} + \left(1-\delta_h\right) E_t Q\left(s_{t+1}|s^t\right) W_d\left(p_d(s^{t+1}); s^{t+1}\right), \tag{9}$$

$$J_d\left(p; s^t\right) = \max\left\{0,\, P_d\left(s^t\right) - p\right\} + \left(1-\delta_h\right) E_t Q\left(s_{t+1}|s^t\right) J_d\left(p_d(s^{t+1}); s^{t+1}\right), \tag{10}$$

where W_d is the value of the domestic producer selling in the domestic country and J_d is the value for the domestic retailer matched with a domestic producer.

[24] Under some assumptions this disconnect would not matter. Two other features of the model may potentially render it relevant: (1) long-lasting matches modeled by $\delta_H < 1$ and (2) market-share adjustment friction, $\phi > 0$. In our quantitative specification, most action will come from (2). However, it should be noted that (1) with low enough depreciation of customer base and *directed search* of retailers can also give rise to similar dynamics of prices, but in an environment like ours it is infeasible to solve (as it requires global solution methods due to corner $h = 0$). The intuition is that when matches are expected to be persistent, even if search of retailers can be directed, retailers' search intensity will depend on the present discounted value of future surpluses, and not only on the current surplus. This will generate PTM (unless shocks are permanent). The view of downplaying current prices in long-lasting partnerships is consistent with the anecdotal evidence in Egan & Mody (1992).

The flow part of the above Bellman equation for the producer is determined by the difference between the wholesale price of the good, p, and the production cost, v, whereas for the retailer, it is determined by the difference between the retail price (resell price) of the good P_d and the wholesale price paid to the producer p. These equations additionally imply that markups will be necessarily bounded below by zero[25] (as no trade is an option in any given period).

With the values from a match at hand, we are now ready to set up the Nash bargaining problem, which imposes the following restriction on the equilibrium schedule of the wholesale prices[26] $p(s^t)$, given bargaining power $0 < \theta < 1$,

$$p_f(s^t) \in \arg\max_p \{J_f(p;s^t)^\theta W_f\left(p;s^t\right)^{1-\theta}\}, \text{ all } s^t. \tag{11}$$

Note that the threat-points of both sides are zero by three assumptions of the model: (i) search cost and marketing cost cannot be retrieved by breaking the match, (ii) there is free entry and exit to retail sector (zero profit condition), (iii) production, marketing and search are all constant returns to scale activities.

The proposition below establishes that under continual renegotiation, the price schedule resulting from (11) allocates θ fraction of the total instantaneous (static) trade surplus given by $P_f - xv^*$ to the producer and fraction $1-\theta$ to the retailer. Intuitively, this follows from the fact that since it is impossible to split the trade surplus from tomorrow onward in any other proportion than θ and $1-\theta$, the *static* surplus today must be split the same way.

PROPOSITION 1. *Assume that trade takes place at s^t. Then the solution to the bargaining problem stated in (11) is given by*

$$\begin{aligned} p_d(s^t) &= \theta P_d(s^t) + (1-\theta)v(s^t), \\ p_x(s^t) &\equiv x(s^t)p_d^*(s^t) = \theta x(s^t)P_d^*(s^t) + (1-\theta)v(s^t). \end{aligned} \tag{12}$$

[25] On the simulation patch markups never hit zero, but for some parameterization this may be the case (featuring low steady state markups and large shocks).

[26] Other prices are defined by analogy.

Proof. Add both sides of (9) and (10) to obtain a Bellman equation for total surplus $S \equiv W + J$. Multiply both sides of this Bellman equation by θ, and then subtract from both sides of it equation (9). By bargaining, note $W = \theta S$ at each state, and hence (12) follows.

Free entry and exit condition Free entry and exit into the retail sector governs the measures of searching retailers in each country h. It relates the expected surplus for the retailer from matching with a producer from the domestic or the foreign country to the search cost incurred to identify a match opportunity, i.e. $\pi(s^t)J_d(p_d(s^t); s^t) + (1 - \pi(s^t))J_f(p_d(s^t); s^t) \leq \chi(s^t)v(s^t)$, with the condition holding with equality whenever $h > 0$. The state-dependent search cost $\chi(s^t)$, assumed uniformly bounded away from zero, is introduced here only for the sake of the analytical characterization of the sources of deviations from LOP in the model. Again, as with marketing input requirement A, unless explicitly noted, we maintain the assumption $\chi(s^t) = \chi > 0$.

E. Feasibility

Equilibrium must satisfy several market clearing conditions and feasibility constraints. The aggregate resource constraint is given by

$$(13)\quad d(s^t) + d^*(s^t) + A_d(s^t)a_d(s^t) + A_f(s^t)a_f(s^t) + h(s^t)\chi(s^t) \leq z(s^t)F(k(s^{t-1}), l(s^t)),\ \text{all } s^t.$$

By representativeness, all producers and retailers choose the same allocation. The aggregate levels of marketing capital are thus given by $\bar{m}_f(s^t) = m_f(s^t) + \zeta$ and $\bar{m}_d(s^t) = m_d(s^t) + \zeta$ for all s^t, where $\zeta > 0$ is an arbitrarily small constant that renders $\pi(s^t)$ well-defined even for $\bar{m} = 0$ (needed for formal proof of existence). Finally, the contact probability $\pi(s^t)$ is consistent with the average relative marketing capital accumulated by the producers of each type, according to (8), and the world asset market clears, $b(s^t) + x\,(s^t)\,b^*\,(s^t) = 0$.

F. Equilibrium

Equilibrium of this economy is, for all histories, an allocation for the domestic country d, f, a_d, a_f, m_d, m_f, H_d, H_f, c, l, b, i, k, an analogous allocation for the foreign country, prices in the domestic country P_d, P_f, p_d, p_f, v, Q, analogous prices in the foreign country, meeting probabilities π and π^*, the aggregates $\bar{m}_d, \bar{m}_d^*$ and $\bar{m}_f, \bar{m}_f^*$, and the real exchange rate x, such the allocation satisfies the feasibility conditions, and, given prices, the allocation solves the household problem, producer problem, and satisfies the retailer zero profit condition. This equilibrium exists:

PROPOSITION 2. *Under standard neoclassical assumptions on functional forms, and initial conditions guaranteeing non-emptiness of the allocation set, the existence of equilibrium is warranted. (Details in online appendix.)*

IV. Analytical Results

In order to gain intuition on the pricing predictions of our theory, in this section we explore analytically the determinants of the international price differential $xp_d^* - p_d$ in our setup. Since, by bargaining, this key measure of pricing to market is hardwired in our setup to a similar differential on the retail level,

$$xp_d^* - p_d = \theta(xP_d^* - P_d), \tag{14}$$

in what follows, we focus on the analysis of $xP_d^* - P_d$, and refer to the fluctuations of this object as deviations from the law of one price broadly defined.

We establish two sets of results as far as the dynamics of $xP_d^* - P_d$ is concerned. First, using a setup with market expansion friction ($\phi > 0$) and by introducing an auxiliary notion of *law of one price for marketing and search cost*, we provide a set of necessary conditions for deviations from LOP (i.e. $xP_d^* \neq P_d$) in the benchmark model. Second, we characterize the force which plays quantitatively a dominant role (80% of deviations) in the benchmark model —the market expansion friction. In order to do so, we shut down the residual source of deviations from LOP identified in the next section (LOP for marketing and search cost).

A. Sources of Deviations from the Law of One Price

As our first step, we derive the set of conditions under which LOP holds in the model. To this end, we introduce an auxiliary notion of the *law of one price for marketing and search cost*, as defined below. This notion assures that the price of marketing investment and the cost of search are identical across countries. Our main result is summarized in Proposition 3 below. It states that any deviations from LOP in the model comes either from (i) international differences in marketing and search cost, or (ii) the market expansion friction ($\phi > 0$). In our analysis, we determine that the second force is the quantitatively dominant one. Specifically, in the quantitative section, we establish that as much as 80% of the overall deviations from LOP are driven by the market expansion friction.

DEFINITION 1. *The* ***law of one price for marketing and search costs*** *holds iff*

$$\chi(s^t)v(s^t) = \chi^*(s^t)x(s^t)v^*(s^t) \text{ and} \tag{15}$$
$$A_d(s^t)v(s^t) = A_f(s^t)v(s^t) = A_d^*(s^t)x(s^t)v^*(s^t) = A_f^*(s^t)x(s^t)v^*(s^t).$$

PROPOSITION 3. *Suppose (15) holds and $\phi = 0$. Then, law of one price (LOP) holds in the benchmark model.*

Proof. In the appendix.

In what follows next, we turn to the analysis of the sole effect of the market expansion friction, the dominant force in the model, by assuming that the Definition 1 is satisfied.

B. Effects of Market Expansion Friction

To expose the mechanism through which market expansion friction $\phi > 0$ leads to deviations from LOP, here we consider a simplified analytic version of our setup by making the following changes, which are innocuous for the results on prices (but not quantities, see comparison of *Benchmark* model to *Static Friction* in tables with results): (A1) we dispense with physical capital accumulation and labor-leisure choice from the household problem (labor the only input); (A2) we assume (15) holds[27] to isolate the sole effect of ϕ, as implied by Proposition 3; (A3) we simplify the capital theoretic formulation of marketing capital by replacing (7) with a static one[28]

$$m_i = a_i - \frac{\phi a_i^s (a_i/a_i^s - 1)^2}{2}, i = d, f, \tag{16}$$

[27]For example, consider: $A_d(s^t) = \frac{xv^*+v}{2}$ and $\chi(s^t) = \chi\frac{xv^*+v}{2v}$, and combine it with (15).

[28]In terms of analytics, this simpler formulation of market expansion friction emulates the crucial properties of benchmark specification, but assumes away an analytically intractable dynamic link between current marketing expenditures and the future cost of market share expansion. We choose capital theoretic formulation in benchmark model as it allows us to more naturally relate market expansion friction to the elasticity puzzle.

where a_i^s is assumed to be the deterministic steady state value of a_i.

The two central results of this section are stated in Propositions 4 and 5 below. Proposition 4 establishes that, in this setup, whenever the real exchange rate changes (regardless the source of the change), it must necessarily imply deviations from LOP, i.e. firms price to market in which they sell. Proposition 5 completes this result by linking the real exchange rate movements in our model to fundamental driver of uncertainty: the productivity shocks. However, given the uncertainty about the channel generating real exchange rate fluctuations in international economics, we view Proposition 4 as central. It shows that pricing to market occurs independently from the exact forces driving the real exchange rate.

The results in Propositions 4 and 5 essentially follow from a technical result summarized in Lemma 1. It states that, to a first-order approximation, any difference between retailer's valuations between the two markets are linked to market share adjustment. The important implication of Lemma 1, used repeatedly in the proofs of Proposition 4 and 5, is summarized in Corollary 1. It states that in the presence of market expansion friction, the law of one price is only consistent with market shares being constant across all dates and states. To see why this allows us to establish the results listed in Proposition 4 and 5, note that when the real exchange rate moves, for LOP to hold, market shares across all dates and states must be constant (implied by Corollary 1). However, constant market shares, by linearization of (4), imply that all retail prices in local units (i.e. P_d and P_d^*) are constant, leading to a immediate contradiction. This is because $P_d = P_d^*$ (all s^t), and so in the presence of real exchange rate fluctuations, we must have: $P_d \neq xP_d^*$ (in some s^t).

LEMMA 1. *Under A1-A3, producer optimization and retailer zero profit condition together imply:*

$$\hat{\mathcal{P}}_d^*(s^t) - \hat{\mathcal{P}}_d(s^t) = \phi \times \frac{\chi + (1-\theta)}{\chi(1-(1-\delta_H)\beta)} \times \left[\widehat{1-\pi^*}(s^t) - \widehat{\pi}(s^t)\right], \tag{17}$$

where $\widehat{}$ *denotes log deviation from the deterministic steady state, and* $\mathcal{P}_d, \mathcal{P}_d^*$ *are given by*

$$\mathcal{P}_d(s^t) = P_d(s^t) + (1-\delta_H)E\left[Q(s_{t+1}|s^t)\mathcal{P}_d(s^{t+1})\right], \tag{18}$$

$$\mathcal{P}_d^*(s^t) = x(s^t)P_d^*(s^t) + (1-\delta_H)E\left[Q(s_{t+1}|s^t)\mathcal{P}_d^*(s^{t+1})\right], \tag{19}$$

Proof. In the appendix.

COROLLARY 1. *In the presence of home-bias in trade in the steady state (i.e. $\pi^s > 1/2$),* $\widehat{xP_d^*}(s^t) = \widehat{P_d}(s^t)$ **iff** $\hat{\pi}(s^t) = \widehat{\pi}^*(s^t) = 0$.

Proof. In the appendix.

PROPOSITION 4. *Under A1-A3, and in the presence of home-bias in trade, the real exchange rate fluctuations lead to deviations from LOP.*

Proof. In the appendix.

PROPOSITION 5. *Under A1-A3, and in the presence of home-bias, the equilibrium response to a relative productivity shock $z \neq z^*$, to a first order approximation, leads to real exchange rate fluctuations, both under perfect risk sharing (benchmark case) and under financial autarky.*

Proof. In the appendix.

The intuitive content of these results is fairly straightforward. Each firm in our model effectively faces an increasing marginal cost of matching with an additional buyer in any given market, and so it expands its market share only when the surplus from trading is higher in one market versus the other (Lemma 1). As a result, after real exchange rate depreciation or appreciation, a price differential between markets necessarily arises and is sustained in the short- and the medium-run by the rising cost of matching.

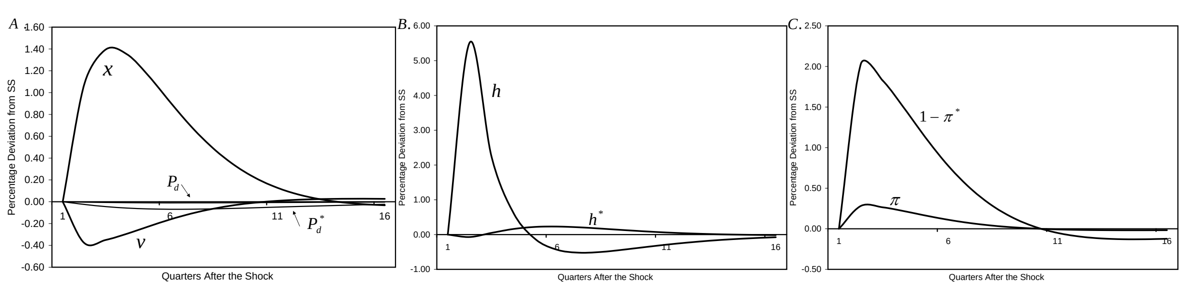

Figure 1: Impulse response in a simplified economy.

To complete the analysis of this static setup, in Figure 1, we present the responses of prices following a 1% positive productivity shocks in the domestic country. The parameterization is described in the next section, and it is not particularly relevant at this point, except for one key feature: We choose a high value of elasticity γ, and set $\phi > 0$, to make the model consistent with the high long-run and the low short-run trade elasticity estimates.

As is clear from panel A in Figure 1, the main predictions are consistent with our analytical results presented earlier. The key implication of the model is that the shock results in real exchange rate depreciation, and due to a muted offsetting response of the local retail prices P_d^*, P_d, it drives a wedge between retailer valuations $xP_d^* > P_d$. The muted response of retail prices is a critical element of our model, as it allows for the market share volatility in the model to be consistent with the one in the data (i.e., the low short-run trade elasticity). In fact, as we can see from panels A and C, the response of import share in this case is close to the response of the real exchange rate to the shock. Factoring in how real exchange rate translates into import price movements, this makes our model consistent the estimates of the short-run trade elasticity in the range of 0.5-1.0.

To understand the intuition behind these responses, consider the sequence of the events that follows the productivity shock. From panel A, observe that the domestic production cost v falls, which, *ceteris paribus*, must raise the surplus from trading the domestic good in matches, i.e. S_d and S_d^* both go up—at this point symmetrically. Second, due to home-bias in trade ($\pi > 1/2$), relative to the foreign country retailers, the domestic country retailers have an advantage in meeting the domestic country producers. Not surprisingly, this creates a gap in the relative search intensity $\widehat{h} - \widehat{h}^* > 0$ seen in panel B, which leads to the real exchange rate depreciation visible in panel A (this last implication follows from risk sharing condition). The price differential created by that feeds into wholesale prices by 14 and creates an incentive for the domestic country producers to expand their market share abroad more than at home (see panel C). However, as implied by Lemma 1, in the short- and medium-run the adjustment of market shares will not close the initial price gap completely due to increasing cost of matching. As a result, the equilibrium response of prices features persistent deviations from LOP in our model.

V. Quantitative Results

We now proceed with the presentation of our main quantitative findings.

A. Parameterization

The two key parameters in our model are the elasticity of substitution γ and the marketing friction parameter ϕ. We first describe the methodology we use to choose these parameters, and then proceed with the description of the remaining targets and parameters. We distinguish here between a *Benchmark:* $\theta = 1/2$ and *Benchmark: Fitted* θ cases, depending on the method of calibrating the bargaining power θ. For the *Static Friction* case (assumptions A1-A3), which we report to give a feel of the importance of the static friction forces described in the Analytical Results section for the quantitative model, we use the parameter for long-run friction from the *Benchmark:* $\theta = 1/2$ case, and parameterize the rest according to the method described below.

Elasticity puzzle and the choice of ϕ and γ To choose the elasticity of substitution and the marketing friction parameter, we use the fact that our model with *dynamic friction* specification has different predictions for the long-run and the short-run response of imports to the relative price fluctuations. Evidence of a similar discrepancy has been documented in the data and is termed the *elasticity puzzle* in the literature.[29] Below we show how we use long-run and short-run measurements to set calibration targets for these two key parameters.

Long-run elasticity measurement In our model, when the adjustments of quantities are extended in time, it can be shown that the response of the import ratio $\frac{f}{d}$ to the relative price of the domestic good d to the foreign good f is equal to the elasticity γ. That is, just as in the frictionless models underlying the measurement of the so called long-run elasticity, for an underlying log change in the tariff rate of ΔT, we have that[30] $\Delta \log \frac{f}{d} \approx -\gamma \Delta T$.

Intuitively, in the long-run[31], the market expansion friction is slack, and thus the response of trade to tariff change depends solely on the intrinsic elasticity of substitution between the domestic and the foreign goods. In terms of the values of this elasticity in the data, the estimates in the literature range from 6 to about 16. We adopt a middle-of-the-pack number

[29] See, for example, Ruhl (2008) for a detailed discussion of this puzzle and an overview of the literature.

[30] We derive this equation in the technical appendix available online.

[31] Due to the market expansion friction, the response of trade to a trade liberalization, instantaneous in the frictionless model, takes multiple quarters in our setup. In response to a complete removal of a 1% symmetric tariff, half of the adjustment is completed in about 20 quarters. For details, see technical appendix.

of 7.9, reported by Head & Ries (2001)[32]

Short-run elasticity measurement Over the business cycle, the adjustment of trade flows in response to prices is dampened in our model. This is because in the short-run, the market expansion friction limits the instantaneous response of quantities to price fluctuations. Since a similar discrepancy has been identified in the data and our model can replicate it, we use it to quantitatively discipline the value of the market expansion friction parameter ϕ.

To this end, we use our own measurement of the short-run elasticity estimated from the aggregate time series (defining feature of this concept). Specifically, we compute the business cycle volatility[33] of the ratio of imports to domestic absorption of domestic good ($\approx \frac{f}{d}$ in the model) relative to the volatility of the ratio of the underlying price deflators ($\approx \frac{p_d}{p_f}$ in the model). We label the ratio of these volatilities the *Volatility Ratio*[34] and compute it for a cross-section of 12 major OECD countries[35]

This methodology of measuring short-run elasticity is motivated by the fact that in a large class of models, the demand for domestic and foreign good is modeled by a CES aggregator (2). In such case, it is straightforward to show that the import ratio is tied to the relative price of domestic and imported goods by $\log \frac{f_t}{d_t} = \gamma \log \frac{p_{d,t}}{p_{f,t}} + \log \frac{\omega_t}{1-\omega_t}$. Under normal conditions, i.e., when the supply curve is an upward-sloping function of the price and the supply shocks are not correlated with the ω_t-demand shocks, we should expect to see a positive correlation

[32]Other long-run oriented studies give similar estimates. See, for example, Hummels (2001) or Eaton & Kortum (2002).

[33]We consider also linear detrending for robustness. Results are the same for HP filter and linear detrending.

[34]To construct the *volatility ratio*, we use series on constant and current price values of imports and domestic absorption, where domestic absorption is defined by the sum of domestic expenditures less imports, $DA = (C+G)+I-IM$. We then identify the prices of imports and domestic absorption with the corresponding price deflators (defined as the ratio of current to constant price values). Denoting the deflator price of domestic absorption of good d by P_{DA} and the deflator price of imports by P_{IM}, the *volatility ratio* is then defined as $\sigma(\frac{IM}{DA})/\sigma(\frac{P_{DA}}{P_{IM}})$, where σ refers to the standard deviation of the logged and H-P-filtered quarterly time series (we consider also linear detrending and results are the same). Note that our *volatility ratio* places an upper bound on the regression coefficient between the two variables underlying its construction. The regression coefficient, typically used in short-run studies, is the *volatility ratio* rescaled by the correlation coefficient ρ ($\text{reg}(x,y) = \frac{\sigma_y}{\sigma_x}\rho_{x,y}$.)

[35]Countries included in the sample are: Australia, Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Switzerland, Sweden, UK and the U.S.

between $\log \frac{\omega_t}{1-\omega_t}$ and $\log \frac{p_{d,t}}{p_{f,t}}$. Then, the *volatility ratio* defined by

$$VR \equiv \sigma(\log \frac{f_t}{d_t})/\sigma(\log \frac{p_{d,t}}{p_{f,t}}) \quad (20)$$

places an upper bound on the value[36] of the intrinsic price elasticity of trade flows γ. Clearly, for our purposes the upper bound estimate is sufficient.

The computed values of the volatility ratio in our sample range from 0.44 (the Netherlands) to 1.27 (Canada), with a median of 0.71[37], which confirms the low values of the short-run price elasticity of trade flows typically found in the literature.[38] In the model, we use a value of 0.71 as a target for the market expansion parameter ϕ, which, as we describe below, is determined jointly with other parameters.

Choice of Parameter Values and Functional Forms Next, we describe in detail the choice of the functional forms and parameter values[39]. We report our choices in Table 1.

We assume a Cobb-Douglas production function, $F(k,l) = k^\alpha l^{1-\alpha}$, and a constant relative risk aversion (CRRA) utility function, $u(c,l) = \frac{(c^\eta(1-l)^{1-\eta})^{1-\sigma}}{1-\sigma}$ ($\sigma > 0, 0 \leq \eta \leq 1$).

Consider first the parameters that can be selected independently from all other parameters by targeting a single moment from the data. This group includes: (i) the discount factor β, (ii) capital share parameter α, (iii) depreciation rate of physical capital δ, and (iv) Armington elasticity γ. We choose standard value of β to give the average annual risk-free real interest rate of 4%, and a standard value of α to match the constant share of labor income in GDP of 64%. We follow BKK and choose the value of δ to target the investment to GDP ratio of 25%.[40] Following the business cycle literature, we choose the value of σ equal to 2. Finally, as explained in the previous subsection, we choose the value of γ equal to 7.9. The parameter

[36]To see this, evaluate the previous equation to get $\gamma = \sigma(\log \frac{f_t}{d_t})/\sigma(\log \frac{p_{d,t}}{p_{f,t}} + \frac{1}{\gamma}\log \frac{\omega_t}{1-\omega_t}) \leq \sigma(\log \frac{f_t}{d_t})/\sigma(\log \frac{p_{d,t}}{p_{f,t}}) = VR$.

[37]For linearly detrended data, the median is 0.73, with a range 0.43 to 1.16. The table with moments for all countries is available in an online Appendix.

[38]E.g., Blonigen & Wilson (1999) or Reinert & Roland-Holst (1992). In contrast to our approach, this literature uses disaggregated data and regression analysis.

[39]The exercises: *BKK* and *Convex Friction* have been parameterized analogously, whenever applicable.

[40]In the updated data we find a slightly smaller ratio. For example, 20% in the United States, 28% in Japan, 22% in Germany, and 21% in France. The OECD median is close to 20%. We adopt a bit higher number to make the model comparable to the results documented in the literature.

Table 1: Parameter Values in the Model Economies.

Model	Parameter Values (if different than Benchmark: $\theta = 1/2$)
Benchmark: $\theta = 1/2$	Preferences: $\beta = 0.99,\ \sigma = 2.0,\ \eta = 0.3385,\ \gamma = 7.9,\ \omega = 0.562$; Technology: $\alpha = 0.36,\ \delta = 0.025\ \theta = 0.5,\ \chi = 0.92,\ \delta_m = 0.098,\ \delta_h = 0.1,\ \phi = 33.4$; Productivity: $\psi = 0.78,\ \sigma^2(\epsilon) = 0.00007,\ corr(\epsilon, \epsilon^*) = 0.21$
Benchmark: Fitted θ	Preferences: $\eta = 0.3384,\ \omega = 0.5605$; Technology: $\theta = 0.3825,\ \chi = 1.48,\ \delta_m = 0.2,\ \phi = 31.2$; Productivity: $\psi = 0.74,\ \sigma^2(\epsilon) = 0.000093,\ corr(\epsilon, \epsilon^*) = 0.21$
Static Friction	Technology: $\chi = 0.95\ \phi = 1.83$; Productivity: $\psi = 0.82,\ \sigma^2(\epsilon) = 0.000069,\ corr(\epsilon, \epsilon^*) = 0.28$
BKK	Preferences: $\gamma = 0.71,\ \eta = 0.3354,\ \omega = 0.943$; Productivity: $\psi = 0.91,\ \sigma^2(\epsilon) = 0.000037,\ corr(\epsilon, \epsilon^*) = 0.28$
Convex Friction	Preferences: $\gamma = 1.5,\ \eta = 0.3354,\ \omega = 0.79,\ \phi = 95.0$; Productivity: $\psi = 0.91,\ \sigma^2(\epsilon) = 0.000037,\ corr(\epsilon, \epsilon^*) = 0.28$

δ_H is arbitrarily chosen to be equal to 0.1 —implying that the matches in the economy last on average 2.5 years (10 quarters). In the technical appendix we present sensitivity analysis that shows that this parameter, except for increasing the persistence of prices, has a negligible effect on the results.

The remaining parameters (except for θ, discussed below) need to be jointly determined because there is no one-to-one mapping between their values and moments in the data. This group includes: (i) the marketing friction parameter ϕ, (ii) the up-front search cost χ, (iii) depreciation of marketing capital δ_m, (iv) the home-bias ω, and (v) the consumption share parameter η. We choose the values of these parameters to target jointly the following moments: (i) volatility ratio of 0.71 (ii) producer markups of 10% as estimated by Basu & Fernald (1997), (iii) standard value for the share of market activities in total time endowment of households equal to 30%, (iv) imports to GDP ratio of 12% (U.S. data 1980–2004), and finally, (v) the share of marketing expenditures to sales on the industry level of 7% (dropped[41] in *Static Friction* case due to lack of δ_m), as reported by Lilien & Little (1976) (also Lilien & Weinstein (1984)), and (vi) moments of the productivity process as discussed in the next paragraph.

[41]In the *Static Friction* case, marketing relative to GDP is equal to 7.61 %

In terms of our choice for the bargaining power θ, for the *Benchmark:* $\theta = 1/2$ and *Static Friction* cases, we take a parsimonious approach and select the value of the bargaining parameter equal to $\frac{1}{2}$, by which we make sure that the key bargaining power is selected independently of any price statistics. For the *Benchmark: fitted* θ exercise, to demonstrate the ability of the model to match the price statistics very closely, we add an extra target to the ones described above: the relative volatility of the real export price p_x to the real exchange rate x of 37% (U.S. data 1980–2004). This improves the fit of the model in terms of the relative volatilities of the terms of trade and the import price. In the online appendix, we establish that this parameterization is unique given the values of θ and ϕ and the data targets.

Productivity process We follow a procedure similar to Heathcote & Perri (2004) to back out the total factor productivity (TFP) residuals z from the data. However, because the model-implied TFP residuals are different from the assumed ones,[42] we modify the correlation and volatility of the assumed disturbances $\varepsilon, \varepsilon^*$, and the AR(1) persistence parameter so that the model implied residuals match the following targets from the data: (i) volatility of model-generated TFP residuals of 0.79%, (ii) the correlation of model-generated TFP residuals of 0.3, and (iii) autoregressive coefficient of 0.91. The exact values of parameters used in the model economies are reported in Table 1.

B. Findings

In this section, we confront our model's quantitative predictions with the data. We identify the United States with the domestic country and the aggregate of 18 major OECD countries with the foreign country.[43] Unless otherwise noted, all reported statistics are based on logged and HP-filtered quarterly time series. The standard model (BKK model), with which we contrast our results, has been parameterized analogously whenever applicable. Table 1 reports parameter choices[44].

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[42]Marketing expenditures are not treated as investment in national accounts, which is reflected in measured TFP.

[43]In the online Appendix, we describe how we map actual national accounting procedures onto our model economy and give a detailed list of countries used.

[44]We solve the models using perturbation methods.

International prices Table 2 reports the business cycle statistics for international prices. It shows that both versions of the benchmark model successfully account for the aggregate patterns discussed in Section II.: (i) real export and real import prices are *positively* correlated (and positively correlated with the real exchange rate), (ii) the volatility of the terms of trade is lower than volatility of the real exchange rate (for the *Fitted* θ case, closely matching the value of 27% for U.S. data after cleaning import price data from the influence of volatile fuel prices[45]) and (iii) producers price-to-market to which they sell—the relative price $\frac{p_x}{p_d}$ is no longer constant and comoves positively with the real exchange rate. None of these features are reproduced by the BKK model. The implied persistence of both the real exchange rate is and of the term p_x/p_d measuring deviations from LOP is 0.94 for the benchmark specifications (both computed on a series of the length of the data)—close to our data estimate of persistence of the real exchange rate of 0.93.

The last row of the Table 2 presents the contribution of the deviations from LOP for marketing and search cost to the model's pricing to market predictions (variable LOPD). This contribution is computed by constructing a counterfactual series $xP_d^* - P_d$ (using (A1) and (A2) from the Appendix), under the assumption that aggregate prices v, v^* and x are taken from the simulation path of the benchmark model (with $\phi > 0$). The ratio of the volatility of the counterfactual series to the actual one is then reported as LOPD in the table[46]. As we can see, in both versions of the benchmark model, sluggish market shares play a dominant role, with the relative volatility of the distribution services cost component equal to 20% and 26% in the $\theta = 1/2$ and *Fitted* θ cases.

[45] To arrive at this estimate, we use the indices of export and import prices disaggregated to a one-digit SITC level by the BLS. We next remove from both indices the classification SITC-3 (fuels). We then measure by how much it reduces the standard deviation of the logged and H-P-filtered overall terms of trade $(1983-2005)$ constructed from the BLS price indices. The result is that the volatility of terms of trade falls from about 1.94% with fuels to about 1.32% without fuels. We next obtain the non-fuel statistics for the United States by multiplying the volatility of the terms of trade measured from the deflator prices of exports and imports (as in Table 2) by the correcting ratio derived from the BLS data: $1.32/1.94 \approx 0.68$. A slightly larger estimate of about 35% would be obtained from the BLS data directly (the BLS estimate refers to a fixed weight index, not a deflator price).

[46] In other words, this exercise asks what the pricing behavior of (atomless) producers would be if they were presented a sequence of aggregate prices from the benchmark model, but counterfactually faced $\phi = 0$. By Proposition 3, any pricing to market implied by this procedure must thus be attributed to the violation of the law of one price for marketing and search cost, as the only other channel, $\phi > 0$, is counterfactually shut down. This procedure allows us to abstract from general equilibrium of the model.

The fourth column of Table 2 presents the results from our numerical exercise of the model with *Static Friction* specification, which we use in the analytical section. The results are qualitatively very close to the benchmark results, which confirms that the analytical results discussed by us indeed capture the main forces of the full quantitative model.

Table 2: International Prices: Theory versus Data[a]

Statistic	Data[b]	Benchmark $\theta = \frac{1}{2}$	Benchmark Fitted θ	Static Friction	BKK	Convex Friction
A. Correlation						
p_x, p_m	0.75	0.99	0.98	1.00	-1.00	0.39
p_x, x	0.46	1.00	0.98	0.99	-1.00	0.51
p_x, x_{-1}	0.61	0.84	0.87	0.80	-0.80	0.04
p_m, x	0.69	1.00	0.99	1.00	1.00	0.52
p, x	0.61	0.47	0.95	0.98	1.00	0.00
B. Volatility relative to[c] x						
p_x	0.37	0.49	0.37	0.40	0.16	1.00
p_m	0.61	0.52	0.64	0.61	1.16	0.99
p (no fuels[d])	0.27	0.06	0.28	0.21	1.32	1.11
p_x/p_d	0.53	0.55	0.44	0.53	0.00	1.06
p_d	0.13	0.08	0.10	0.14	0.16	0.14
C. Standard deviation of x						
	3.60	0.44	0.42	2.02	0.45	0.29
D. Correlation of c/c^* *with* x						
	-0.71	0.95	0.92	1.00	0.98	0.28
E. Price elasticity of trade						
Long-run	7.9	7.90	7.90	n.a.	0.71	1.50
Short-run	0.71	0.71	0.71	0.71	0.71	1.50
F. Decomposition of sources of PTM						
LOPD	n.a.	20%	26%	n.a.	n.a.	n.a.

[a]Statistics based on logged and Hodrick-Prescott filtered time-series with a smoothing parameter $\lambda = 1600$.
[b]Unless otherwise noted, data column refers to US data for the time period 1980:1-2004:1.
[c]Ratio of corresponding standard deviation to the standard deviation of the real exchange rate x.
[d]Refers to the terms of trade cleaned from fuels (SITC3); for overall index it is .41.
[e]We use PPI for manufacturing goods as a proxy measure for p_d (time period 1984:1-2004:1).

Finally, Panel C shows that both the benchmark model and the BKK model fail to replicate the volatility of the real exchange rate by an order of magnitude, and both models imply a positive correlation between the real exchange rate and the consumption ratio (the

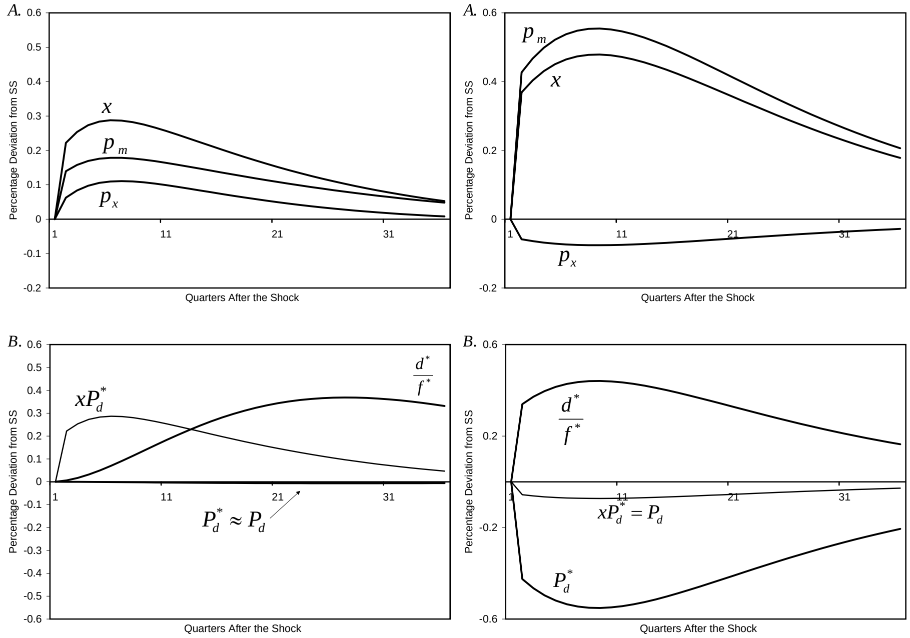
Table 3: Quantities - Comovement and Relative Volatility[a]

Statistic	Data[b]	Benchmark $\theta = \frac{1}{2}$	Benchmark Fitted θ	Static Friction	BKK	Convex Friction
A. Correlation						
domestic with foreign						
TFP (measured[c])	0.30	0.30	0.30	0.30	0.30	0.30
GDP	0.40	0.35	0.36	0.32	0.35	0.30
Consumption	0.25	0.25	0.27	0.27	0.33	0.46
Employment	0.21	0.34	0.30	n.a.	0.49	0.29
Investment	0.23	0.04	0.05	n.a.	0.19	0.28
GDP with						
Consumption	0.83	0.94	0.93	0.99	0.95	0.87
Employment	0.85	0.86	0.79	n.a.	0.98	-0.10
Investment	0.93	0.81	0.83	n.a.	0.67	0.88
Net exports	-0.49	-0.55	-0.54	-0.07	-0.57	-0.28
Terms of trade with						
Net exports	-0.17	-0.60	-0.88	-0.19	-0.84	-0.92
B. Volatility relative to GDP[d]						
Consumption	0.74	0.31	0.32	1.02	0.33	0.32
Investment	2.79	3.62	3.59	n.a.	3.25	3.06
Employment	0.81	0.61	0.71	n.a.	0.47	1.88
Net exports	0.29	0.18	0.19	0.15	0.14	0.06

[a]Statistics based on logged and Hodrick-Prescott filtered time series with a smoothing parameter $\lambda = 1600$.
[b]Unless otherwise noted, data column refers to US data for the period 1980:1-2004:1.
[c]Calculated using the actual national accounting formulas; see technical appendix for further details.
[d]Ratio of corresponding standard deviation to the standard deviation of GDP.

Backus-Smith puzzle)[47]. As Proposition 4 and Corollary 1 in the analytical section show, these problems do not affect the pricing predictions of our model. For example, as we report in the working paper version of the paper (Drozd & Nosal (2008)), even under financial autarky, when the volatility of the real exchange rate is much closer to the data, all of our results still stand.

[47]The observed persistence difference between the models is solely attributed to the fact that shock dies earlier in the benchmark model due to different specification of the AR process. See values of coefficients in Table 1.



Benchmark: Fitted θ *Standard Model*

Figure 2: Benchmark model and BKK model: Impulse response to a positive productivity shock in the domestic country.

Quantities Table 3 reports the quantity statistics. The benchmark model implies a bit too low international comovement of investment[48] (0.04−0.05 models vs. 0.23 data), but matches the rest of the statistics well. Note that the benchmark model is additionally consistent with the fact that output is more internationally correlated than consumption (0.4 and 0.25 in the data vs 0.35 and 0.25 in the model), addressing the so-called quantity puzzle (in BKK these are roughly equal). Because most of the quantitative discrepancies can be fixed by incorporating additional features (e.g., convex adjustment cost or home production), both the BKK model and the benchmark model are relatively successful in the quantity dimension.[49]

[48]For the most recent subperiod (1986–2000), Heathcote & Perri (2004) report an international correlation of investment equal to zero.

[49]An additional prediction of our richer framework pertains to the behavior of marketing expenditures over the business cycle. The evidence on the behavior of marketing expenditures over the business cycle is scant. However, annual aggregate figures for advertising expenditures on the national level are readily available from the *Statistical Abstract of the United States* published by the U.S. Census Bureau. These figures reveal that advertising expenditures and the share of advertising expenditures in GDP are both highly procyclical. This observation is qualitatively consistent with the predictions of our model.

Impulse functions in the benchmark model The different behavior of prices in our model versus the standard frictionless model is exposed in the impulse responses of key variables to a positive productivity shock in the domestic country, presented in Figure 2. The main conclusion from the figure is that import ratios in our framework move in similar magnitudes as in the frictionless model with low elasticity of substitution, while the implied behavior of retail prices (P_d and P_d^*) is dramatically different (Panel B). Similar movements in quantities translate into similar movements in the real exchange rate in both models, but in the benchmark model it results in $xP_d^* > P_d$, while in the standard model an offsetting drop of P_d^* preserves LOP $xP_d^* = P_d$ after the shock. This exercise confirms that the characterization of the mechanism in the analytic section applies also to the benchmark model.

VI. Exchange Rate Pass-Through Coefficient

Our model implies exchange rate movements which are incompletely passed through to real export prices (and real import prices), and it thus offers a new theory of incomplete (real) pass-through. To relate our results to the pass-through measurement in the data, we derive the theoretical pass-through coefficient predicted by the model by using the bargaining equation (12), from which we can compute the elasticity of the real export price (evaluated at the deterministic steady state values[50]) with respect to the real exchange rate:

$$PT \equiv \frac{d\log p_x}{d\log x} \equiv \theta\mu^s + \theta\mu^s \frac{d\log P_d^*}{d\log x}, \tag{21}$$

where μ^s here means the deterministic steady state retailer's markup given by $\mu^s = P_d^s/p_d^s$.

The theoretical pass-through coefficient to export prices (PT hereafter) derived above has two important properties which are specific to our model.

First, it exhibits a dynamic pattern: it is high in the short-run and low in the long-run[51], which is qualitatively consistent with the observations in the data of higher short-run and lower long-run pass through of exchange rates to export prices. In particular, Goldberg

[50]Since the empirical pass-through regressions in principle control for all cost components, we abstract from marginal cost movements.

[51]To see that this is the case, observe (see Figure 2) that the dynamic pattern of $\frac{d\log P_d^*}{d\log x}$, due to sluggish market shares, will be a small and negative initial value that grows more negative over time, lowering the long-run predicted pass-through relative to short-run pass-through.

& Campa (2005) find values of the pass-through coefficient to *import prices* that can be translated to the values of the short-run and long-run analog of our PT coefficient to export prices of 0.4 and 0.2, respectively. This pattern is in line with the predictions of our model.

Second, the short-run PT coefficient in our model is to a first order approximation bounded from below by the bargaining power θ. This follows from the fact that the dynamic component $\frac{d \log P_d^*}{d \log x}$ is very small in the short run, and hence the short-run pass-through predicted by the model is equal to $\theta\mu^s$. Interestingly, even if we approach zero markups in the model ($\mu^s \to 1$), the PT coefficient will never fall below θ (assuming trade takes place).

As a consequence of the short-run pass-through depending on the bargaining power, our model can in principle match a broad range of short-run pass-through coefficients[52]. This sets apart our theory from others in the literature, because, as we point out in Drozd & Nosal (2010), matching a low level of pass-through to import prices and high to export prices is a challenge for many models of pricing to market . Specifically, in Drozd & Nosal (2010), we show that for several leading pricing to market mechanisms in the literature, matching empirically relevant levels of the short-run PT coefficient (50%-70% to import prices, 30%-50% to export prices) often requires producer markups of 50% or above[53]. Such level of markups requires high pure profitability of firms in equilibrium, or high profitability that covers a large level of fixed cost associated with production. Both features, if incorporated to a model, may crucially restrict the the choice of parameters / affect dynamics of quantities in possible DSGE applications. In this context, our results demonstrate that the benchmark model can be consistent with the empirical range of pass-through coefficient even for producer markups in the modest range of 10%-20% (surplus from trade has to be positive to guarantee trade takes place in matches, and so markups can not be too low.)

VII. Comparison to Adjustment Cost Model

In this section, we answer the question whether a simple adjustment cost suggested by Krugman (1986), and further explored in Kasa (1992) and Lapham (1995), could generate

[52]Quantitatively, even though it will decay over time, this will take quite a while (see impulse response figures).

[53]See Drozd & Nosal (2010) for a detailed discussion of the importance of steady state markup levels for pricing to market in several leading pricing to market models.

quantitatively similar behavior of prices as our micro-founded frictions. We report our quantitative results for this exercise, labeled *Convex Friction*, in Tables 2 and 3. Parameters are listed in Table 1. While these frictions are related to ours, there are important differences in performance, which we discuss below.

To keep the exercise in the same spirit as the benchmark model, we introduce a quadratic adjustment cost directly on the quantity sold by producers into the standard BKK model. Formally, the domestic producers maximize the standard profit function subject to

$$d(s^t) + d^*(s^t) = f\left(k(s^t), l(s^t)\right) - \phi\left(\frac{d(s^t)}{d(s^{t-1})} - 1\right)^2 - \phi\left(\frac{d^*(s^t)}{d^*(s^{t-1})} - 1\right)^2, \tag{22}$$

where ϕ is the adjustment costs for changing sales.

The first crucial difference between our approach and this approach is that here, unlike in the benchmark model, producer prices are equal to the retail prices and therefore are tied to the import ratio $\frac{f}{d}$ and $\frac{f^*}{d^*}$ through the consumer first order conditions (4). These conditions imply that this model is not capable of disconnecting the short-run and the long-run measured elasticities, and hence our calibration strategy used for the benchmark model is not applicable. Wholesale and retail prices have also identical properties.

Lacking independent discipline for the key parameters, we select the standard business cycle value for the elasticity of substitution γ equal to 1.5, and search over all values of the adjustment cost to best fit the price data. An extensive search over the entire parameter space reveals that when choosing the parameter ϕ, we are constrained by a trade-off between matching (i) the correlation between terms of trade and the real exchange rate, and (ii) the persistence of the pass-through of exchange rate movements into export prices. Increasing the adjustment cost lowers (i) and increases (ii), and makes it difficult for the model to generate both to be high, as in the data. Apart from this trade-off, high cost ϕ helps the other price statistics, and hence, we select the adjustment cost to be the highest value which matches both of the signs of (i) and (ii). This tension is independent of the value of γ.

Results for prices for our selection of the adjustment cost are presented in Table 2. As we can see, the model does not perform as well as the benchmark model. It implies that the correlation between the still very persistent real exchange rate lagged by one quarter and the real export price is almost zero, as is the correlation of the terms of trade and the real exchange rate. We thus conclude that bargaining and a different specification of the process of adjusting sales are crucial elements, which allow our theory to match the price data.

VIII. Conclusions

In this paper, we have demonstrated that dynamic frictions of building market shares have the potential to account for pricing-to-market observations, and the observed discrepancy between the short-run and the long-run price elasticity of trade flows. Given the abundance of evidence about the importance of switching costs and the long-lasting nature of producer-supplier relations in international trade, as well as evidence on the slow growth of firms following entry to a new market, we believe that the mechanism proposed by us is an important step towards an endogenous theory of the failure of arbitrage disciplined by evidence on the dynamics of quantities.

Appendix

Extended proofs, additional derivations and notes can be found in the online Technical Appendix. Wherever possible, state dependent notation is suppressed for clarity of exposition.

Proof of Proposition 3.

By bargaining, under continual renegotiation, the surplus from a match that goes to the producer is given by θS_d, and the surplus that goes to retailer is $(1-\theta)S_d$, where $S_i \equiv W_i + J_i$, $i = d, f$. Furthermore, given (15) and $\phi = 0$, we know that both the domestic producer and the foreign importer face the same marginal cost of matching in the domestic country, which is given by $\frac{\bar{m}}{h}(Av - (1-\delta_m)E\left[Q_{+1}A_{+1}v_{+1}\right])$. Given equality of this cost, we conclude that the surplus from a match must be equal: $S_d = S_f$ and $S_d^* = S_f^*$ (all expressed to domestic numéraire[54]). By (15) and domestic and foreign retailer zero profit condition, we have

$$\text{(A1)} \qquad S_d(s^t) = \frac{\chi(s^t)v(s^t)}{1-\theta}\ , \ S_d^*(s^t) = \frac{\chi^*(s^t)x(s^t)v^*(s^t)}{1-\theta},\ \text{all } s^t,$$

which implies that surpluses across the border are equal (by (15)). Now, using the Bellman equations defining total surpluses from a match,

$$\text{(A2)} \qquad x(s^t)P_d^*(s^t) - P_d(s^t) = [S_d^*(s^t) - S_d(s^t)] - (1-\delta_H)E[Q(s^{t+1}, s^t)(S_d^*(s^{t+1}) - S_d(s^{t+1}))],$$

we obtain $x(s^t)P_d^*(s^t) = P_d(s^t)$, all s^t.

Proof of Lemma 1 and Corollary 1.

(**Lemma**) Log-linearization of the retailer zero profit conditions, equations (8), (16), and the producer first order condition (given by $\theta S_i = (\frac{\bar{m}}{h})Av/(1-\phi(a_i/a_i^s - 1))$, implies $\hat{S}_d = \phi\widehat{\pi}$, $\hat{S}_d^* = \phi\left(\widehat{1-\pi^*}\right)$. To derive (17), note that $S_d(s^t)$ can be equivalently expressed as $\mathcal{P}_d(s^t) - \mathcal{V}_d(s^t)$, where $\mathcal{V}_d(s^t) \equiv v(s^t) + (1-\delta_H)E\left[Q(s_{t+1}|s^t)\mathcal{V}_d(s^{t+1})\right]$. Log-linearizing the differential, and noting that $\mathcal{V}_d(s^t)$ drops out, we thus conclude $\hat{S}_d^* - \hat{S}_d = \left(\hat{\mathcal{P}}_d^* - \hat{\mathcal{P}}_d\right)\frac{\mathcal{P}_d^s}{S^s}$, and calculate from deterministic steady state relations $\mathcal{P}_d^s/S^s = \frac{\chi+(1-\theta)}{\chi(1-(1-\delta_H)\beta)}$.

(**Corollary**) ($\Rightarrow$) Suppose, by contradiction, that, to a first order approximation, LOP holds in all markets, but market shares do adjust after the shock. Since LOP (in all states and dates) implies $\hat{\mathcal{P}}_d^* - \hat{\mathcal{P}}_d = 0$, by (17), the only possibility is that $\widehat{1-\pi^*} = \widehat{\pi} = \Delta \neq 0$. Since (17) applies also to the foreign country by symmetry, and thus $\widehat{1-\pi} = \widehat{\pi^*}$, by home-bias ($\pi^s > 1/2$), we obtain $|\widehat{1-\pi}| > \Delta$, $|\widehat{\pi^*}| > \Delta$, and $|\widehat{1-\pi^*}| > \Delta$, which is a contradiction. ($\Leftarrow$) By (17), we must show that $\hat{\mathcal{P}}_d^*(s^t) = \hat{\mathcal{P}}_d(s^t)$ for all s^timplies LOP (to a first order approximation). The conclusion follows from the following evaluation derived from (18) and (19)

$$\text{(A3)} \qquad \underbrace{\mathcal{P}_d^*(s^t) - \mathcal{P}_d(s^t)}_{\text{all 1st order terms=0}} \equiv x(s^t)P_d^*(s^t) - P_d(s^t) + (1-\delta_H)E\{\underbrace{Q(s_{t+1}|s^t)[\mathcal{P}_d^*(s^{t+1}) - \mathcal{P}_d(s^{t+1})]}_{\text{all 1st order terms = 0}}\}.$$

Proof of Proposition 4.

Suppose LOP holds, implying $x\hat{P}_d^* = \hat{P}_d = 0$. Since Lemma 1 applies, by (17), we conclude $\widehat{1-\pi^*(s^t)} = \widehat{\pi}(s^t) = 0$. Furthermore, by log-linearization of (4), given $\hat{\pi} = \hat{\pi}^* = 0$ we know

[54] To save on notation, we abuse our usual convention and define valuations of foreign producers and retailers in domestic country numéraire, i.e. $S_d^* \equiv W_d + xJ_d^*$, $S_f^* \equiv xW_f + J_f$ and $S_f^* \equiv xW_f^* + xJ_f^*$. Note that these definitions also explain our use of the domestic discount Q in all of the equations.

$\hat{P}_d^* = \hat{P}_d = 0$. Since $\hat{x} \neq 0$ and $\widehat{1 - \pi^*(s^t)} = \widehat{\pi}(s^t) = 0$, we obtain $x\hat{P}_d^* \neq 0$ and $\hat{P}_d = 0$, which is a contradiction.

Proof of Proposition 5.

Assume LOP holds.

(**Complete markets**) By Lemma 1, we know $\hat{\pi} = \hat{\pi}^* = 0$ and $\hat{S}_d - \hat{S}_f = 0$ (see the proof of Lemma 1). Furthermore, note that it implies $\widehat{xv^*} = \hat{v} = 0$ by an evaluation analogous to (A3), but applied to the differential $S_d - S_f$. By Lemma 2 stated below, we know that $\hat{z} \neq \hat{z}^*$ implies $\hat{h} \neq \hat{h}^*$. This is a contradiction for the following reason. Under efficient risk sharing, the log-linearization of (2), and steady state relations $H_d^s = \pi^s h^s/\delta_H$, $H_f^s = (1 - \pi^s)h^s/\delta_H$, imply $\hat{c} = \hat{h}$, $\hat{c} \neq \hat{c}^*$ and hence $\hat{x} \neq 0$ by (5). Furthermore, by the log-linearization of (4) and the fact that $\hat{\pi} = \hat{\pi}^* = 0$, we have that $\hat{P}_d^* = \hat{P}_d = 0$. Combining the two, we obtain $\widehat{xP_d^*} \neq \widehat{P_d}$, which is a contradiction.

(**Financial autarky**) In this case, $\hat{h} \neq \hat{h}^*$ violates the requirement that current account is zero at all states and dates if LOP is to be preserved, namely $xp_d^*(1-\pi^*)h^* - p_f(1-\pi)h + Av(a_d^* - a_f) = 0$. Specifically, for the market shares to be constant, it must be true that $\hat{m}_i = \hat{m}_i^*$. Since $\hat{m}_i = \hat{a}_i$, as implied by the log-linearization of (16), from the producer's first order conditions, we obtain $\hat{a}_d = \frac{\hat{h} - \widehat{Av}}{(1+\phi)}$, and $\hat{a}_f^* = \frac{\hat{h}^* - \widehat{Av}}{(1+\phi)}$ (see proof of Lemma 2). Plugging in these equations to the log-linearized current account condition, we establish that for the current account to be zero, the response of prices p_d^*, p_f or the real exchange rate x is necessary. This, however, is a contradiction: as noted above, $\widehat{xv^*} = \hat{v} = 0$, $\hat{P}_d^* \equiv \hat{P}_d \equiv 0$, and by the hypothesis: $xP_d^* \equiv P_d$.

LEMMA 2. *Suppose in equilibrium $\hat{\pi} = \hat{\pi}^* = 0$. Then, $z > (<) z^*$ implies $h > (<) h^*$.*

Proof of Lemma 2.

To derive the lemma, we subtract log-linearized foreign feasibility (13) (first multiplied by $\frac{xv^*}{v}$ to use (15)) from each side of the of the log-linearized domestic feasibility (13)[55], and use the fact that $\hat{a}_i = \frac{\hat{h} - \widehat{Av}}{1+\phi}, \hat{a}_i^* = \frac{\hat{h} - \widehat{Av}}{1+\phi}, i = d, f$, as derived from: (i) first part of the proof of Lemma 1 implying $\hat{S}_i = \hat{S}_i^* = 0$, (ii) log-linearized producer FOCs given by $\theta S_i = (\frac{m_d + m_f}{h}) Av/(1 - \phi(a_i/a_i^s - 1))$, and (iii) log-linearized equation (16), which together with $\hat{\pi} = \hat{\pi}^* = 0$, implies $\hat{m}_d = \hat{a}_d = \hat{m}_f = \hat{a}_f$.

[55] We assume feasibility conditions holds with equality, which is wlog in the given context. See online appendix for details.

References

Alessandria, G. 2004. "International Deviations from the Law of One Price: The Role of Search Frictions and Market Share." *International Economic Review* 45(4):1263–1291.

Alessandria, George. 2009. "Consumer Search, Price Dispersion and International Relative Price Volatility." *International Economic Review* .

Armington, P. S. 1969. "A Theory of Demand for Products Distinguished by Place of Production." *International Monetary Fund Staff Papers* 16(1):159–178.

Atkeson, Andrew & Ariel T. Burstein. 2008. "Pricing-to-Market, Trade Costs, and International Relative Prices." *American Economic Review* .

Backus, D. K., P. J. Kehoe & F. E. Kydland. 1995. International Business Cycles: Theory and Evidence. In *Frontiers of Business Cycle Research*, ed. T. Cooley. Chapter 11, pp. 331-356, Princeton, NJ: Princeton University Press.

Basu, S. & J. G. Fernald. 1997. "Returns to Scale in U.S. Production: Estimates and Implications." *Journal of Political Economy* 105(2):249–283.

Blonigen, B. A. & W. W. Wilson. 1999. "Explaining Armington: What Determines Substitutability Between Home and Foreign Goods?" *Canadian Journal of Economics* 32(1):1–22.

Burstein, Ariel & Nir Jaimovich. 2009. "Understanding Movements in Aggregate and Product-Level Real Exchange Rates." *manuscript* .

Chari, V. V., P. J. Kehoe & E. McGrattan. 2002. "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?" *Review of Economic Studies* 69(3):533–563.

Corsetti, G., L. Dedola & S. Leduc. 2008. "International Risk-Sharing and the Transmission of Productivity Shocks." *Review of Economic Studies* 75:443–473.

Drozd, L. A. & J. B. Nosal. 2008. "Understanding International Prices: Customers as Capital." *Federal Reserve Bank of Minneapolis Staff Report 411* .

Drozd, Lukasz A. & Jaromir B. Nosal. 2010. "Pricing to Market in Business Cycle Models." *unpublished manuscript* .

Eaton, J. & S. Kortum. 2002. "Technology, Geography, and Trade." *Econometrica* 70(5):1741–1779.

Eaton, Jonathan, Marcela Eslava, Maurice Kugler & James Tybout. 2007. "Export Dynamics in Colombia: Firm-Level Evidence." *NBER Working Paper* .

Egan, M. L. & A. Mody. 1992. "Buyer Seller Links in Export Development." *World Development* 20(3):321–334.

Foster, Lucia, John Haltiwanger & Chad Syverson. 2008. "The Slow Growth of New Plants: Learning about Demand?" *Working Paper* .

Froot, K. A. & P. D. Klemperer. 1989. "Exchange Rate Pass-Through When Market Share Matters." *American Economic Review* 79(4):637–654.

Ghironi, F. & M. J. Melitz. 2005. “International Trade and Macroeconomic Dynamics with Heterogeneous Firms.” *Quarterly Journal of Economics* 120(3):865–915.

Goldberg, L. & J. Campa. 2005. “Exchange Rate Pass-Through into Import Prices.” *Review of Economics and Statistics* (8934).

Goldberg, P. K. & M. M. Knetter. 1997. “Goods Prices and Exchange Rates: What Have We Learned?” *Journal of Economic Literature* 35(3):1243–1272.

H. Hakansson, ed. 1982. *International Marketing and Purchasing of Industrial Goods*. IMP Project Group. Chichester: Wiley and Sons.

Head, K. & J. Ries. 2001. “Increasing Returns Versus National Product Differentiation as an Explanation for the Pattern of U.S.-Canada Trade.” *American Economic Review* 91(4):858–876.

Heathcote, J. & F. Perri. 2004. “Financial Globalization and Real Regionalization.” *Journal of Economic Theory* 119(1):207–243.

Hummels, D. 2001. “Toward a Geography of Trade Costs.” unpublished manuscript, Purdue University.

International Financial Statistics. 2005. International Monetary Fund.

Johri, Alok & Amartya Lahiri. 2008. “Persistent Real Exchange Rates.” *Journal of International Economics* 76:223–236.

Kasa, Kenneth. 1992. “Adjustment Costs and Pricing-to-Market: Theory and Evidence.” *Journal of International Economics* 32:1–30.

Knetter, M. M. 1993. “International Comparisons of Price-to-Market Behavior.” *American Economic Review* 83(3):473–486.

Krugman, P. 1986. “Pricing-to-Market When the Exchange Rate Changes.” NBER Working Paper 1926, National Bureau of Economic Research.

Kydland, F. E. & E. C. Prescott. 1982. “Time to Build and Aggregate Fluctuations.” *Econometrica* 50(6):1345–1370.

Lapham, Beverly. 1995. “A Dynamic General Equilibrium Analysis of Deviations from the Laws of One Price.” *Journal of Economic Dynamics and Control* 19:1355–1389.

Lilien, G. L. & D. Weinstein. 1984. “An International Comparison of the Determinants of Industrial Marketing Expenditures.” *Journal of Marketing* 48(1):46–53.

Lilien, G. L. & J. D. C. Little. 1976. “The ADVISOR Project: A Study of Industrial Marketing Budgets.” *Sloan Management Review* 17(3):17–31.

Marston, R. C. 1990. “Pricing to Market in Japanese Manufacturing.” *Journal of International Economics* 29(3/4):217–236.

OECD Main Economic Indicators. 2007. SourceOECD.org.

Pissarides, C. A. 1984. “Efficient Job Rejection.” *Economic Journal* (supplement) 94(376a):97–108.

Ravn, Morten, Stephanie Schmitt-Grohe & Martin Uribe. 2007. "Pricing to Habits and the Law of One Price." *American Economic Review Papers and Proceedings* 97:232–238.

Reinert, K. A. & D. W. Roland-Holst. 1992. "Armington Elasticities for United States Manufacturing Sectors." *Journal of Policy Modeling* 14(5):631–639.

Rogoff, M. 1996. "The Purchasing Power Parity." *Journal of Economic Literature* 34(2):647–668.

Ruhl, K. J. 2008. "The International Elasticity Puzzle." *University of Texas-Austin* unpublished manuscript.

Ruhl, K. J. & J. L. Willis. 2008. "Convexities, Nonconvexities, and Firm Export Behavior." Manuscript, Midwest Macro Conference, Philadelphia.

Stockman, A. C. & L. Tesar. 1995. "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements." *American Economic Review* 85(1):169–185.

Turnbull, P. W. & eds. M. T. Cunningham. 1981. *International Marketing and Purchasing*. Houndmills, Basingstoke, Hampshire, U.K: Palgrave Macmillan.